UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

State • 2015

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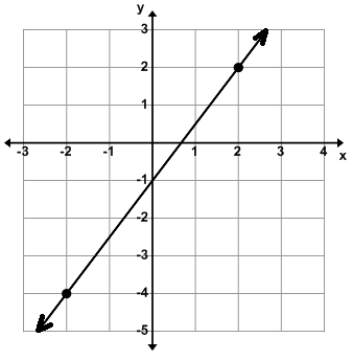
1. Evaluate: $5 + 2 \times 6 - 5 \div 2 \times 6 + 5^2 - 6$

(A) 130 **(B)** 55 **(C)** 51 **(D)** 21 **(E)** 17

2. The *Parr Fore* golf store sells a sleeve of 3 balls for $3.85, and a box of a dozen balls for $12.25. How much money would Ty Gerr save before sales tax if he bought 36 balls by the dozen instead of by the sleeve?

(A) $8.40 **(B)** $9.45 **(C)** $9.55 **(D)** $10.55 **(E)** $10.95

3. Which of the following is an equation of the line shown?

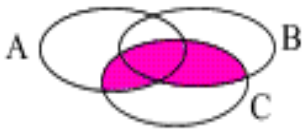


(A) $2x - 3y = -2$ **(B)** $2x + 3y = 2$ **(C)** $3x - 2y = -1$ **(D)** $3x + 2y = -1$ **(E)** $3x - 2y = 2$

4. Find the sum of the arithmetic mean, median, mode, and range of 5, 2, 6, 2, 8, 2, 0, 1, & 5.

(A) $15\frac{4}{9}$ **(B)** $15\frac{7}{8}$ **(C)** $16\frac{4}{9}$ **(D)** $18\frac{7}{8}$ **(E)** $18\frac{4}{9}$

5. The shaded region of the Venn diagram shown represents which of the following sets:



(A) $(A \cup B) \cap (C \cup B)$ **(B)** $(A \cap B) \cup C$ **(C)** $(A \cup B) \cap C$
(D) $(A \cap C) \cup B$ **(E)** $(A \cap B) \cup (C \cap B)$

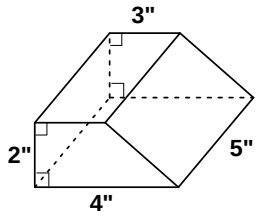
6. Simplify: $\left(\frac{4x^2 - 8x - 5}{x^3 + 2x^2 - 5x - 6}\right) \times \left(\frac{x^2 - x - 2}{5 - 2x}\right) \div \left(\frac{1 + 2x}{3 - x}\right)$

(A) $\frac{3 - x}{x + 3}$ **(B)** $\frac{2x^2 - 11x + 15}{2x^2 + x - 15}$ **(C)** $\frac{x - 3}{x + 3}$ **(D)** $x^2 - 9$ **(E)** 1

7. Ten years ago Tu Yung's father was seven times as old as she. In five years, she will be half of her father's age. What is the sum of their ages now?

(A) 62 **(B)** 56 **(C)** 48 **(D)** 44 **(E)** 40

8. Find the volume of the trapezoidal prism shown. (nearest cu. in). Drawing is not to scale.



(A) 40 cu. in (B) 24 cu. in (C) 39 cu. in (D) 45 cu. in (E) 35 cu. in

9. The ratio of the length to the width of a rectangle is 2.5:1. If 7 units are added to both the length and the width, then the ratio of the length to the width is now 4:3. What is the difference in the perimeters of the two rectangles? (units)

(A) 56 (B) 49 (C) 28 (D) 21 (E) 14

10. The *Ice T* hockey team has 8 forwards, 9 defensemen, and 3 goaltenders. How many 6-member squads can be formed if each team needs 3 forwards, 2 defensemen, and 1 goaltender?

