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M A T H E M A T I C S
T E S T   # 1   ©
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Academic Excellence in Mathematics and Science through Competition

GENERAL DIRECTIONS

1. About this test:
  A. You will be given 40 minutes to take this test.
  B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate: $6! \div 36^{\frac{1}{2}} - 6 \times 36^{-1} + 6 \times 36^{0}$.

A) Undefined   B) 26   C) $\dfrac{155}{6}$ D) $\dfrac{683}{6}$ E) $\dfrac{755}{6}$

2. The quantity y varies directly with the quantity x. Which of the following equations could represent the relationship?

    I. $5x + 3y = 0$ II. $2xy = 28$ III. $2x - 5y = 4$ IV. $y = \dfrac{1}{3}x$

A) I and IV   B) II only   C) I, III and IV   D) IV only   E) III and IV

3. Mr. Green's statistics class passed out a survey to 132 seniors. Forty-two students did not return the survey. Of the students who returned the survey, 72 agreed with proposition one and 39 agreed with proposition two. How many students agreed with both propositions?

A) 33   B) 60   C) 21   D) 11   E) 49

4. If $\dfrac{x^3 + 3x^2 - 16x - 48}{x^2 + 2ax - 12} = \dfrac{x^2 - x - 12}{x - 3}$, then $a =$

A) 1   B) $-\dfrac{1}{2}$ C) $\dfrac{1}{2}$ D) -1   E) $\dfrac{1}{3}$

5. Kara currently has an average of 87 in her math class. The only remaining grade is the final. What minimum grade does she need to make on the final to earn a 90 in the class if the final is 30% of her final grade?

A) 93   B) 95   C) 91   D) 97   E) 99

6. A large conical tank holds 5331 gallons of liquid. If the radius of the tank is 70 in, what is the height of the tank?

A) 20 ft.   B) 30 ft.   C) 28 ft.   D) 366 ft.   E) 22 ft.

7. The equation of a line $\overleftrightarrow{AB}$ is $y = \dfrac{2}{3}x + \dfrac{11}{3}$. The line $\overleftrightarrow{CD}$ is parallel to $\overleftrightarrow{AB}$ and includes the point $(2, -8)$. What is the x-intercept of $\overleftrightarrow{CD}$?

A) $\left(0, -\dfrac{28}{3}\right)$ B) $(14, 0)$ C) $(-6, 0)$ D) $(0, -4)$ E) $(-10, 0)$

8. D is the center of the circle shown. Given that AB = $5\sqrt{3}$ in. and AC = 10 in. calculate the area of triangle ABD.

A) $\dfrac{10\sqrt{3}}{4}$ in^2 B) $\dfrac{25\sqrt{3}}{2}$ in^2 C) $\dfrac{25\sqrt{6}}{2}$ in^2 D) $\dfrac{25\sqrt{6}}{4}$ in^2 E) $\dfrac{25\sqrt{3}}{4}$ in^2

9. A triangle has side lengths of 7 cm, 11 cm and 6 cm. Which of the following is a classification of the triangle?

A) Isosceles   B) Obtuse   C) Acute   D) Right   E) Undefined

10. x is 20% less than y and y is 75% greater than z. x is what part of z?

A) 70%   B) 15%   C) 35%   D) 140%   E) 90%

11. How many distinguishable arrangements can be made with the letters in the word PARAMETER?

A) 362880   B) 720   C) 90720   D) 366 ft.   E) 45360

12. Two standard dice are rolled. What is the probability that the difference in the numbers on the dice have a difference greater than 2?

A) $\dfrac{5}{9}$ B) $\dfrac{1}{3}$ C) $\dfrac{4}{9}$ D) $\dfrac{1}{6}$ E) $\dfrac{2}{9}$

13. A small motor boat travelling from point A to point B along the river can make the 14 mile trip in 6 hours. The return trip takes 8 hours and 24 minutes. If the speed of the boat and the current remained constant throughout both trips, what is the speed of the current?

