



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 1 • 2014



DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $[2 - 3(5 - 7)] + 11 \times (13 + 17 - 19)$

- (A) 123 (B) 129 (C) 132 (D) 149 (E) 167

2. Let $S = \{s,e,c,o,n,d\}$, $M = \{m,i,n,u,t,e\}$, $H = \{h,o,u,r\}$, and $T = \{t,i,m,e\}$. The number of distinct elements in $(S \cap H) \cup (M \cap T)$ is _____.

- (A) 2 (B) 6 (C) 10 (D) 4 (E) 5

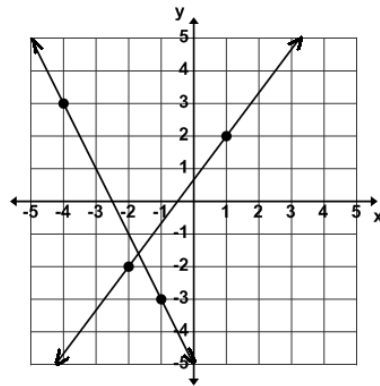
3. Penni Lesse got a \$2000.00 student loan to help pay her dorm rent. She has to pay it back in 24 equal monthly payments. What will her monthly payments be if the annual interest rate of 3.5% is compounded monthly? (nearest cent)

- (A) \$84.80 (B) \$85.63 (C) \$86.30 (D) \$89.37 (E) \$90.17

4. If P is 130% of Q and R is 70% of P, then R is what percent of Q?

- (A) 100% (B) 60% (C) 91% (D) 86% (E) 90%

5. The two lines shown intersect at the point (x, y) . Find $x + y$.



- (A) $-3.777\dots$ (B) -3.5 (C) $-3.222\dots$ (D) -3.3 (E) -3.275

6. Let $\frac{x+2}{3x-4} - \frac{4x-1}{2x+3} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$. Find $\frac{A+B+C}{P+Q+R}$.

- (A) 2.4 (B) 0.4 (C) -1 (D) -2.4 (E) -3.6

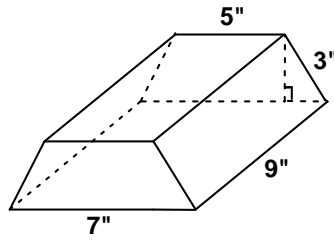
7. Which of the sets of numbers given is not closed under the operation of addition?

- (A) Even numbers (B) Integers (C) Irrationals (D) Primes (E) Rationals

8. Saul T. Baye is fishing in his kayak at Baffin Bay. He starts paddling towards the shore at a rate of 4 miles per hour. How far from shore was he if he makes it to shore in 12 minutes?

- (A) 1,184 yds (B) 1,408 yds (C) 1,420.8 yds (D) 1,440.2 yds (E) 1,712 yds

9. Find the base area of the isosceles trapezoid prism. (nearest sq. in). Drawing is not to scale.



- (A) 17 sq. in (B) 18 sq. in (C) 24 sq. in (D) 34 sq. in (E) 63 sq. in

10. Find the shortest distance from the point (1, 1) to the line $5x + 12y = 60$.

- (A) $1\frac{7}{13}$ (B) $2\frac{2}{5}$ (C) $3\frac{4}{13}$ (D) 5 (E) $5\frac{12}{13}$

11. A pentagonal rotunda has 17 faces and 35 edges. How many vertices does it have?

- (A) 15 (B) 20 (C) 33 (D) 37 (E) 50

12. Phil Whitwhatter has an empty rectangular water tank that is 10' long, 8' wide, and 6' deep. If Phil puts 2500 gallons of water in the tank, what percent of the tank's capacity contains water? (nearest percent)

- (A) 48% (B) 52% (C) 60% (D) 63% (E) 70%

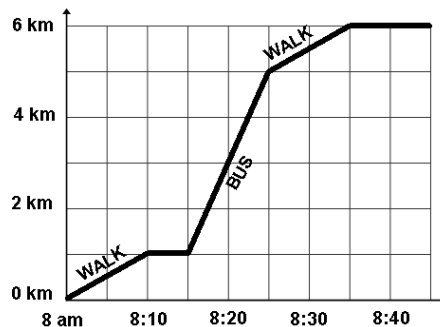
13. The heronian mean of 8 and 18 is: (nearest tenth)