(A) 7,056 (B) 6,048 (C) 95 (D) 2,016 (E) 38,760

11. Simplify: $\left(a^{-5} \times b^{2}\right)^{-6} \div \left(a^{8} \times b^{-2}\right)^{5} \times a^{20} \div b^{15}$

(A) $a^{10}b^{-17}$ **(B)** $a^{-4}b^{-22}$ **(C)** a^{29} **(D)** $a^{10}b^{-9}$ **(E)** $a^{11}b^{5}$

12. The equation $4x^2 - 8x + k = 0$ **always has two positive roots when which of the following is true?**

(A) $0 < k < 4$ **(B)** $8 > k > 4$ **(C)** $k > -2$ **(D)** $k > 4$ **(E)** $k < 0.5$

13. If $\frac{A}{5x+2} + \frac{B}{3x-1} = \frac{41x+1}{15x^2+x-2}$**, where A and B are constants, then** $A + B$ **equals:**

(A) 4 (B) 6.9 (C) 7 (D) 11 (E) 13.8

14. Let $a_1 = 5$, $a_2 = -2$, $a_3 = 6$ **and** $a_n = (a_{n-2}) \times \left[(a_{n-3}) - (a_{n-1})\right]$ **for** $n \geq 4$**. Find** a_6**.**

(A) 312 (B) 60 (C) 1,152 (D) 2 (E) 68

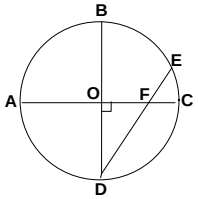
15. What are the odds of randomly selecting a number that is divisible by 3 from set of the triangular numbers less than 60?

(A) 1.5:1 (B) 3:5 (C) 3:1 (D) 5:3 (E) 6:5

16. Let $A = \begin{bmatrix} 0 & 5 \\ 2 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 0 \\ -1 & -5 \end{bmatrix}$**. Find** $\left|A^T + B\right|$**.**

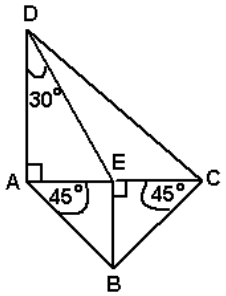
(A) -20 **(B)** -11 **(C)** -6 **(D)** -3 **(E) 0**

17. Given the circle with center O, perpendicular diameters and a chord, find the perimeter of △DFO if DE = 14 cm and DF = 10 cm. (nearest tenth)



(A) 32.4 cm (B) 29.5 cm (C) 27.8 cm (D) 23.8 cm (E) 18.7 cm

18. Find m∠DCE, nearest degree, if AD = $\sqrt{48}$ inches.



(A) 11° (B) 30° (C) 41° (D) 49° (E) 52°

19. Use the Fibonacci characteristic sequence ..., — 1, p, q, r, 4, 7, ... to find p + q + r.

(A) 5 (B) 6 (C) 9 (D) 10 (E) 16

20. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 2nd, 26th, and 50th numbers in row 26.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 1,875 (B) 1,914 (C) 1,953 (D) 1,991 (E) 2,028

21. A particle is moving along the straight line with a function of $f(t) = t^2 - t + 2$, where f(t) is the distance in meters per second. Find the instantaneous rate of change at a time of 2 seconds.

(A) 6 m (B) 5 m (C) 4 m (D) 3 m (E) 2 m

22. The directrix of the conic given by the equation $x^2 - 2y + 4x = -8$ is:

(A) y = — 1.5 (B) y = — 0.5 (C) y = 0.5 (D) y = 1.5 (E) y = 2.5

23. Let k be a positive integer less than or equal to 120 such that k is not a multiple of 2 and not a multiple of 3. How many such numbers exist?

(A) 100 (B) 80 (C) 60 (D) 40 (E) 20

24. Kandy Packer has gumballs, suckers, gumdrops, chocolate kisses, and bubble gum. She puts 6 pieces of candy in each pack to give to her students. How many different packs of candy can Kandy pack?

(A) 2,310 (B) 462 (C) 210 (D) 120 (E) 30

25. The first term of an arithmetic sequence is 2 and the common difference is 6. How many terms are in the sequence if the sum of the terms is 420.

