



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 2 • 2014



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $[1 - 4 \times (6 + 8)] - 9 \div (10 - 12) + 14$

- (A) -36.5 (B) -28.4 (C) -23.5 (D) 0.4 (E) 26

2. Les Cash had \$225.00 in cash to spend on Spring break. He spent 30% of his cash on food, one-fourth on lodging, 15% on travel, and \$40.00 on entertainment. How much did he have left?

- (A) \$60.40 (B) \$27.50 (C) \$24.75 (D) \$22.50 (E) \$15.00

3. Find the number of positive integral divisors of 464.

- (A) 10 (B) 16 (C) 12 (D) 8 (E) 14

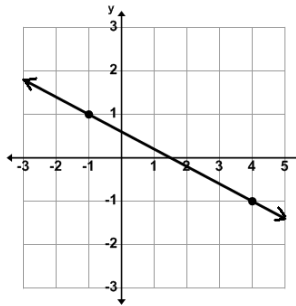
4. On a map legend, $\frac{3}{4}$ inch represents 75 miles. How far is it on the map, from Slippery Slope, Colorado to Busted Buns, Utah, if the distance in miles is 637.5?

- (A) $8\frac{1}{2}$ " (B) $8\frac{1}{4}$ " (C) $7\frac{3}{4}$ " (D) $7\frac{1}{8}$ " (E) $6\frac{3}{8}$ "

5. Let $\frac{3x-2}{2x-3} - \frac{4x+5}{5x+4} = \frac{Ax^2+Bx+C}{Dx^2+Ex+F}$. Find $\frac{A+B+C}{D+E+F}$.

- (A) -2 (B) $-1.333\dots$ (C) $-0.888\dots$ (D) 0.5 (E) 9

6. Line m is perpendicular to the line shown and contains the point $(-5, 2)$. Which of the following points is on line m ?



- (A) $(0, 4)$ (B) $(-7, 3)$ (C) $(7.5, -3)$ (D) $(5, -2)$ (E) $(-1, 12)$

7. A smaller pulley with a radius of 4" is connected with a fan belt to a larger pulley with a radius of 6". Find the speed of the larger pulley if the speeds of the pulleys are in inverse proportion to their diameters and the smaller pulley runs at 1,452 rpm.

- (A) 2,420 rpm (B) 2,178 rpm (C) 1,210 rpm (D) 968 rpm (E) 645 rpm

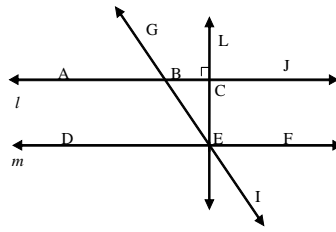
8. These three lines, $x + 2y = 3$, $3x - 2y = 1$, and $7x + 8y = k$, are concurrent. Find k .

- (A) -1 (B) 1 (C) 6 (D) 9 (E) 15

9. How many distinguishable arrangements can be made from the letters "STATEMEET"?

- (A) 120 (B) 3,024 (C) 10,080 (D) 90,720 (E) 181,440

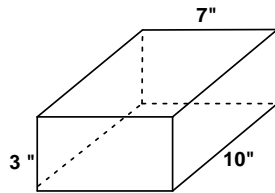
10. The four lines in the figure are coplanar with $m \parallel l$. Which of the following are true statements?



- 1. $\angle CEG$ & $\angle FEI$ are congruent
- 2. $\angle GBA$ & $\angle CEB$ are alternate exterior angles
- 3. $m\angle JBG = m\angle BEF$
- 4. $\angle FEG$ & $\angle BED$ are supplementary angles

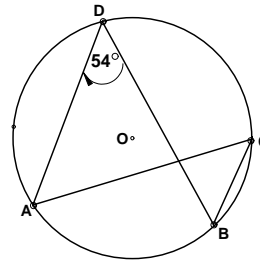
- (A) 1, 2, 3, & 4 (B) 2 & 3 (C) 1, 2, & 4 (D) 3 & 4 (E) 1 & 4

11. Find the total surface area of the rectangle shown. (nearest sq. in).



- (A) 200 sq. in (B) 210 sq. in (C) 221 sq. in (D) 234 sq. in (E) 242 sq. in

12. Given the circle with center O, find $m\angle DBC$. (Drawing not to scale.)



- (A) 72° (B) 54° (C) 46° (D) 36° (E) 27°

13. The ratio of the distance from the center to a focus to the distance that focus is to the endpoint of the semiminor axis of an ellipse is the _____.