- (A) 12.7 (B) 13 (C) 11.1 (D) 13.6 (E) 12

14. One of the three most famous and well known Zeno paradoxes involves trying to reach a specific point by covering only half of the remaining distance each time. Hence not ever reaching that specific point. This paradox is known as the:

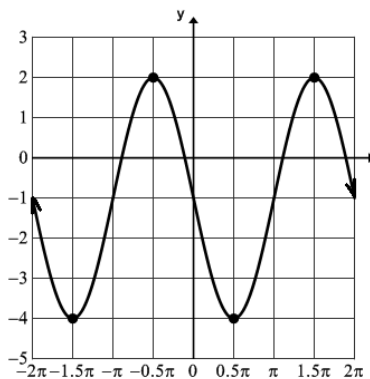
- (A) Achilles and Tortoise (B) Arrow flight (C) Dichotomy (D) Stadium (E) Grain of Millet

15. Willie Makette has to walk and ride a bus to get to school each day. Using the graph below determine the average speed of the bus.



- (A) 14 kmph (B) 24 kmph (C) 32 kmph (D) 40 kmph (E) 48 kmph

16. The probability of scoring at least 120 on the district 1 test in class 1A is 64%. Based on this probability, if 500 take this test what are the odds of scoring less than 120?
- (A) $\frac{9}{25}$ (B) $\frac{3}{10}$ (C) $\frac{8}{15}$ (D) $\frac{9}{16}$ (E) $\frac{6}{25}$
17. The Millersview *Old-Goats* football team consists of three 25¢ backs, five lopers, seven kneelers, four pass-outers, and a kicker of the bucket. In how many ways can a group of eleven Old Goats be formed such that there is one 25¢ back, three lopers, five kneelers, and two pass-outers?
- (A) 40 (B) 2,943 (C) 3,780 (D) 12,600 (E) 59
18. The first five numbers in row 11 of Pascal's triangle are 1, 11, k, 165, 330, Find k?
- (A) 121 (B) 110 (C) 88 (D) 55 (E) 44
19. If $3^{(3x - y)} = 27$ and $5^{(x + 5y)} = 3,125$ then $4^{(2x + 4y)} =$ _____.
- (A) 4,096 (B) 256 (C) 512 (D) 2,048 (E) 65,536
20. Tye Purr and Ed Ittur need to read and edit a 500 page book before it is published. Tye can do it alone in 6 hours 50 minutes. Ed can do it alone in 5 hours 20 minutes. How long would it take them to complete the edits working together? (nearest minute)
- (A) 2 hrs 53 min (B) 3 hrs (C) 5 hrs 45 min (D) 6 hrs 2 min (E) 6 hrs 5 min
21. How many integral values of n exist such that $n > 1$ and $\frac{(n + 1)!}{(n - 1)!} \leq 110$
- (A) 22 (B) 20 (C) 11 (D) 10 (E) 9
22. Let $y = D + A\sin(Bx - C\pi)$ will produce this graph. If $C > 0$ then $A + B + C + D = ?$



- (A) 6 (B) 4 (C) 2 (D) -2 (E) -3
23. $\sin(2\theta) \times \tan(\theta) + \cos(2\theta)$ is equivalent to:
- (A) $\sec(\frac{\pi}{4})$ (B) $\tan(\frac{\pi}{4})$ (C) $\cos(\frac{\pi}{2})$ (D) $\sin(\frac{\pi}{4})$ (E) $\cot(\frac{\pi}{2})$

