UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

District 2 • 2013

7 8 9

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: 0.333... ÷ 0.1666... + 0.142857142857148527... — 0.111... ÷ 0.090909...

   (A) $22\frac{22}{63}$ (B) $1\frac{23}{63}$ (C) $\frac{58}{63}$ (D) $-\frac{41}{42}$ (E) $-1\frac{1}{42}$

2. Cal Q. Lait is buying scientific calculators for his math team. The cost of each calculator is $125.00. If he buys a set of 6 he gets 25% off of the regular price for 4 and 40% off the regular price for the other 2. How much money will he save if he buys a set of 6?

   (A) $81.25   (B) $150.00   (C) $187.25   (D) $212.50   (E) $225.00

3. $3 \times (6 + 10) = 3 \times 16 = 48$ and $3 \times 6 + 3 \times 10 = 18 + 30 = 48$ are examples of the
   _____ property of equality.

   (A) commutative   (B) closure   (C) associative   (D) transitive   (E) distributive

4. All triangular numbers and all odd numbers are considered to be _____ numbers?

   (A) perfect   (B) polite   (C) pretty   (D) prime   (E) primeval

5. A line perpendicular to the line shown intersects the line at point $(-2, -1)$ and has an x-intercept at point (x, y) and a y-intercept at point (x_1, y_1). Find $x + x_1 + y + y_1$.
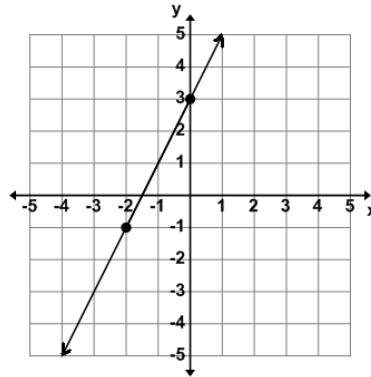
   (A) -9 (B) -8 (C) -6 (D) -3 (E) -2

6. Simplify: $\left(\dfrac{6x^3 + x^2 - 2x}{9x^2 + 12x + 4}\right) \div \left(\dfrac{2x^2 - x}{6x^2 + 7x + 2}\right)$

   (A) $2x + 1$ (B) $3x - 1$ (C) $4x^2 - 1$ (D) $4x^2 - 4x + 1$ (E) $x^2 + 3x - 6$

7. The number of hours it takes to pick up an orchard of pecans varies inversely to the number of pecan pickers. If it takes 30 hours for 5 pickers to pickup all of the pecans, how long would it take 8 pickers to pickup all of them?   (nearest minute)

   (A) 22 hrs 30 min   (B) 40 hrs   (C) 18 hrs   (D) 10 hrs 15 min   (E) 18 hrs 45 min

8. If $y = x - 5$ and $xy = 2$ then $x^3 - y^3 = ?$

   (A) 155   (B) 133   (C) 117   (D) 106   (E) 95

9. Find the total surface area of the right circular cone shown. (nearest sq. in.)
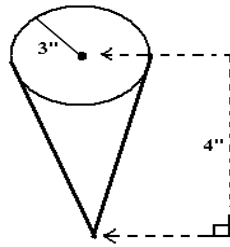
    (A) 38 sq. in.   (B) 47 sq. in.   (C) 66 sq. in.   (D) 75 sq. in.   (E) 113 sq. in.

10. Corresponding sides of two similar triangles are proportional to the corresponding:

    (1) altitudes   (2) medians   (3) perimeters

    (A) 1 only   (B) 1 & 2   (C) 1 & 3   (D) 1, 2, & 3   (E) 2 only

11. A tourist looked up at the Big Ben circular clock in Westminster and saw the time to be 8:40 a.m. What was the measure of the smaller angle between the big hand and the little hand at that time?