- (A) asymptote (B) directrix (C) eccentricity (D) focal distance (E) locus

14. Let $f(x) = 2x^2 - 3x - 4$ and $g(x) = 5x + 1$. Find $g\left(f\left(g^{-1}(6)\right)\right)$.

- (A) -24 (B) -9 (C) -6 (D) 3 (E) 6

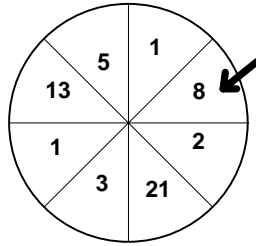
15. If $a_1 = -1$, $a_2 = 0$, $a_3 = 1$, and $a_n = a_{n-3} - a_{n-2} + a_{n-1}$, where $n \geq 4$, then a_{11} equals:

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

16. The graph of the polar equation $r = 3 - 3\cos(\theta)$ is a:

- (A) cardioid (B) lemniscate (C) dimpled limaçon (D) looped limaçon (E) convex limaçon

17. Willie When spins the spinner shown, where all sectors are the same size. If it lands on a prime number he gets \$5.00 and if it lands on a composite number he gets \$3.00. If it doesn't land on a prime or composite number, he loses \$2.00. Assuming it does not land on a line, what is the mathematical expectation on any one spin?



- (A) \$2.25 (B) \$2.50 (C) \$2.75 (D) \$3.75 (E) \$4.00
18. Frank Lynn's kite is flying 30 feet above the ground. He anchors the kite's string to the ground. Find the angle of elevation of the kite string if the string is 50 feet long. (nearest degree)

- (A) 31° (B) 37° (C) 40° (D) 53° (E) 59°

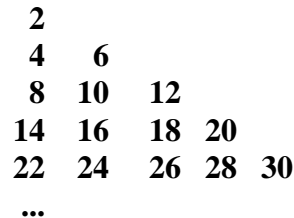
19. Simplify: $\frac{\sec x}{\sin x} - \frac{\cos x}{\sin x}$

- (A) 1 (B) $\csc x$ (C) $\tan x$ (D) $\sin x \cos x$ (E) $\cot x$

20. Simplify to the form $a + bi$: $(2 - \sqrt{-98})(2 + \sqrt{-72})$

- (A) $88 - 2\sqrt{2}i$ (B) $46 - 13\sqrt{2}i$ (C) $46 - 2\sqrt{2}i$ (D) $88 - 13\sqrt{2}i$ (E) $-80 - 2\sqrt{2}i$

21. If the set of even numbers continue in the triangular array shown below, the sum of the digits in the 9th row would be?



- (A) 729 (B) 810 (C) 720 (D) 819 (E) 738

22. If $\begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} \times \begin{bmatrix} -\frac{2}{3} & -\frac{3}{4} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a \times d + b \times c = ?$

- (A) 1 (B) 2 (C) 3 (D) 10 (E) 18

23. Let $f(x) = x^2 - 3x - 5$ and $g(x) = 2x^2 + 3x - 5$. Find $f'(g'(x + 1))$

- (A) $4(x + 2)$ (B) $8x - 1$ (C) $4(2x + 1)$ (D) $6(x + 1)$ (E) $8x + 11$

24. Lotta Space is building a rectangular parking lot for her *Little Lots* store. The lot will be bordered on one side by a road. She has 1500 feet of fence to enclose the lot. What is the maximum area of her parking lot?