24. Tim Burr needs to know how tall a tree is before cutting it down to be sure it doesn't hit a shed. The angle of elevation to the top of the tree is 29° from where is standing. From a point 50 feet closer, the angle of elevation is 34° . How tall is the tree? (nearest inch)
- (A) 189' 5" (B) 121' 1" (C) 155' 6" (D) 180' 2" (E) 127' 10"
25. The sum of the coefficients of the 2nd term in the expansion of $(x + 1)^2$, the 3rd term of $(x + 1)^3$, the 5th term of $(x + 1)^5$, and 7th term of $(x + 1)^7$ is:
- (A) 10 (B) 14 (C) 17 (D) 18 (E) 22
26. Find $C + D$ if the the triangular pattern shown below continues.
- | | | | | | | |
|---|----|----|-----|---|---|--|
| | | | | | | |
| | | | | 1 | | |
| | | | 2 | 1 | | |
| | | 4 | 4 | 1 | | |
| | | 8 | 12 | 6 | 1 | |
| | 16 | 32 | 24 | 8 | 1 | |
| A | B | C | D | E | F | |
| | | | ... | | | |
- (A) 160 (B) 112 (C) 50 (D) 72 (E) 120
27. Let $f(x) = 4x^2 - 8x + 1$ and $g(x) = 2x + 1$. Find $g(f'(x - 1))$
- (A) $16x - 31$ (B) $4x^2 - 16x + 13$ (C) $8x - 7$ (D) $8(x^2 - 2x + 1)$ (E) $16(x - 1)$
28. Evaluate: $\int_{-n}^n (3 - 2x^3) dx$
- (A) n^4 (B) $6n$ (C) $6n + n^4$ (D) $6n - n^4$ (E) $n^4 - 6n$
29. I. C. Cleerly has 25 students in his Algebra 1 class. Each student is assigned a letter of the alphabet, except for the letter "I" which is reserved for Mr. Cleerly. Two of the student's letters are drawn at random without replacement. If the probability that both of the letters drawn are students that wear glasses is 22%, how many of the 25 students do not wear glasses?
- (A) 10 (B) 11 (C) 13 (D) 14 (E) 15
30. The number 90 is considered to be a *polite number*. What is it's *politeness*?
- (A) 12 (B) 9 (C) 6 (D) 5 (E) 3
31. $111_2 + 222_4 + 333_8 = \underline{\hspace{2cm}}_{10}$
- (A) 1,934 (B) 268 (C) 222 (D) 206 (E) 108
32. Which of the following lines is concurrent to the lines $2x + 3y = -1$ and $x - 2y = 3$?
- (A) $4x + 5y = 1$ (B) $3x + 2y = -1$ (C) $3x - 4y = -7$ (D) $2x + y = -3$ (E) $5x - 6y = 11$

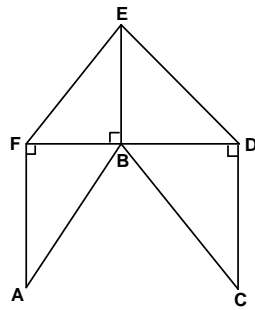
33. I. M. Cheep wants to buy four shirts for his Spring Break trip. There is a "No-Tax Discount" sale going on. Which of the following is the cheapest deal?

- (A) \$13.90 each (B) \$14.50 each and he has a 10% off coupon
 (C) buy 3 at \$17.85 each and get 1 free (D) buy 2 at \$19.95 each and get 2 half off
 (E) buy 3 at \$15.60 and get 25% off of the 4th one

34. The Real value solution set for $|4 - 5x| + 6 > 10$ is?

- (A) $\{x \mid \{x < 0\} \cup \{x > 1.6\}\}$ (B) $\{x \mid \{x < 0\} \cup \{x > 4\}\}$ (C) $\{x \mid \{x > 0\} \cup \{x < -1.6\}\}$
 (D) $\{x \mid 0 < x < 4\}$ (E) $\{x \mid 0 < x < 1.6\}$

35. Given: $m\angle BFE = 45^\circ$, $m\angle BED = 60^\circ$, $m\angle CBD = 30^\circ$, $m\angle ABF = 60^\circ$, and $BE = 4$ cm. Find the perimeter of the hexagon shown. (nearest tenth).