A)   1760 ft/hr   B)   12320 ft/hr   C)   10560 ft/hr   D)   4928 ft/hr   E)   8800 ft/hr

14. If $5^x \cdot 25^{2y} = 1$ and $3^{5x} \cdot 9^y = \dfrac{1}{9}$ then $x + y =$

A) $-\dfrac{1}{3}$ B) $-\dfrac{4}{9}$ C) $\dfrac{1}{9}$ D) $\dfrac{1}{3}$ E) $-\dfrac{1}{9}$

15. If the set of even numbers continues as shown, the sum of the digits in the 9$^{\text{th}}$ row would be:

A)   738   B)   108   C)   90   D)   864   E)   990

```
  2    4
  6    8    10
 12   14   16   18
 20   22   24   26   28
 30   32   34   36   38   40
```

16. The length of one side of a regular dodecagon is 13.8 cm. Find the length of the apothem (nearest mm.).

A)   258 mm   B)   120 mm   C)   267 mm   D)   71 mm   E)   81 mm

17. Working alone, Jim and Joe can plaster a wall in 22 min and 32 min respectively. How fast can they plaster a wall twice as high and twice as long together? (nearest minute)
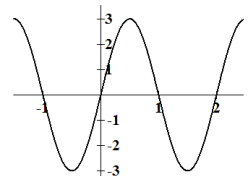
A)   52 min   B)   13 min   C)   26 min   D)   104 min   E)   54 min

18. Craig is building a ramp up to a porch which sits 4 ft. and 3 in. off the ground. If the planks he is using are 12 ft. long, calculate the angle of elevation. (nearest degree)

A)   69°   B)   30°   C)   21°   D)   63°   E)   27°

19. If the equation of the function graphed below is $y = a\sin bx + c$, find the value of $a \cdot b \cdot c$.

A) $-\dfrac{\pi}{3}$ B) -3π C)   0   D) $\dfrac{\pi}{3}$ E) 3π

20. $0.242424..._6 =$ _____$_{10}$.

A) $\dfrac{24}{35}$ B) $\dfrac{8}{33}$ C) $\dfrac{12}{35}$ D) $\dfrac{4}{15}$ E) $\dfrac{16}{35}$

21. $\left(2 + \sqrt{-27}\right)\left(7 - \sqrt{-75}\right) =$

A) $14 + 45i$ B) $59 + 11\sqrt{3}i$ C) $14 - 45i$ D) $59 - 11\sqrt{3}i$ E)   undefined

22. What is $\displaystyle\sum_{k=-2}^{0}(kx+2)^2$?

A) $4x^2 + 8$ B) $5x^2 - 12x + 12$ C) $4x^2 - 12x + 8$ D) $5x^2 + 12$ E) $5x^2 - 8x + 12$

23. Find the median of the numbers in the 12$^{\text{th}}$ row of Pascal's triangle.

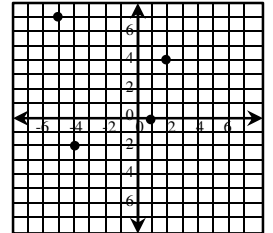A)   924   B)   495   C)   220   D)   143   E)   358

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24. Two events A and B are such that $p(A)=0.2$, $p(A'\cap B)=0.22$ and $p(A\cap B)=0.18$. Evaluate $p(A|B)$.

A) $\dfrac{9}{20}$

B) $\dfrac{9}{11}$

C) $\dfrac{1}{11}$

D) $\dfrac{3}{7}$

E) $\dfrac{10}{21}$

25. If $\displaystyle\int_{2}^{k}\dfrac{1}{x+8}dx=\ln 2$, find the value of k.

A) 4

B) 8

C) -4

D) 12

E) 0

26. If the points on the coordinate plane are joined in a clockwise direction beginning in the third quadrant they form a quadrilateral. What is the area of the quadrilateral?

A) 78

B) 10

C) 39

D) 20

E) 54

27. The ratio of the width to the length of a rectangle is 5:7. The perimeter is 124.8 cm. What is the area of the rectangle?

A) 35 cm^2

B) 3785.6 cm^2

C) 946.4 cm^2

D) 876.6 cm^2

E) 182 cm^2

28. $11_2 + 123_4 + 66_8 = $ _____$_{10}$

A) 200

B) 84

C) 1110

D) 124

E) 68

29. What is the sum of the coefficients of the expansion of $(x-3)^4$?

A) -2

B) -108

C) 81

D) -12

E) 16

30. Triangle PQR is such that $m\angle R = 60°$, $PR = 8$ and $PQ = 7$. There are two possible values of QR. Find the sum of the two values.