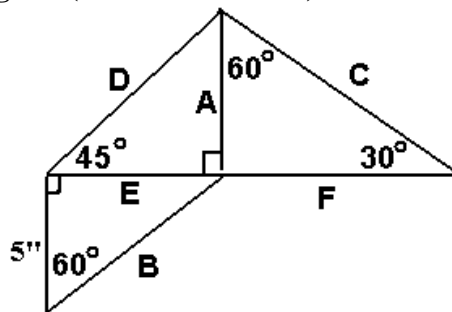   (A) 24 °   (B) 20 °   (C) 16 °   (D) 5 °   (E) 0 °

12. If you connect the centers of all the faces of an octahedron you will form a(n):

    (A) tetrahedron   (B) cube   (C) icosahedron   (D) square pyramid   (E) decahedron

13. Find the circumference of the circle, $x^2 + y^2 - 10x - 12y = 60$. (nearest tenth)

    (A) 71.0   (B) 69.1   (C) 59.6   (D) 52.9   (E) 48.7

14. Find the perimeter of the pentagon. (nearest half inch).

    (A) $4'8''$ (B) $4'10''$ (C) $4'11\frac{1}{2}''$ (D) $5'\frac{1}{2}''$ (E) $5'2''$

15. The graph of $10x - 4y^2 + 12y = 21$ is a(n):

    (A) circle   (B) cardiod   (C) ellipse   (D) hyperbola   (E) parabola

16. Let $f(x) = 3x - 1$ and $g(x) = 3 - 2x$ and $h(x) = 4x + 5$. Find $g\left(f\left(h(-x)\right)\right)$.

    (A) $24x + 35$ (B) $24x - 21$ (C) $24x - 25$ (D) $24x - 9$ (E) $24x + 7$

17. Admiral C. Weed leaves port and sails his ship 3 hours on a bearing of 60° at 15 knots. Then he changes course and sails on a bearing of 150° for 2 hours at 12 knots. Then he turns to sail back to port. What bearing should he set his course for to sail directly back to port? (nearest degree)

(A) 298°   (B) 300°   (C) 332°   (D) 268°   (E) 272°

18. Which of the following equations will have a graph with an amplitude of 4, a frequency of $\frac{1}{2}$, a phase shift of -2, and a displacement of $\frac{1}{4}$?

(A) $\frac{1}{4} + 4\cos(\pi x + 2\pi)$ (B) $4 + \frac{1}{4}\cos(\frac{1}{2}\pi x - 2)$ (C) $\frac{1}{4} + 4\cos(\pi x - 2\pi)$

(D) $\frac{1}{2} + 4\cos(4\pi x + 8\pi)$ (E) $\frac{1}{4} + 4\cos(2\pi x + \frac{\pi}{2})$

19. Simplify: $\sin(\theta) + \sin(\theta)\cot^2(\theta)$

(A) $\sec(\theta)$ (B) $\sin^3(\theta)$ (C) $\tan^2(\theta)$ (D) $\csc(\theta)$ (E) $1 - \sin(\theta)$

20. In the expansion of $(2x + 3)^6$, the sum of the coefficients of the 3rd and the 6th term is:

(A) 900   (B) 5,076   (C) 5,113   (D) 5,436   (E) 15,625

21. Suhn Wi Noh and Wen Wi Kan solve all of the advanced math test questions in 3 hours and 45 minutes. Suhn can solve all of them in 10 hours by himself. How long would it take Wen to solve all of them by himself?

(A) 6 hrs   (B) 6 hrs 15 min (C) 6 hrs 52 min (D) 7 hrs 30 min (E) 7 hrs

22. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix}$. Find the sum of the elements of $A(A - B)$.