- (A) 140,625 ft.² (B) 281,250 ft.² (C) 421,875 ft.² (D) 500,000 ft.² (E) 1,125,000 ft.²

25. Integrate: $\int \sin(\frac{\theta}{2})\cos(\frac{\theta}{2}) d\theta$




- (A) $\cos(2\theta) + C$ (B) $2\sin(\theta) + C$ (C) $-\frac{\cos \theta}{2} + C$ (D) $-\frac{\sin \theta}{2} + C$ (E) $2\cos(\frac{\theta}{2}) + C$

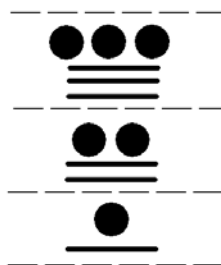
26. The Millersview Swatters and the Fifi Flies are playing a five games series. The Swatters are three times as likely to beat the Flies. What is the probability that the Swatters will win all 5 games? (nearest percent)

- (A) 85% (B) 75% (C) 33% (D) 30% (E) 24%

27. The state math test result sheets show that 70 of the 125 students taking the test scored above 120. Based on these results, what were the odds that a student scored 120 or less?

- (A) $\frac{11}{14}$ (B) $\frac{3}{5}$ (C) $\frac{11}{24}$ (D) $\frac{7}{12}$ (E) $\frac{11}{25}$

28. The Mayan number system consists of three symbols,   . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. What base 10 number would the following Mayan number be?



- (A) 2,656 (B) 1,296 (C) 7,446 (D) 13,126 (E) 966

29. Which of these mathematicians proved that the infinite set of real numbers are "more numerous" than the infinite set of natural numbers?

- (A) Georg Cantor (B) John Napier (C) George Boole (D) Theano (E) Benoit Mandelbrot

30. Find k when $4 - 5(3 + k) \div 2 = 6 - k$

- (A) $-14\frac{1}{4}$ (B) $-11\frac{1}{3}$ (C) $-6\frac{1}{3}$ (D) $-1\frac{3}{4}$ (E) $-1\frac{2}{3}$

31. Let X be a two-digit number such that when the sum of the digits is subtracted from X , the difference is 63. Let Y be the two digit number when the digits of X are reversed. When the sum of the digits of Y is subtracted from Y , the difference is 9. Find $X + Y$.

- (A) 36 (B) 72 (C) 88 (D) 104 (E) 132

32. Find the sum of all of the two-digit numbers whose units digit is divisible by 3 or 9.

- (A) 1,008 (B) 1,369 (C) 1,458 (D) 1,962 (E) 3,069

33. Find the average of the arithmetic mean, the median, the mode, and the range of these quiz grades: 85, 90, 75, 100, 85, 90, 70, & 85.

- (A) 66.666... (B) 68.75 (C) 70.5 (D) 71.25 (E) 71.875

34. $10001_2 + 3003_4 + 707_8 = \underline{\hspace{2cm}}_{10}$

- (A) 13,711 (B) 99 (C) 667 (D) 226 (E) 3,592

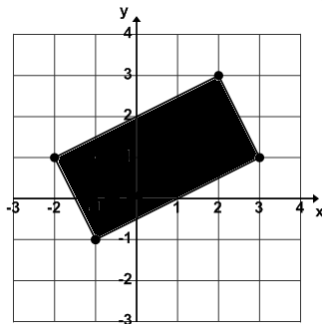
35. The Real value solution set for $|1 + 2x| - 3 < 4$ is?

- (A) $\{x \mid \{x < -4\} \cup \{x > 3\}\}$ (B) $\{x \mid -3 < x < 4\}$ (C) $\{x \mid -4 < x < 3\}$
 (D) $\{x \mid -4 < x < 3\}$ (E) $\{x \mid \{x > 4\} \cup \{x < -3\}\}$

36. Kandy Kane wants to mix 5 pounds of peppermint bits with some mint chips for a special blend called K&K's. If peppermint bits sell for 85¢ a pound and mint chips sell for \$1.12 a pound, how many pounds of mint chips will she need to make K&K's that will sell for \$1.00 per pound?

- (A) 6.6 lbs (B) 6.25 lbs (C) 5.27 lbs (D) 4.5 lbs (E) 3.8 lbs

37. Desi Cortez drew the rectangle on his Cartesian Coordinate plane below. The vertices of the rectangle have integral coordinates. What is the area of his rectangle? (nearest sq. unit)



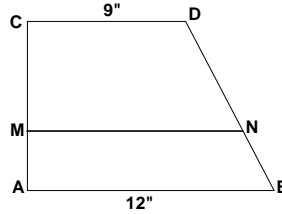
- (A) 10 units² (B) 10.25 units² (C) 10.5 units² (D) 11.2 units² (E) 12.2 units²