A) 8

B) 7

C) 5

D) 15

E) 12

31. A circle circumscribed about a triangle. The center of the circle is the ____ of the triangle.

A) Centroid

B) Incenter

C) Foci

D) Orthocenter

E) circumcenter

32. Use the Fibonacci type sequence $a,b,-1,c,-5$ to find the value of $a+b+c$.

A) -3

B) -7

C) 7

D) -5

E) 4

33. If $a-b=7$ and $ab=-4$ then $a^3-b^3 = $?

A) 427

B) 259

C) 315

D) 448

E) 592

34. There are seven boys and eight girls on the student council. Their sponsor is asked to select three boys and three girls to attend an elementary school pep rally. How many different groups could the sponsor choose?

A) 91

B) 70560

C) 546

D) 448

E) 1960

35. Given $a_{n+2} = a_n(a_{n+1})$, $a_1 = 3$ and $a_2 = -4$ find a_6.

A) 27648

B) -576

C) 15925248

D) -27648

E) 576

36. What is the reference angle for 2015°?

A) 125°

B) 35°

C) -145°

D) 215°

E) 55°

37. Find the slope of the tangent line to the function $f(x)=5x^3-2x^2+11x-1$ when $x=-\dfrac{3}{2}$.

A) $-\dfrac{311}{8}$

B) $\dfrac{203}{4}$

C) $\dfrac{25}{8}$

D) $\dfrac{67}{4}$

E) $\dfrac{115}{4}$

38. What is the area of a sector with a central angle of $\dfrac{3}{8}\pi$ in a circle with a diameter of 22 cm? (nearest cm^2)

A)   286 cm^2 B)   91 cm^2 C)   23 cm^2 D)   191 cm^2 E)   71 cm^2

39. Let $\|v_1\|=7$ and $\|v_2\|=24$, when the direction angles of v_1 and v_2 are 36° and 77° respectively. Find the direction angle of $\|v_1+v_2\|$. (nearest degree)
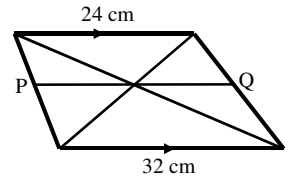
A)   22°   B)   113°   C)   41°   D)   66°   E)   68°

40. The school day at Austin Elementary School begins at 8:25 am and ends at 3:20 pm. How many degrees has the hour hand moved during the school day?

A)   207.5°   B)   150°   C)   152.5°   D)   210°   E)   187.5°

41. Find PQ.

A) $\dfrac{192}{7}$ cm   B) $\dfrac{56+16\sqrt{3}}{3}$ cm   C)   28 cm   D) $\dfrac{200}{7}$ cm   E) $16\sqrt{3}$ cm

42. Given $f(x)=2x+5$ and $g(x)=x^2-1$ find $g(f(x))$.

A) $4x^2+24$ B) $2x^2+4$ C) $2x^2+3$ D) $4x^2+20x+24$ E) $4x^3+10x^2+6x+15$

43. Given $y=-2\cos x$, find the value of x for which $\dfrac{dy}{dx}=\dfrac{dx}{dy}$, where $0\le x\le\dfrac{\pi}{2}$.

A)   0   B) $\dfrac{\pi}{4}$ C) $\dfrac{\pi}{3}$ D) π E) $\dfrac{\pi}{6}$

44. A fair 5-sided die with sides labeled 1, 1, 5, 8, 9 is rolled and the roll is recorded. What is the expected value of the roll?

A)   5   B)   5.2   C)   4.6   D)   4.8   E)   4.5

45. $\dfrac{3x+5}{(x+3)(x-1)}=\dfrac{A}{x+3}+\dfrac{B}{x-1}$. Calculate $A+B$.