- (A) 55.5 cm (B) 45.9 cm (C) 42.2 cm (D) 40.6 cm (E) 38.4 cm

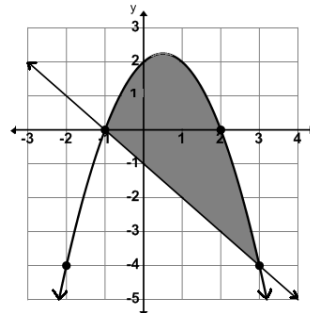
36. The roots of the equation $x^3 + kx^2 - 23x = 60$ are R, 5, and -3 . Find k.

- (A) 2 (B) 1 (C) -12 (D) -6 (E) -4

37. If $a_1 = -2$, $a_2 = 1$, $a_3 = -3$, and $a_n = a_{n-3}(a_{n-2} + a_{n-1})$, where $n \geq 4$, then a_7 equals:

- (A) -15 (B) -4 (C) 64 (D) 72 (E) -56

38. Find the area of the shaded region in square units.

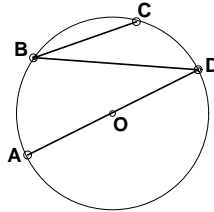


- (A) 10 (B) $10\frac{1}{3}$ (C) $10\frac{2}{3}$ (D) 11 (E) $11\frac{1}{3}$

39. The probability that statement P is true is 48%, and the probability that statement Q is true is 75%. Determine the probability that $P \rightarrow Q$ is false.

- (A) 88% (B) 64% (C) 36% (D) 27% (E) 12%


40. Given the circle with center O. Find $m\widehat{CD} + m\angle ADB + m\angle CBD$, if $m\widehat{AB} = 70^\circ$ and $m\angle BOC = 60^\circ$. (Drawing not to scale.)



- (A) 180° (B) 135° (C) 110° (D) 105° (E) 90°

41. How many 3-digit numbers can be made using the digits 0, 2, 2, 4, and 6?

- (A) 39 (B) 36 (C) 30 (D) 26 (E) 24

42. The Mayan number system consists of three symbols, \bullet — . The total number of these symbols needed to write 2014 in base 10 as a Mayan number is:

- (A) 12 (B) 10 (C) 8 (D) 7 (E) 5

43. Let K be a two-digit number such that the tens digit is divisible by 3 and units digit is a positive multiple of 4. What is the sum of all of these two-digit numbers?

- (A) 396 (B) 72 (C) 384 (D) 184 (E) 408

44. A box contains five rods whose lengths are 4 in, 7 in, 10 in, 1 ft 1 in and 1 ft 6 in. How many different acute triangles can be made using only three rods at a time.

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

45. Use the Fibonacci characteristic sequence $\dots - 8, p, q, r, - 5, \dots$ to Find $p + q + r$.

- (A) $-1\frac{1}{3}$ (B) -1 (C) $-\frac{2}{3}$ (D) $1\frac{2}{3}$ (E) $2\frac{1}{3}$

46. If $(1 - 2i) - (2 + 3i) \times (3 - 4i) = a + bi$ then $a + b$ is:

- (A) 14 (B) 4 (C) -34 (D) -22 (E) -20

47. The eccentricity of the ellipse $16x^2 + 25y^2 = 400$ is: (nearest hundredth)

- (A) $\frac{25}{3}$ (B) $\frac{9}{16}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5}$ (E) $\frac{3}{4}$

48. Find the slope of the line tangent to the curve $y = 7 - 6x - x^2$ at the point $(-1, 12)$.

- (A) $-4\frac{1}{3}$ (B) -4 (C) $-3\frac{1}{4}$ (D) $-\frac{1}{4}$ (E) $-\frac{1}{12}$

49. Willis A. Nutt bought 5 pounds of walnuts at \$4.50 per pound and 7 pounds of filberts at \$3.75 per pound. He mixed them together. The cost per pound of the mixture would be? (nearest cent)

- (A) \$3.81 (B) \$3.94 (C) \$4.06 (D) \$4.13 (E) \$4.16

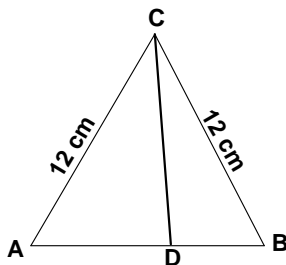
50. Points P, Q, and R lie on a circle with center C and chord PQ is the diameter. Point X is on chord PR. Find the perimeter of $\triangle CPX$ if $\overline{CX} \parallel \overline{QR}$, $PQ = 13$ cm and $QR = 5$ cm.