(A) 12 (B) 18 (C) 20 (D) 24 (E) 35

26. If P, Q, and R represent digits then $RPQ_8 - QRP_4 - PQR_2$ has a numeric value in base 10 of:

(A) $3P - 9Q + 11R$ (B) $8P - 13Q + 69R$ (C) $3P - 17Q + 59R$
(D) $11P - 13Q + 61R$ (E) $5P - 10Q + 11R$

27. Lotta Dough had a bag of pennies. She gave her brother $\frac{1}{5}$ of her pennies. Then she gave her sister 20% of what she had left. Then she used her pennies to buy a 30¢ sucker. She put the remaining 50 pennies in her piggy bank. How many pennies did Lotta have in the bag, originally?

(A) 120 (B) 125 (C) 130 (D) 134 (E) 156

28. Which of the following is not a solution to $5 + |2x - 6| \leq 15$?

(A) $-1.333...$ (B) -0.7 (C) $3\sqrt{8}$ (D) $2\sqrt{7}$ (E) $\frac{50}{7}$

29. If the roots of $2x^3 + bx^2 + cx + d = 0$ are -3, 1, and 2, then $b + c + d$ equals:

(A) -2 (B) 26 (C) 0 (D) 4 (E) -13

30. Which of the following points of concurrency lies on the vertex of the right angle of a right triangle? (1) circumcenter (2) centroid (3) orthocenter (4) incenter

(A) 1 & 2 (B) 1 only (C) 3 only (D) 2, 3, & 4 (E) none of these

31. Find the shortest distance from the point (2, 8) and the line $y = 1.25 - 0.75x$.

(A) 8.6 (B) $5\frac{3}{14}$ (C) 7.4 (D) 4 (E) 6.6

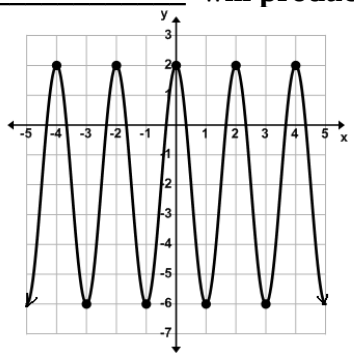
32. How many integral values of n exist such that $n > 1$ and $\frac{(n+1)!}{(n-1)!} \leq 26$?

(A) 25 (B) 13 (C) 9 (D) 3 (E) 2

33. If (1, 1) and (2, -2) are members of the function $\{(x, y) \mid y = ax - 2b\}$, then $a + b = ?$

(A) -5 (B) -4 (C) -3 (D) -2 (E) -1

34. The equation y = _____ will produce this graph.



(A) $2 - 4\cos(\pi x - 4\pi)$ **(B)** $2 + 4\cos(\pi x - 3\pi)$ **(C)** $4\sin(\pi x - 3\pi) - 2$
(D) $4\cos(\pi x - 4\pi) - 2$ **(E)** $2 - 4\sin(\pi x - 2\pi)$

35. The graph of the parametric equations $x = 2\sin^2(t)$ and $y = \sin(t)$ is a(n) _____.

(A) circle **(B) ellipse** **(C) hyperbola** **(D) cycloid** **(E) parabola**

36. In the expansion of $(5x + 2)^6$, the sum of the coefficients of the 2nd, 3rd, 4th, 5th, and 6th terms is:

(A) 117,647 **(B) 117,640** **(C) 109,804** **(D) 101,967** **(E) 101,960**

37. Simplify to the form a + bi: $(5 - 2i)(6 + i) \div (5i)$

(A) $-0.4 + 6i$ **(B)** $7 + 32i$ **(C)** $2.2 - 0.2i$ **(D)** $-5.6 - 3.4i$ **(E)** $-1.4 - 6.4i$

38. $F(x) = \frac{10}{x^2} + \frac{10}{x}$ has an inflection point at :

(A) $(-3\frac{1}{2}, -2\frac{1}{25})$ **B)** $(-3, -2\frac{2}{9})$ **(C)** $(-2, -2\frac{1}{2})$ **(D)** $(3, 4\frac{4}{9})$ **(E)** $(4, 3\frac{1}{8})$

