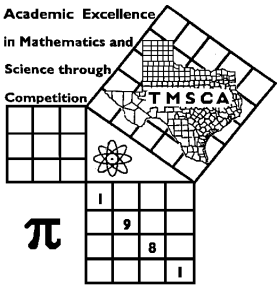

TMSCA HIGH SCHOOL MATHEMATICS TEST# 10©

FEBRUARY 4, 2017

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2016-2017 TMSCA Mathematics Test Ten

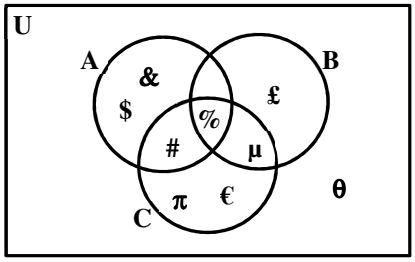
1. **Evaluate $\left(5!+2^2\times 3\right)\div\left(11\div 5\right)+2$.**

   (A) 4.4 (B) 39.4 (C) 10.25 (D) 59.8 (E) 62

2. **Jennifer borrowed \$1,250 at 6.8% annual simple interest for 24 months. What will her monthly payments be?**

   (A) \$55.63 (B) \$59.17 (C) \$59.41 (D) \$55.63 (E) \$54.87

3. **Use the Venn diagram to identify the set $\left(A\cup B\right)\cap C$.**



   (A) {θ, π, €} (B) {#, %, μ} (C) {$, &, £} (D) {π, €} (E) {$, &, £, θ}

4. **The line $ax+7y=13$ is perpendicular to the line $25x+28y=32$. Find the value of *a*.**

   (A) $-\frac{196}{25}$ (B) $\frac{25}{4}$ (C) $-\frac{25}{4}$ (D) $\frac{4}{25}$ (E) $-\frac{147}{4}$

5. **The binomials $\left(3x+5\right)$, $\left(x-8\right)$ and $\left(3x-5\right)$ are all factors of**

   (A) $9x^3-72x^2-25x+100$ (B) $9x^3-97x^2+200$ (C) $9x^3-72x^2-25x+200$

   (D) $9x^3-72x^2-25x-200$ (E) none of these

6. **Find the greatest common divisor of $2^5\times 3^2\times 5^3$, $2^3\times 3^3\times 5^4$ and $2^2\times 3^5\times 5^2$.**

   (A) 180 (B) 450 (C) 90 (D) 900 (E) 150

7. **Susan plans on buying 5 shirts for \$27.95 each, 2 skirts for \$32.99 each and a pair of shoes for \$42.95. If the local tax rate is 8.75%, how much money will Susan save by making her purchases on tax-free weekend.**

   (A) \$21.76 (B) \$19.98 (C) \$8.57 (D) \$19.04 (E) \$18.21

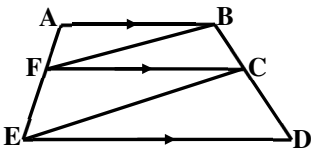
8. **What is the sum of the arithmetic sequence 9.8, 11, 12.2, 13.4, …, 25.4?**

   (A) 228.2 (B) 264.6 (C) 254.2 (D) 246.4 (E) 262

9. **A box contains 4 black marbles, 7 red marbles and 8 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 black marbles?**

   (A) 4:969 (B) 55:768 (C) 32:2875 (D) 64:6795 (E) 4:965

10. **On the diagram, FC is the geometric mean of AB and ED. AF = 7.5 cm, FE = 10.5 cm and EC = 19.8 cm. Find FB. (nearest tenth)**



   (A) 17.7 (B) 13.9 (C) 18.2

   (D) 15.2 (E) 14.1

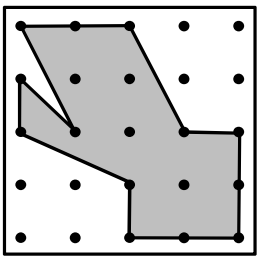
11. Simplify: $\left(\frac{x^2+x-56}{x^2+6x-16}\right)\div\left(\frac{x^2+4x-21}{x^2+12x+35}\right)$.