38. If $\frac{3x-1}{2x+3} + \frac{3x+1}{2x-3} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $(A + B + C) - (P + Q + R)$ equals:

- (A) 10 (B) 13 (C) 17 (D) 23 (E) 27

39. Let O be the origin on a Cartesian Coordinate plane. Point O is reflected across the line $y = x + 4$ to point P . Point P is translated vertically 3 units up to point Q . Point Q is reflected back across the line $y = x + 4$ to point R . Point R is translated vertically 3 units down to point S . If the coordinates of S are (x, y) then $x + y$ equals:
- (A) -4 (B) -2 (C) 0 (D) 3 (E) 4
40. How many integral values of n exist such that $n > 3$ and $\frac{(n-1)!}{(n-3)!} \leq 12$
- (A) 7 (B) 5 (C) 2 (D) 1 (E) none
41. Let $f(x - 1) = 1 - 2f(x)$. Find $f(-2)$ if $f(1) = -1$.
- (A) -5 (B) -3 (C) 2 (D) 4 (E) 11
42. Let vector $u = (1, -4)$ and vector $v = (5, -3)$. Find the measure of the angle with initial side u and terminal side v . (nearest second)
- (A) $14^\circ 2' 10''$ (B) $30^\circ 57' 50''$ (C) $45^\circ 0' 0''$ (D) $59^\circ 2' 10''$ (E) $75^\circ 57' 50''$
43. The altitude of $\triangle ABC$ forms two right triangles, $\triangle ABD$ and $\triangle CBD$. Find BC if $AB = 15''$, $m\angle BCD = 55^\circ$, and $m\angle BAD = 45^\circ$. (nearest tenth)
- (A) $18.5''$ (B) $15.0''$ (C) $10.6''$ (D) $12.9''$ (E) $7.4''$
44. The sum of the coefficients of the 3rd term in the expansion of $(x + 1)^2$, the 3rd term of $(x + 1)^3$, the 3rd term of $(x + 1)^4$, and 3rd term of $(x + 1)^5$ is:
- (A) 12 (B) 14 (C) 15 (D) 17 (E) 20
45. If $\log_2(15) = x$ and If $\log_{15}(32) = y$, then $xy = ?$
- (A) 0.0625 (B) 0.2 (C) 4 (D) 5 (E) 16
46. Cliff D. Kemist has 20 ounces of a 15% solution of argyle in water. How much water should he add to reduce his solution to a solution that is 90% water?
- (A) 5 oz (B) 10 oz (C) 15 oz (D) 20 oz (E) 25 oz
47. Nick Ohl tossed a nickel four times and recorded the results. What is the probability of at least two consecutive heads? (nearest per cent)
- (A) 50% (B) 33% (C) 28% (D) 25% (E) 13%
48. $\triangle ABC$ and $\triangle PQR$ exist such that $\angle BCA \cong \angle PQR$, $\angle CBA \cong \angle QPR$, $AB = 20 \text{ cm}$, $BC = 25 \text{ cm}$, $PQ = 75 \text{ cm}$, and $RQ = 48 \text{ cm}$. Find the ratio of AC to PR .
- (A) $\frac{25}{48}$ (B) $\frac{4}{15}$ (C) $\frac{4}{5}$ (D) $\frac{5}{12}$ (E) $\frac{1}{3}$

49. Given the trapezoid shown where the area of trapezoid ABNM is equal to the area of trapezoid CDN M, find MN. (nearest hundredth)