39. Let $f(x) = 5x^2 - 2x - 6$ and $g(x) = 5x^2 + 2x - 8$. Find $f'(g'(1 + 5x))$.

(A) $500x + 82$ **(B)** $100x + 34$ **(C)** $100x + 20$ **(D)** $100x + 6$ **(E)** $500x + 118$

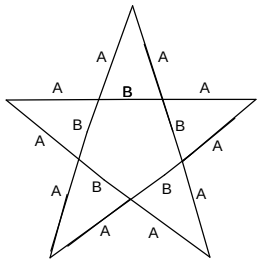
40. A star gazer is watching the night sky. The probability that he will see a satellite is 40%, the probability that he will see a shooting start is 25%, and the probability that he will see both is 15%. What is the probability that he will see either a satellite, a shooting star, or both? (nearest percent)

(A) 80% **(B) 65%** **(C) 55%** **(D) 50%** **(E) 30%**

41. Saul DeRod had 5 wooden rods with lengths of 5", 2", 6", 1" and 5". How many acute triangles can he form using only 3 rods at a time?

(A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) 5**

42. Given the pentagram shown, find the ratio of A to B. (nearest tenth)



(A) 0.5 (B) 0.6 (C) 1.6 (D) 1.9 (E) 2.0

43. The harmonic mean, nearest tenth, of the real roots of $x^3 - 13x^2 + 52x - 60 = 0$ is:

(A) 4.7 (B) 4.3 (C) 3.9 (D) 3.5 (E) 3.1

44. The graph of the polar equation $r = 7 + 4\cos(\theta)$ is a _____.

(A) dimpled limacon (B) convex limacon (C) inner loop limacon
(D) lemniscate (E) cardioid

45. Let $f(x) = \frac{5x^3 - 8}{x^2 + 3x - 1}$ and s(x) be the slant asymptote of f. Find the value of s(3).

(A) 0 (B) $7\frac{8}{17}$ (C) 17 (D) 127 (E) undefined

46. A standard deck of 52 cards is shuffled. The top 4 cards are dealt face up. What is the probability that they are all face cards (Jacks, Queens, and/or Kings)? (nearest hundredth)

(A) 0.03% (B) 0.18% (C) 0.23% (D) 0.31% (E) 0.83%

47. The square root of 11661 in base 8 is what in base 10:

(A) 77 (B) 76 (C) 73 (D) 72 (E) 71

48. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find GCD(f_m, f_n).

(A) $f_{(m+n)}$ (B) $f_m + f_n$ (C) $f_{(mn)}$ (D) $f_m \times f_n$ (E) $f_{GCD(m,n)}$

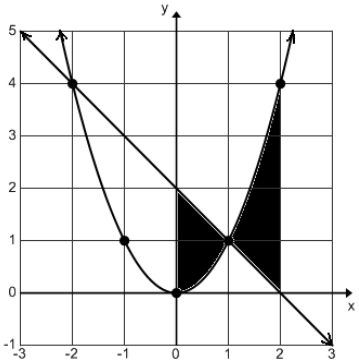
49. The number 60 is considered to be a "polite" number. The "politeness" of 60 is ____.

(A) 5 (B) 3 (C) 2 (D) 1 (E) 0

50. The *Slo-Poke* freight train leaves the station at 8:00 a.m. traveling at 35 mph. Later, the *Super-Speed* Amtrak left the same station traveling in the opposite direction at a speed of 75 mph. At 12:00 p.m. the two trains were 350 miles apart. At what time did the *Super-Speed* leave the station? (nearest minute)

(A) 8:45 a.m. (B) 9:12 a.m. (C) 9:45 a.m. (D) 10:12 a.m. (E) 10:48 a.m.

51. Find the area of the shaded regions. (square units).



(A) 4 **(B) 3.5** **(C) 3.333...** **(D) 3** **(E) 2.666...**

52. Bill Meelator borrowed $750.00 for his first semester books. Part of the loan was at the rate of 3% per year and the rest of the loan was at 8% per year. If the interest was $19.50 at the end of 6 months, how much of the loan was at 3%?