(A) $\frac{x^2-2x-35}{x^2-5x+6}$ **(B)** $\frac{x^2+2x-35}{x^2-x+6}$ **(C)** $\frac{x^2-2x-35}{x^2-x+6}$ **(D)** $\frac{x^2+2x-35}{x^2-5x+6}$ **(E)** $\frac{x^2-2x-35}{x^2+x+6}$

12. Events A and B are independent such that $P(A)=6P(B)$ **and** $P(A\cup B)=0.915$**. Find** $P(B)$**.**

(A) 0.75 **(B)** 0.20 **(C)** 0.15 **(D)** 0.125 **(E)** 0.375

13. A rubber band was stretched on the geoboard to form this 10-sided figure. What is the area?



(A) 8.5 units^2 **(B)** 8 units^2 **(C)** 9 units^2

(D) 9.5 units^2 **(E)** 10.5units^2

14. $-2(8+12)=-16-24$ **and** $(15-6)\div 3=5-2$ **are examples of _____property of equality.**

(A) Distributive **(B)** Commutative **(C)** Associative **(D)** Transitive **(E)** Identity

15. The graph of $y=6-5\sin(3x-105^\circ)$ **reaches a maximum value at:**

(A) $(-65^\circ,1)$ **(B)** $(65^\circ,11)$ **(C)** $(0^\circ,11)$ **(D)** $(-115^\circ,11)$ **(E)** $(65^\circ,1)$

16. Let the "1" at the top of Pascal's triangle be row 0. Determine the third number in row 27.

(A) 378 **(B)** 325 **(C)** 338 **(D)** 351 **(E)** 343

17. Let $f(x)=\frac{x+4}{2x-7}$**, where** $x\neq 7$**. Find** $f^{-1}(x)$**.**

(A) $\frac{7x+4}{2x-1}$ **(B)** $\frac{2x-7}{x+4}$ **(C)** $\frac{9}{x-1}$ **(D)** $\frac{7x+4}{2x+1}$ **(E)** $\frac{4x-7}{2x+1}$

18. Given that $x-y=-10$ **and** $xy=28$**, find** x^3-y^3**.**

(A) −1560 **(B)** −1840 **(C)** −720 **(D)** −1280 **(E)** −440

19. How many distinct 4-letter arrangements can be made from the letters in "ROUND ROCK"?

(A) 1320 **(B)** 840 **(C)** 3024 **(D)** 1050 **(E)** 1306

20. Find $\lim_{x\to 3/2}\frac{2x^2-x-3}{2x^2-9x+9}$

(A) 0 **(B)** $\frac{1}{9}$ **(C)** $-\frac{5}{3}$ **(D)** $-\frac{5}{6}$ **(E)** does not exist

21. Let $f_0=0, f_1=1, f_2=1, f_3=2, f_4=3,\ldots$ **be the terms of the Fibonacci sequence. Find** f_{32}**.**

(A) 1,346,269 **(B)** 2,178,309 **(C)** 3,524,578 **(D)** 2,787,840 **(E)** 3,256,209

22. The fraction 0.656565... in base 8 can be written as which of the following fractions in base 8?

(A) $\frac{53}{63}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{32}{33}$ **(D)** $\frac{65}{77}$ **(E)** $\frac{31}{33}$

23. If the pattern of the sequence 15, 55, 123, 225, 367, 555, 795, … continues, find the 20th term.

(A) 12,975 **(B)** 9,915 **(C)** 11,377 **(D)** 6968 **(E)** 7704

24. If $\left[(5+3i)(2-3i)\right]\div(2-i)=a+bi$ **, then** $a+b=$ **?**

(A) 9.6 **(B)** −1.6 **(C)** 9.4 **(D)** 0.2 **(E)** 9.2

25. The population standard deviation of the set of numbers $\{4,6,6,7,9,9,14,15,17\}$ **is___. (nearest tenth)**

(A) 4.3 **(B)** 4.6 **(C)** 5.0 **(D)** 4.7 **(E)** 4.8

26. If f **is continuous on the closed interval** $[a,b]$ **and** k **is any number between** $f(a)$ **and** $f(b)$**, then there is at least one number** c **in** $[a,b]$ **such that** $f(c)=k$**. This is the _____.**

(A) Rolle's Theorem **(B)** Sandwich Theorem **(C)** Fundamental Theorem of Calculus