A)   8   B)   5   C)   3   D)   1   E)   2

46. What is the 6th harmonic number?

A)   2.45   B)   0.408   C) $2.28\overline{3}$ D)   0.438   E)   3.5

47. What are the coordinates of the relative maximum of the function $f(x)=1-3x+x^3$?
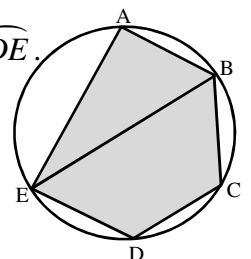
A) $(1,-3)$ B) $(-1,1)$ C) $(1,-1)$ D) $(1,0)$ E) $(-1,3)$

48. Given $\det\begin{bmatrix} 2 & -1 & 3 \\ 7 & k & 1 \\ -2 & 1 & 5 \end{bmatrix}=120$ find the value of k.

A)   4   B)   8.625   C)   8.375   D)   6   E)   -4

49. On the picture shown right, $\overline{EB}$ is a diameter, $AE=30$ cm, $AB=16$ cm and $\overset{\frown}{BC}\cong\overset{\frown}{CD}\cong\overset{\frown}{DE}$. The area of the shaded region is _____ cm^2. (nearest whole number)

A)   548   B)   615   C)   751   D)   741   E)   365

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50. What is the range of the function $f(x) = -3\tan\left[\frac{1}{2}(x-6)\right] + 7$.

A) $(-\infty, \infty)$ B) $[4,10]$ C) $[-4,10]$ D) $[-10,4]$ E) $[-3,3]$

51. Find the value of C if the remainder of $2x^5 + 3x^3 + Cx^2 + 8$ divided by $x+2$ is -96.

A) 2   B) -4   C) -48   D) -64   E) 32

52. If $f(x) = \frac{x^2+x}{x-3}$ then $f''(-2) =$

A) $\frac{244}{125}$ B) $-\frac{12}{125}$ C) 0   D) 2   E) $-\frac{124}{125}$

53. A ship is initially at a position A and travels 6 km on a bearing 55° followed by 8 km on a bearing of 150° to reach a final position B. Find the bearing of A to B. (nearest degree)

A) 143°   B) 95°   C) 111°   D) 205°   E) 127°

54. Express the polar equation $r = 3$ in Cartesian form.

A) $x^2 - y^2 = 3$ B) $x^2 - y^2 = 9$ C) $x^2 + y^2 = \sqrt{3}$ D) $x^2 + y^2 = 9$ E) $x^2 + y^2 = 3$

55. The function f is such that $\int_{-1}^{8} f(x)dx = 7$. What is the value of $\int_{-1}^{8}(f(x)+3)dx$?

A) 34   B) 10   C) 28   D) 27   E) 32

56. The point $(2,7)$ is reflected over the x-axis, reflected over the line $y = x$, rotated 180° clockwise around the origin, then shifted down three units to the point (a,b). Find $a+b$.

A) 2   B) 6   C) 1   D) -3   E) 4

57. How many solutions are there to $6x + 8y = 212$ such that $x, y \in \mathbb{Z}^+$.

A) 8   B) 27   C) 7   D) 9   E) 36

58. The volume of a spherical balloon is increasing at a rate of 415 cm³ per second. How fast is the radius changing when the radius is 6 cm? (nearest thousandth)

A) 2.882 cms⁻¹   B) 0.459 cms⁻¹   C) 2.459 cms⁻¹   D) 0.917 cms⁻¹   E) 1.670 cms⁻¹

59. Two numbers are in a ratio of 7 to 9. If the lesser number is reduced by 36 and the larger number is reduced by 27, the resulting ratio is 1 to 2, what is the product of the numbers?

A) 972   B) 1215   C) 5103   D) 3281   E) 1458

60. Six circles are tangent to each other and an equilateral triangle is inscribed around them as shown. What percent of the area of the triangle is not shaded?

A) 78.13%   B) 31.98%   C) 21.87%   D) 40.00%   E) 40.31%
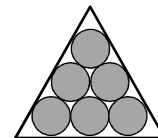
2014-2015 TMSCA Mathematics Test One Answers

1. E	21. B	41. A
2. A	22. B	42. D
3. C	23. C	43. E
4. C	24. A	44. D
5. D	25. D	45. C
6. A	26. C	46. A
7. B	27. C	47. E
8. E	28. B	48. A
9. B	29. E	49. B
10. D	30. A	50. A
11. E	31. E	51. B
12. B	32. D	52. E
13. A	33. B	53. C
14. A	34. E	54. D
15. E	35. D	55. A
16. A	36. B	56. A
17. A	37. B	57. D
18. C	38. E	58. D
19. C	39. E	59. C
20. E	40. A	60. C

2013-2014 TMSCA Mathematics Test One Select Solutions

8. AB and AC indicate that triangle ABC is a 30-60-90. The altitude from B to $\overline{AC}$ has a length of $\frac{5\sqrt{3}}{2}$, so the area of triangle ABC is $\frac{1}{2}(5)\left(\frac{5\sqrt{3}}{2}\right) = \frac{25\sqrt{3}}{4}$

11. There are 9 letters and three of them appear twice, so the number of distinct arrangements is $\frac{9!}{(2!)(2!)(2!)}$.