(A) $468.75 **(B) $330.00** **(C) $112.50** **(D) $281.25** **(E) $420.00**

53. Ranger Saul D. Smoke sees two fires from his ranger station. He uses a Triangulation Device to mark the point of each fire on his map. Then, using his protractor, he computes fire A to be 15 miles from his station on a bearing of 75° degrees and fire B to be 10 miles from his station on a bearing of 245°. How far apart are the two fires? (nearest mile)

(A) 20 mi **(B) 21 mi** **(C) 23 mi** **(D) 24 mi** **(E) 25 mi**

54. The Ceehahks and the Paytritts play two games during the Foosball season. The Ceehahks are one and a half times as likely to win any game as is the Paytritts. What is the probability that the Ceehahks will win both games?

(A) $55\frac{5}{9}\%$ **(B) 36%** **(C)** $44\frac{4}{9}\%$ **(D) 16%** **(E) 52%**

55. The point (— 2, 6) lies on a circle whose center is (1, 5). Which of the following points lie on the circle? P (4, 4) Q (— 2, 4) R (0, 2)

(A) P only **(B) P & Q** **(C) P & R** **(D) Q & R** **(E) P, Q, & R**

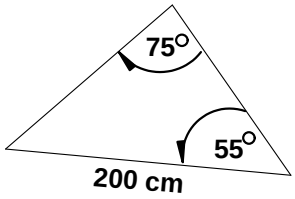
56. Let f(x) = $x^2 + bx + c$**. If f(x) is divided by x — 3 the remainder is 2 and if f(x) is divided by** $x + 2$ **the remainder is 3. Find** $b + c$**.**

(A) $-4\frac{3}{5}$ **(B)** $-3\frac{1}{2}$ **(C)** -1 **(D)** $1\frac{1}{2}$ **(E)** $2\frac{3}{5}$

57. If the two-digit number 3Q is subtracted from the two-digit number P2 the difference is 27. Find the sum of the two-digit numbers PQ and QP, where P and Q are single digits.

(A) 130 **(B) 121** **(C) 112** **(D) 111** **(E) 97**

58. Find the perimeter of the triangle shown (nearest cm).

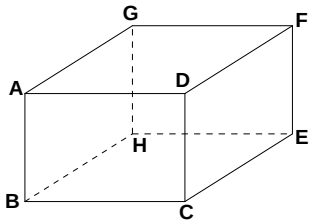


(A) 516 cm (B) 528 cm (C) 546 cm (D) 560 cm (E) 623 cm

59. Max Space needs two adjacent rectangular holding pens to separate his three cows from his bull. He has twenty 100' rolls of fencing. What is the maximum area that Max can fence in?

(A) 125,000 ft.^{2} (B) $133{,}333\frac{1}{3}$ ft.^{2} (C) $166{,}666\frac{2}{3}$ ft.^{2} (D) 175,000 ft.^{2} (E) 250,000 ft.^{2}

60. Given the rectangular solid shown, find AE if AF = y, BG = x and FH = z.



(A) $x^2 + y^2 + z^2$ **(B)** $\frac{x^2 + y^2 + z^2}{2}$ **(C)** $\sqrt{\frac{x^2 + y^2 + z^2}{2}}$

(D) $2(x^2 + y^2 + z^2)$ **(E)** $\frac{\sqrt{x + y + z}}{2}$

University Interscholastic League
MATHEMATICS CONTEST
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Answer Key

1. D	**21. D**	**41. C**
2. B	**22. D**	**42. C**
3. E	**23. D**	**43. D**
4. A	**24. C**	**44. A**
5. C	**25. A**	**45. A**
6. C	**26. C**	**46. B**
7. D	**27. B**	**47. E**
8. E	**28. C**	**48. E**
9. C	**29. A**	**49. B**
10. B	**30. C**	**50. B**
11. A	**31. E**	**51. D**
12. A	**32. D**	**52. E**
13. D	**33. A**	**53. E**
14. B	**34. D**	**54. B**
15. A	**35. E**	**55. E**
16. C	**36. E**	**56. A**
17. D	**37. E**	**57. B**
18. C	**38. B**	**58. B**
19. B	**39. E**	**59. C**
20. C	**40. D**	**60. C**