- (A) 10.71 (B) 10.61 (C) 10.57 (D) 10.50 (E) 10.39
50. The coordinates of the vertices of $\triangle RST$ are $(-2, 6)$, $(5, -3)$ and (x, y) . The coordinates of the centroid of $\triangle PQR$ are $(2, 2)$. Find xy .
- (A) $2\frac{7}{9}$ (B) 9 (C) $2\frac{1}{4}$ (D) 6 (E) 25
51. The harmonic mean of the real roots of $3x^3 - 10x^2 + kx - 2 = 0$ is $\frac{2}{3}$. Find k .
- (A) 12 (B) 9 (C) 6 (D) 3 (E) 2
52. Let $f(x) = \frac{x-1}{x+1}$. Which of the following is true about $f(x)$?
- (A) It is an odd function. (B) It is an even function. (C) It is not a function.
 (D) It is neither an even nor an odd function (E) It is a logarithmic function.
53. If $3x^2 + 6y = 10$ and $x > 0$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when $x = ?$
- (A) 0.5 (B) 1 (C) 1.5 (D) 1.666... (E) 3.333...
54. Find the area of the region bounded by $x = 10 - y^2$ and $x = y^2 - 4y + 4$.
- (A) 10.666... (B) 14.666... (C) 18.333... (D) 21.333... (E) 24.0
55. Les Speed's car is traveling at a rate of 30 ft/sec approaching an intersection. When Les is 120 feet from the intersection, Joy Ryder's truck traveling at a rate of 40 ft/sec crosses the intersection. Both vehicles are on roads that are at right angles to each other. How fast are the vehicles separating 2 seconds after Joy leaves the intersection? (nearest tenth)
- (A) 27.7 ft/sec (B) 7.1 ft/sec (C) 13.8 ft/sec (D) 12.0 ft/sec (E) 14.0 ft/sec
56. The probability that Larry will ask Shemp to go to the movies is 25%, that Moe will ask him is 50%, and that Curley will ask him is 87.5%. What is the probability that Moe and Curley will ask Shemp, but Larry will not? (nearest percent)
- (A) 33% (B) 11% (C) 78% (D) 44% (E) 16%

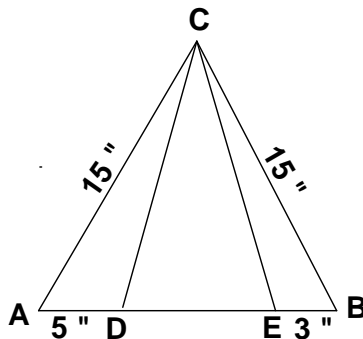
57. The repeating decimal $0.676767\dots$ in base 8 can be written as which of the following fractions in base 8?

- (A) $\frac{15}{17}_8$ (B) $\frac{45}{51}_8$ (C) $\frac{61}{64}_8$ (D) $\frac{67}{77}_8$ (E) $\frac{55}{63}_8$

58. Find k , given: $1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, k, \frac{49}{20}, \dots$

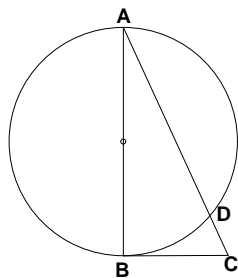
- (A) $\frac{35}{18}$ (B) $\frac{47}{24}$ (C) $\frac{75}{48}$ (D) $\frac{137}{60}$ (E) $\frac{145}{72}$

59. $\triangle ABC$ is an equilateral triangle. Find the perimeter of $\triangle CDE$. (nearest tenth)



- (A) 34.6" (B) 34.0" (C) 33.0" (D) 32.4" (E) 29.8"

60. Given: \overline{AB} is a diameter of the circle shown, \overline{BC} is tangent to the circle, $m\angle BAC = 30^\circ$, and $CD = \sqrt{3}$ cm. Find the perimeter of $\triangle ABC$. (nearest tenth)



- (A) 21.6 cm (B) 15.6 cm (C) 10.4 cm (D) 20.8 cm (E) 16.4 cm

**University Interscholastic League
MATHEMATICS CONTEST
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Answer Key**

- | | | |
|-------|-------|-------|
| 1. A | 21. E | 41. E |
| 2. B | 22. C | 42. C |
| 3. A | 23. E | 43. D |
| 4. E | 24. B | 44. E |
| 5. A | 25. C | 45. D |
| 6. E | 26. E | 46. B |
| 7. D | 27. A | 47. A |
| 8. E | 28. C | 48. B |
| 9. C | 29. A | 49. B |
| 10. D | 30. C | 50. B |
| 11. E | 31. C | 51. B |
| 12. B | 32. D | 52. D |
| 13. C | 33. D | 53. B |
| 14. A | 34. C | 54. D |
| 15. D | 35. D | 55. E |
| 16. A | 36. B | 56. A |
| 17. C | 37. A | 57. D |
| 18. B | 38. D | 58. D |
| 19. C | 39. C | 59. B |
| 20. A | 40. C | 60. E |