14. the original two equations can be rewritten as $5^x \cdot 5^{4y} = 5^0$ and $3^{5x} \cdot 3^{2y} = 3^{-2}$. These can be used to generate the system of equations $x + 4y = 0$ and $5x + 2y = -2$. Addition yields $6x + 6y = -2$, so $x + y = -\frac{1}{3}$

17. Combined rate for Jim and Joe is $\left(\frac{1}{22} + \frac{1}{32}\right)$. The new wall has an area 4 times greater than the original, so the time is the solution to the equation $\left(\frac{1}{22} + \frac{1}{32}\right)t = 4$ so $t \approx 52$ minutes

20. still in base 6,
$$100n = 24.\overline{24}$$
$$n = 0.\overline{24}$$
$$55n = 24$$
to finish $\frac{24}{55_6} = \frac{16}{35_{10}}$.

23. There are 13 numbers in the 12th row, but all appear twice except 924. If the numbers are arranged in order, the middle number is 220.

24. Using the diagram, $p(A|B) = \frac{0.18}{0.40} = \frac{9}{20}$

U
A   B
0.02 0.18 0.22
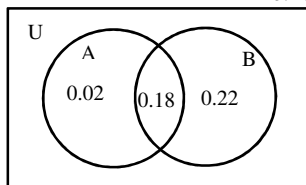
25. $\ln(x+8)\big]_2^k = \ln\left(\frac{k+8}{2}\right) = \ln 2$ so $\frac{k+8}{2} = 2$ and $k = 4$

30. Use law of cosines to set up the quadratic $7^2 = 8^2 + x^2 - 2(x)(8)\cos 60$. This can be simplified and rearranged to form $0 = x^2 + 8x + 15$. The sum of the roots is 8.

33. $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)((a-b)^2 + 3ab) = 7(49 + 3(-41)) = 259$

38. If the angle measure is radians, then the area of the sector is $\frac{r^2\theta}{2} = 121\left(\frac{1}{2}\right)\left(\frac{3\pi}{8}\right) \approx 71$

39. $\arctan\left(\frac{7\sin 36 + 24\sin 77}{7\cos 36 + 24\cos 77}\right) \approx 68°$

43. $\frac{dy}{dx} = \frac{dx}{dy}$ when $\left(\frac{dy}{dx}\right)^2 = 1$, $\left(\frac{dy}{dx}\right)^2 = 4\sin^2 x = 1$ when $\sin x = \pm\frac{1}{2}$ so $x = \frac{\pi}{6}$

45. $3x + 5 = A(x-1) + B(x+3)$ so $A + B = 3$

49. The triangle AEB is a right triangle because it is inscribed in a semicircle, and the trapezoid EDCB can be divided into three equilateral triangles because the central angle for each of the three arcs is 60°. The area of the whole figure is $\frac{1}{2}(30)(16) + 3\left(\frac{17^2\sqrt{3}}{4}\right) \approx 615$

51. The root associated with the factor is -2 and $f(-2) = -64 - 24 + 4C + 8 = 96$. Solving the equation yields $C = -4$.

52. $r = 3$ is the polar equation for a circle with center $(0,0)$ and a radius of 3. The Cartesian equation for the circle is $x^2 + y^2 = 9$.

55. $f(x) + 3$ is just a vertical translation of the function, so the integral is the original plus $3(\Delta x)$. $\int_{-1}^{8}(f(x)+3)dx = 7 + 3(9)$.

60. If the radius of each circle is 1, then the base of the triangle has a length of $s = 4 + 2\sqrt{3}$ and the area of the triangle is $A = \frac{s^2\sqrt{3}}{4}$. So the percent that is not shaded is $\frac{A - 6\pi}{A} \approx 21.87\%$