(A) 7   (B) 5   (C) 3   (D) 1   (E) 0

23. Determine the concavity of the graph of $f(x) = \sin(x) + \cos(x)$ at $x = \frac{4\pi}{3}$.

(A) $\frac{\sqrt{3} - 1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{-1 - \sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{1 + \sqrt{3}}{2}$

24. Let $f(x) = \frac{2}{x + 5}$. At which of these intervals is function f not continuous?

(A) $(-\infty, 0)$ (B) $[-10, -5)$ (C) $(-5, \infty)$ (D) $(3, 7)$ (E) all of these

25. Five ping pong balls are numbered using the first five "lucky" numbers, 1, 3, 7, 9, and 13. Two balls are randomly selected. What is the probability that the sum of the "lucky" numbers selected is an "unlucky" number?

(A) 100%   (B) 60%   (C) 40%   (D) 20%   (E) 10%

26. The five circles in the quadrilateral are congruent. The four outer circles are tangent to the quadrilateral and to the center circle. Amos Goode throws a dart that hits somewhere in the quadrilateral. What is the probability that the dart hits inside a circle? (nearest whole percent)


18 cm (vertical), **18 cm** (horizontal)
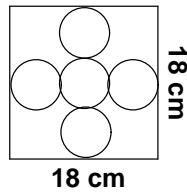
   (A) 65%   (B) 57%   (C) 51%   (D) 44%   (E) 29%

27. An icosahedron has an edge length of 2". The length of its midradius is: (nearest hundredth)

   (A) 2.62"   (B) 0.81"   (C) 3.24"   (D) 2.29"   (E) 1.62"

28. Which of the following numbers will be in the 18th row of Pascal's triangle?

   (A) 1,820   (B) 3,003   (C) 5,239   (D) 6,188   (E) 19,455

29. A women mathematician credited with writing the first book discussing both differential and integral calculus had a special curve named after her. Her name is:

   (A) Agnesi   (B) Hypatia   (C) Ada Byron   (D) Freda Porter   (E) Alecia Stott

30. Lucas folds the net shown into a cube. He multiplies the numbers on each pair of opposite sides. What is the sum of the three products?
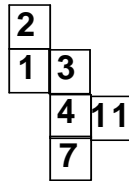

   (A) 28   (B) 29   (C) 40   (D) 47   (E) 51

31. $456_9 - 1221_3 = $ _____ $_3$

   (A) 102222   (B) 112110   (C) 121012   (D) 120222   (E) 122210

32. P and Q are the real roots of $4x^2 - 8x - 5 = 0$. Find $P^4 + 4P^3Q + 6P^2Q^2 + 4PQ^3 + Q^4$.

   (A) 44   (B) 28.125   (C) 16   (D) 12.5   (E) 9

33. $\left\{ (x,y) \mid x,y \in \{\text{Integers}\}, -7 \leq x \leq 7, \text{and} -11 \leq y \leq 11 \right\}$ is the solution set of $5x - 3y = 2$. How many such ordered pairs exist?

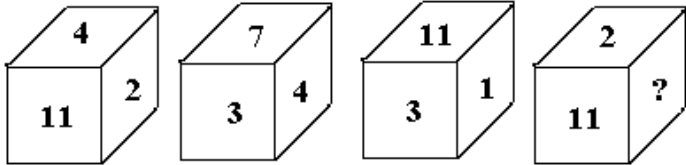   (A) 6   (B) 5   (C) 4   (D) 3   (E) 2

34. Find the circumference of the inscribed circle. (nearest tenth)
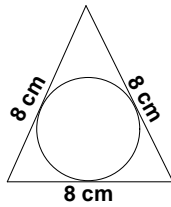

(A) 25.1 cm   (B) 12.6 cm   (C) 13.9 cm   (D) 16.8 cm   (E) 14.5 cm

35. The eccentricity of $18x^2 - 9y^2 = 162$ is:

(A) $\frac{3\sqrt{2}}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{6}}{2}$ (E) $\sqrt{3}$

36. Otto Mobill has 16 quarts of a 20% solution of antifreeze in his radiator. How much will he have to drain out of his radiator and replace with pure antifreeze to make a 25% solution?