(D) Intermediate Value Theorem **(E)** Fundamental Theorem of Algebra

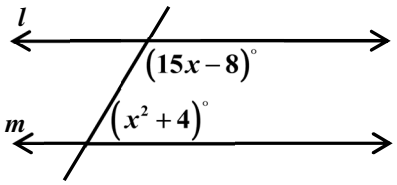
27. The ellipse $25x^2+49y^2+150x-196y=804$ **has area _____square units. (nearest square unit)**

(A) 141 **(B)** 126 **(C)** 154 **(D)** 113 **(E)** 110

28. In the diagram, $l \parallel m$ **. Find the value of** x**.**

(A) 5 **(B)** 7 **(C)** 8

(D) 4 **(E)** 3



29. The length of one edge of a regular tetrahedron is $8\sqrt{3}$ **cm. The surface area of the tetrahedron is _____cm². (nearest square centimeter)**

(A) 1330 **(B)** 272 **(C)** 303 **(D)** 208 **(E)** 333

30. P and Q are the roots of $f(x)=2x^2+13x-7$**. Calculate** $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$**.**

(A) $-\frac{28561}{16}$ **(B)** $\frac{50,625}{16}$ **(C)** $\frac{83521}{16}$ **(D)** $-\frac{50,625}{16}$ **(E)** $\frac{28561}{16}$

31. How many positive 3-digit numbers exist such that the sum of their digits equals 15?

(A) 67 **(B)** 69 **(C)** 74 **(D)** 56 **(E)** 63

32. Let $f(x)=3x^2-5x+2$**,** $g(x)=x+5$ **and** $s(x)$ **be the slant asymptote of** $\frac{f(x)}{g(x)}$ **. Find** $s(-2)$**.**

(A) −26 **(B)** −14 **(C)** 24 **(D)** −16 **(E)** 14

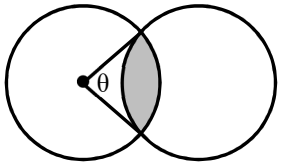
33. In a survey of 75 students, 33 listened to country music, 45 listened to pop music and 18 listened to both. How many students listened to neither?

(A) 22 **(B)** 16 **(C)** 19 **(D)** 21 **(E)** 15

34. Blackbeard sailed from his hideout at bearing 275° for 100 miles to island A, then on to island B at a bearing of 122° for 144 miles. How far would Blackbeard sail to go directly back to his hideout? (nearest mile)

(A) 219 mi (B) 145 mi (C) 165 mi (D) 71 mi (E) 102 mi

35. The illustration shows two congruent circles each with a radius of 48 cm and $\theta = 1.5$ radians. Find the area of the shaded region. (nearest square centimeter)



(A) 579 cm^2 (B) 869 cm^2 (C) 290 cm^2 (D) 1158 cm^2 (E) 886 cm^2

36. The point of concurrency of the altitudes of a triangle is called the_____.

(A) Incenter (B) Centroid (C) Orthocenter (D) Circumcenter (E) Euler Line

37. If the pattern of the sequence 5, 9, 16, 26, 39, 55, …continues, find the 30th term.

(A) 1339 (B) 1251 (C) 1430 (D) 1166 (E) 1298

38. The Real value solution set of $|2x+4|+5<9$ is

(A) $\{x|\{x<-4\}\cup\{x>0\}\}$ **(B)** $\{x|x<0\}$ **(C)** $\{x|0<x<4\}$

(D) $\{x|\{x<0\}\cup\{x>4\}\}$ **(E)** $\{x|-4<x<0\}$

39. Calculate the angle between the vectors $v_1=\langle -27,15\rangle$ and $v_2=\langle 18,3\rangle$. (nearest degree)

(A) 129° (B) 42° (C) 141° (D) 122° (E) 36°

40. Use the function below to find $f(2)+f(-4)+f(8)$.

(A) 11 (B) 9 (C) 13

(D) 17 (E) 27

$$f(x)=\begin{cases} x-3, & x<0 \\ (2x)^2, & 0\le x\le 4 \\ \frac{x}{2}, & x>4 \end{cases}$$

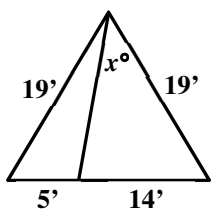
41. What is the constant term in the expansion of $\left(3x^2-\frac{1}{x}\right)^6$?