- (A) 32.5 cm (B) 30 cm (C) 22.5 cm (D) 16.25 cm (E) 15 cm

51. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 1 & 3 & 6 \end{bmatrix}$. Find $|A|$.

- (A) -4 (B) -6 (C) 3 (D) 2 (E) 1

52. $\triangle ABC$ is an equilateral triangle. The ratio of AD to DB is 2 to 1. Find $m\angle CDB$. (nearest degree)



- (A) 101° (B) 104° (C) 108° (D) 112° (E) 120°

53. $(3\text{cis}(\frac{\pi}{3}))^3$ equals:

- (A) $9 - 9i$ (B) $-6.25 + 4.5i$ (C) $3 - i\sqrt{3}$ (D) $1 - i$ (E) -27

54. Rusty Yaht sets sail from his personal dock at 8:00 am on a bearing of 65° . After sailing 5 km, he changes course and sails 10 km on a bearing of 120° . How far away from his personal dock is he at the end of the 15 km voyage? (nearest tenth)

- (A) 11.6 km (B) 12.5 km (C) 13.2 km (D) 13.5 km (E) 14.4 km

55. Given the geometric sequence $-4, a, b, -\frac{27}{128}, c, \dots$, find $a + b + c$.

- (A) $-2\frac{181}{512}$ (B) $-2\frac{145}{1024}$ (C) $-4\frac{81}{256}$ (D) $-6\frac{35}{128}$ (E) $-6\frac{361}{1024}$

56. If $\log(x + 1) + \log(x + 2) = \log(2x + 22)$, solve for x .

- (A) 2 (B) 4 (C) 7 (D) 11 (E) no solution

57. Which of the following series are divergent? 1. $\sum_{n=0}^{\infty} \frac{2^n - 1}{3^n}$ 2. $\sum_{n=0}^{\infty} \cos(n\pi)$ 3. $\sum_{n=0}^{\infty} \frac{n!}{1000^n}$

- (A) 1 only (B) 2 only (C) 3 only (D) 1 & 2 (E) 2 & 3

58. A light bulb is hung 15 feet above a straight horizontal path. A man is walking away from the light at a rate of 5 feet per second. If the man is 6 feet tall, at what rate is the tip of the man's shadow moving? (nearest tenth)

- (A) 8.3 ft/sec (B) 8.0 ft/sec (C) 6.0 ft/sec (D) 2.5 ft/sec (E) 1.2 ft/sec

59. Cookie Baykur baked chocolate chip cookies, peanut butter cookies, sugar cookies, carrot cake cookies, and pecan cookies. She packages them in sets of three per package. How many different packages can she make?

- (A) 35 (B) 21 (C) 720 (D) 60 (E) 15

60. Change the base 10 proper fraction $\frac{5}{12}$ to a repeating decimal in base 11.

- (A) 0.464646...₁₁ (B) 0.363636...₁₁ (C) 0.2181818...₁₁ (D) 0.636363...₁₁ (E) 0.8333...₁₁

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. B | 21. E | 41. D |
| 2. E | 22. B | 42. C |
| 3. D | 23. B | 43. A |
| 4. C | 24. C | 44. A |
| 5. D | 25. C | 45. A |
| 6. E | 26. E | 46. E |
| 7. C | 27. A | 47. D |
| 8. B | 28. B | 48. B |
| 9. A | 29. C | 49. C |
| 10. C | 30. D | 50. E |
| 11. B | 31. B | 51. D |
| 12. E | 32. E | 52. A |
| 13. A | 33. B | 53. E |
| 14. C | 34. A | 54. D |
| 15. B | 35. D | 55. B |
| 16. D | 36. A | 56. B |
| 17. C | 37. E | 57. E |
| 18. D | 38. C | 58. A |
| 19. D | 39. E | 59. A |
| 20. B | 40. C | 60. A |