(A) 1 qt   (B) 1.25 qts   (C) 2 qts   (D) 2.5 qts   (E) 5 qts

37. Jack Black throws out all of the cards from a standard deck except for the 4 aces and the face cards, 4 jacks, 4 queens, and 4 kings. He shuffles these cards and deals the top two cards face up on the table. What is the probability that the top card is a face card and the next card is an ace?

(A) 15%   (B) 16.666...%   (C) 18.75%   (D) 20%   (E) 22.333...%

38. A triangle is drawn as shown. Find the m$\angle$XZY, if area of $\triangle$XZY = 66 cm^2, h = 11 cm and YZ = 22 cm.
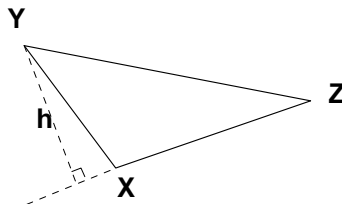
(A) 45°   (B) 15°   (C) 22.5°   (D) 26.6°   (E) 30°

39. Willie Spotette looks down from his deer blind and spots a deer feeding at his corn feeder. The altitude from the ground to Willie's eyes is 20 feet and the distance from his blind to his feeder is 60 yards. What is the angle of depression from Willie to the deer? (nearest minute)

(A) 18° 26'   (B) 1° 26'   (C) 6° 20'   (D) 6° 37'   (E) 19° 28'

40. How many elements are in $\left\{ x \ \middle| 4\sin(x)\cos(x) = \sqrt{3}, x \in (-\pi, \pi) \right\}$?

(A) 1   (B) 2   (C) 4   (D) 6   (E) 8

41. If $\cos(\theta - \pi) < 0$ and $\sin(\theta + \pi) > 0$ then where will θ terminate?

   (A) origin   (B) QI   (C) QII   (D) QIII   (E) QIV

42. Simplify: $\log_b 3x + 2\log_b 2x - 3\log_b x$, if $x > 0$

   (A) $\log_b 12$ (B) $2\log_b 4x$ (C) $-\log_b 6x$ (D) $\frac{2}{3}\log_b 4x$ (E) $\log_b 6$

43. The polar graph of $r = 3\cos 2\theta$ is symmetric to the: (1) polar axis   (2) pole   (3) line $\theta = \frac{\pi}{2}$

   (A) 1 only   (B) 3 only   (C) 1 & 3   (D) 2 & 3   (E) 1, 2, & 3

44. The series $\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots$ converges to _____.

   (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1 + \sqrt{5}}{2}$ (D) ∞ (E) 2

45. Each face of a cube is labeled with the Lucas digits, 2, 1, 3, 4, 7, 11. Four views of the cube are shown. What is the sum of the numbers on the faces containing the "?" and its opposite face? Ignore orientation of the numbers.

   (A) 4   (B) 5   (C) 7   (D) 8   (E) 10

46. Determine the number of non-negative integer solutions to the equation: $p + q + r = 12$.

   (A) 36   (B) 91   (C) 105   (D) 455   (E) 1,728

47. Bunny Wabit dyed a basket of hard boiled eggs for the big egg hunt. She hid 50% of the eggs in open sight. Then she hid $\frac{1}{4}$ of the remaining eggs under bushes. She gave each of her 3 helpers an egg for helping and kept the last 3 eggs for her morning breakfast. How many eggs were in the basket originally?

   (A) 12   (B) 16   (C) 18   (D) 24   (E) 30

48. Find the sum, nearest whole number, of the mean, median, mode, and range of 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, & 31.

   (A) 101   (B) 98   (C) 96   (D) 95   (E) 93

49. The coordinates of the vertices of $\triangle PQR$ are $(-5, -2)$, $(-2, 2)$, and $(1, -2)$. The coordinates of the incenter is (x, y). Find y.