(A) −9 (B) 135 (C) 15 (D) 405 (E) −45

42. Find the value of x in the triangle diagram. (nearest degree)

(A) 40° (B) 43° (C) 44°

(D) 45° (E) 48°



43. The function $f(x) = 8x^3 - 28x^2 - 18x + 63$ is concave up at which of the following values of x.

(A) -1 (B) -1.5 (C) 0.9 (D) 1.75 (E) 1

44. Solve $\log_a(x) + \log_a(x+5) = \log_a(24)$.

(A) 3 (B) $-3, 5$ (C) -3 (D) 5 (E) $-5, 3$

45. The polar coordinates of point P are $\left(-8, \frac{5\pi}{3}\right)$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian plane?

(A) QI (B) QII (C) QIII (D) QIV (E) y-axis

46. $f(x) = ax^5 + bx^3 + cx + 17$. If $f(-5) = 37$ then $f(5) =$

(A) -14 (B) -37 (C) -3 (D) 37 (E) 17

47. $(-8,-20)$ and $(8,12)$ are opposite vertices of a parallelogram. If $(4,-3)$ is the third vertex, then the fourth vertex is:

(A) $(-4,-3)$ (B) $(0,-8)$ (C) $(-4,-5)$ (D) $(0,-4)$ (E) $(-3,-4)$

48. The reaction time of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds. What is the probability that the reaction time of a person chosen at random will be less than 0.67 seconds? (nearest hundredth)

(A) 0.07 (B) 0.43 (C) 1.00 (D) 0.93 (E) 0.57

49. Which of the following statements about $f(x) = \sqrt{x-9}$ is/are true?

I. $f(x)$ is a one-to-one function on its domain
II. $f(x)$ has an inverse function on its domain
III. $f(x)$ has a one-to-one inverse function on its domain

(A) I only (B) I, II & III (C) I & II (D) I & III (E) none of these

50. $\sum_{k=0}^{12}\left[2k(k-2)\right] =$

(A) 650 (B) 988 (C) 338 (D) 676 (E) 884

51. Meredith has eleven school books to fit on her shelf. Four of these books are math books and six of the books are language arts. In how many distinct ways can Meredith arrange the books in a single row on her shelf if she keeps all the math books together and all of the language arts books together?

(A) 103,680 (B) 1,244,160 (C) 414,720 (D) 207,360 (E) 34,560

52. What is the distance between the point $(-7,8)$ and the line $12x+5y=27$? (nearest tenth)

(A) 3.8 (B) 2.7 (C) 1.3 (D) 1.8 (E) 5.5

53. $14641_b =$ _____$_{10}$ when $b>6$.

(A) $b^3+4b^2+6b+4+\frac{1}{b}$ **(B)** $(b+1)^4$ **(C)** $(b^2+1)^2$

(D) $b(b+1)(b+2)(b+3)$ **(E)** $(b^2+1)(b^2-1)$

54. The length of the sides of triangle PQR are the roots of $f(x)=x^3-14x^2+62x-88$. Find the area of triangle PQR. (nearest tenth unit)

(A) 8.5 (B) 9.2 (C) 3.7 (D) 6.0 (E) 4.6

55. How many solutions are there to $5x+8y=432$ such that $x, y \in \mathbb{Z}^+$

(A) 8 (B) 7 (C) 10 (D) 9 (E) 11

56. Find the slope of the curve $x^2+2y^3=17$ at the point $(-1,2)$.

(A) $\frac{1}{12}$ **(B)** $\frac{19}{12}$ **(C)** -12 **(D)** $-\frac{12}{19}$ **(E)** $-\frac{1}{12}$

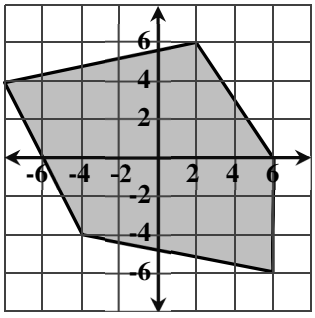
57. Let $\begin{bmatrix} a & 5 \\ 1 & b \end{bmatrix} \times \begin{bmatrix} 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 54 \\ -9 \end{bmatrix}$. $a+b=$

(A) 4 (B) –4 (C) 7 (D) –3 (E) 0

58. The coordinates of the vertices of the figure on the coordinate plane below are all integers. The area of the shaded region in _____square units.

(A) 96 (B) 112 (C) 118

(D) 107 (E) 113



59. What is the digit in the ten-thousandth place of sum $3^2-\frac{3^6}{3!}+\frac{3^{10}}{5!}-\frac{3^{14}}{7!}+\frac{3^{18}}{9!}-\ldots$.

(A) 3 (B) 4 (C) 1 (D) 6 (E) 9

60. Let $f(x)=\frac{x}{3}$ **and** $g(x)=5x+1$. **Find** $f(g(x))+g(f(-x))$.

(A) $\frac{8-10x}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{2-10x}{3}$ **(D)** $\frac{8}{3}$ **(E)** $\frac{2}{3}$

Test Ten Answer Key

1. E	21. B	41. B
2. B	22. D	42. D
3. B	23. C	43. D
4. A	24. A	44. A
5. C	25. A	45. B
6. D	26. D	46. C
7. A	27. E	47. C
8. D	28. C	48. A
9. E	29. E	49. B
10. E	30. B	50. B
11. A	31. B	51. A
12. C	32. A	52. E
13. D	33. E	53. B
14. A	34. D	54. E
15. D	35. D	55. C
16. D	36. C	56. A
17. A	37. A	57. E
18. B	38. E	58. B
19. E	39. C	59. C
20. C	40. C	60. B

Test Ten Select Solutions

10. The parallel inside a trapezoid that is the geometric mean of the two bases divides the trapezoid into two similar trapezoids, so $\frac{7.5}{10.5}=\frac{x}{19.8}$ **for** $x\approx 14.1$ **cm.**

12. Use $p(A\cup B)=p(A)+p(B)-p(A)\times p(B)$ **for independent events and solve for** $0.915=6p(B)+p(B)-6p(B)\times p(B)$ **and** $p(B)=0.15$.

13. $A=\frac{2I+P}{2}-1=\frac{8+13}{2}-1=9.5$

16. The third number in the row is the (n-1)th triangular number, so $\frac{(26)(27)}{2}=351$

18.

$$x^3-y^3=(x-y)\left(x^2+xy+y^2\right)=(x-y)\left[(x-y)^2+3xy\right]=$$

$$-10\left[(-10)^2+3(28)\right]=-1840$$

19. There are 7 distinct letters and two pairs of repeating. Distinct arrangements of four with

No repeats: ${}_7P_4=840$

1 Repeat: $(2)({}_6C_2)\left(\frac{4!}{2!}\right)=360$

2 Repeats: $\frac{4!}{(2!)(2!)}=6$

For a total of 1206.

23. A cubic regression is a perfect fit for this data with the 20th term being 11,377.

30. The desired value is $(P-Q)^4$ **for** $\left(\frac{1}{2}-7\right)^4=\frac{28561}{16}$.

35. $2\left(\frac{48^2\times 1.5}{2}-\frac{48^2}{2}\sin 1.5\right)\approx 1158$.

39. $\cos\theta=\frac{-27(18)+15(3)}{\sqrt{(-27)^2+(15)^2}\times\sqrt{(18)^2+(3)^2}}$ **for 141°.**

41. ${}_6C_2\left(3x^2\right)^2\left(-\frac{1}{x}\right)^4=135$.

42. The larger angle is equilateral. To find the center line, use the law of cosines: $\sqrt{14^2+19^2-2(14)(19)\cos 60}\approx 17.1$ **then** $\frac{\sin 60}{17.1}=\frac{\sin x}{14}$ **for** $x\approx 45^\circ$.

46. Without the 17, $f(x)$ **would be an odd function, so** $37=C+17$ **for** $C=20$. **Then** $f(5)=-20+17=-3$.

50. $2\left[\frac{12(12+1)(2\times 12+1)}{6}\right]-4\left[\frac{12(12+1)}{2}\right]=988$

52. $\frac{|-7(12)+8(5)-27|}{\sqrt{12^2+5^2}}\approx 5.5$

54. $\sqrt{7\times f(7)}\approx 4.6$

59. This is the MacClaurin series expansion of $\sin\left(3^2\right)$ **for a ten-thousandth place digit of 1.**