   (A) -2 (B) -1.5 (C) -1 (D) -0.5 (E) -0.25

50. **Which of the following equations will produce the graph shown here?**
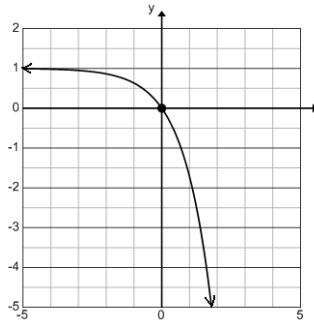
(A) $y = \ln(-x) - 1$ (B) $y = 1 + \ln(x)$ (C) $y = 1 + e^{(\ln(x))}$ (D) $y = 1 - e^{(-x)}$ (E) $y = 1 - e^{(x)}$

51. **Dim Whitt and his brother, Nit Whitt can paint 6 houses in 32 hours. How long would it take them to paint 10 houses if their other brother, Half Whitt, helps them and they all work at the same rate as the Dim and Nit? (nearest minute)**

(A) 16 hrs   (B) 17 hrs 47 min   (C) 26 hrs 40 min   (D) 35 hrs 33 min   (E) 53 hrs 20 min

52. **Find the sum of all two-digit numbers such that reversing the digits results in another two-digit number that is $42\frac{6}{7}\%$ less than the original two-digit number.**

(A) 120   (B) 360   (C) 420   (D) 240   (E) 210

53. **$\triangle ABC$ exists as shown. Let $m\angle AMB = \theta$. Find $\cos(\theta)$.   (nearest hundreth)**
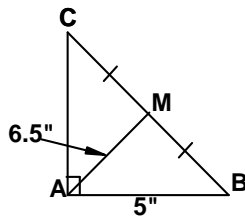
(A) 0.38   (B) 0.92   (C) 0.56   (D) 0.70   (E) 0.77

54. **How many solutions are there for the equation $4x + 5y = 123$ such that both x and y are positive integers?**

(A) 14   (B) 13   (C) 12   (D) 6   (E) 5

55. **The polar coordinates of point P is $(-9, \frac{5\pi}{6})$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian coordinate plane?**

(A) Q1I   (B) QIII   (C) QIV   (D) x-axis   (E) y-axis

56. **Find the area of the region bounded between the graphs of $y = -1$ and $y = x^3$ for $-1 \le x \le 0$.**

(A) 1.5   (B) 1.25   (C) 0.75   (D) 0.666...   (E) 0.333...

57. $F(x) = \frac{x+1}{3-x}$ has an inflection point at x = ?

    (A) -1 (B) 3   (C) 1   (D) -3 (E) does not exist

58. The probability that statement P is true is 2/5, and the probability that statement Q is true is 3/8. Determine the probability that P $\rightarrow$ Q is false.

    (A) $\frac{3}{4}$ (B) $\frac{15}{16}$ (C) $\frac{1}{40}$ (D) $\frac{1}{4}$ (E) $\frac{3}{20}$

59. $110010101111_2 + 303_4 + 1E3_{16} =$ _____ $_{16}$

    (A) EC5   (B) 103E   (C) BAD1   (D) 3E5   (E) ABC

60. P, Q, and R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, and R is $\frac{18}{23}$ and and PQR = 1. Find C.

    (A) $4\frac{1}{6}$ (B) $3\frac{5}{6}$ (C) $2\frac{5}{9}$ (D) $1\frac{5}{18}$ (E) 1

University Interscholastic League
MATHEMATICS CONTEST
HS ● District 2 ● 2013
Answer Key

1. C	21. A	41. E
2. E	22. A	42. A
3. E	23. E	43. E
4. B	24. A	44. B
5. C	25. A	45. B
6. A	26. D	46. B
7. E	27. E	47. B
8. A	28. D	48. D
9. D	29. A	49. D
10. D	30. C	50. E
11. B	31. A	51. D
12. B	32. C	52. E
13. B	33. B	53. D
14. C	34. E	54. D
15. E	35. E	55. C
16. C	36. A	56. C
17. D	37. D	57. E
18. A	38. E	58. D
19. D	39. C	59. A
20. B	40. C	60. B