T M S C A   H I G H   S C H O O L
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Academic Excellence
in Mathematics and
Science through
Competition

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GENERAL DIRECTIONS

1. About this test:
  A. You will be given 40 minutes to take this test.
  B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate $\dfrac{11!}{8!} \bullet \dfrac{18!}{9!} \div \dfrac{16!}{7!}$.

A) 14960   B) $\dfrac{495}{2}$ C) $\dfrac{8415}{2}$ D) 33660   E) 3740

2. The length of the second leg of a right triangle is 1 cm more than the length of the first leg, and the hypotenuse is 11 cm less than twice the length of the first leg. Find the perimeter of the triangle.
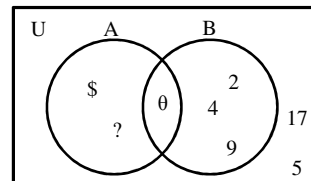
A) 20 cm   B) 210 cm   C) 24 cm   D) 120 cm   E) 70 cm

3. Mrs. Jones is fifteen years older than twice the age of her son. Twenty years ago, her age was nine times that of her son. Find her age.

A) 75   B) 65   C) 50   D) 25   E) 20

4. Use the Venn diagram to determine the set $A' \cup B$.

(A) $\{\theta\}$ (C) $\{2,4,5,9,17\}$ (E) $\{\$,?,\theta,2,4,9\}$

(B) $\{2,4,9\}$ (D) $\{\theta,2,4,5,9,17\}$

5. Karolyn invested $1000 for 4 years in a variable interest account. Her annual interest rates are shown in the table below. What was the average interest for the 4 years?

Year	1	2	3	4
Interest	7% gain	4% loss	6% loss	6% gain

(A) 0.58%   (B) 2.35%   (C) 1.35%   (D) 0.75%   (E) 2.19%

6. Find a if $(3x+5)$ is a factor of $6x^3 + ax^2 - 54x - a$.

A) -9   B) 63   C) -49   D) 17   E) -35

7. The points $P(7,12)$ and $Q(5,20)$ are on $\overrightarrow{PQ}$. The equation of the line parallel to $\overrightarrow{PQ}$ through the point $(-1,3)$ is:

A) $4x + y + 1 = 0$ B) $4x + y - 7 = 0$ C) $4x - y + 5 = 0$ D) $x - 4y + 5 = 0$ E) $4x + y - 40 = 0$

8. $\displaystyle\sum_{k=1}^{7} \left(3k^2 + 2k - 1\right) =$

A) 441   B) 469   C) 357   D) 476   E) 385

9. Carrie has 10 school books. In how many ways can she arrange her books on a single shelf so that she keeps her three math books together?

A) 241920   B) 30240   C) 120960   D) 3628800   E) 40320

10. If $\dfrac{x^4 - 5x^2 + 4}{\left(x^2 + 4x + 4\right)\left(x^2 + 2x + 1\right)} = \dfrac{x^2 + ax + 2}{x^2 + bx + 2}$, find $\dfrac{a}{b}$.

A) 0.5   B) -2   C) 1   D) -0.5   E) -1

11. A community theater holds 1200 patrons. Child and adult tickets cost $12.75 and $17.50 respectively. If a sold-out play brought in $18853, how many adult tickets were sold?

A) 452   B) 657   C) 748   D) 543   E) 572

12. The intersection of the _____ of a triangle is the center of the circumscribed circle.
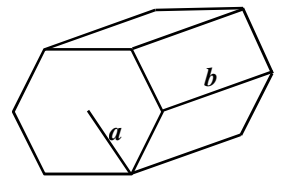
A) Medians   B) Angle bisectors   C) Midpoints   D) Perpendicular bisectors   E) Altitudes

13. Let $f(x) = x^3 + 3x + 1$, and $g(x) = x^2$. Find $g(f'(3))$.

A) 1369   B) 900   C) 225   D) 1225   E) 1521

14. The base of the prism is a regular hexagon. Which of the following is an
expression for the volume of the prism?

A) $\dfrac{a^2 b\sqrt{3}}{2}$ B) $\dfrac{3a^2 b}{2}$ C) $\dfrac{a^2 b\sqrt{3}}{3}$ D) $a^2 b\sqrt{3}$ E) $\dfrac{3a^2 b\sqrt{3}}{2}$
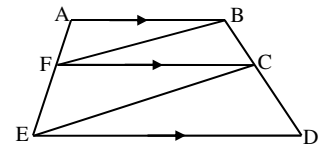
15. $\dfrac{\cot^2 t}{\csc t} =$

A) $\sec t + \cos t$ B) $\sec t - \cos t$ C) $\csc t + \sin t$ D) $\sec t - \sin t$ E) $\csc t - \sin t$

16. On the diagram, FC is the geometric mean of AB and ED, AB = 29.6, FC = 37
and EC = 55.5. Find FB. (nearest tenth)

A) 42.6   B) 40.5   C) 44.4   D) 41.9   E) 46.9

17. The repeating decimal 0.222… in base 11 can be written as what fraction in base 11?

A) $\dfrac{24}{121_{11}}$ B) $\dfrac{11}{60_{11}}$ C) $\dfrac{1}{5_{11}}$ D) $\dfrac{22}{121_{11}}$ E) $\dfrac{2}{11_{11}}$

18. Evaluate: $\displaystyle\prod_{k=2}^{6}(2k+4)$.
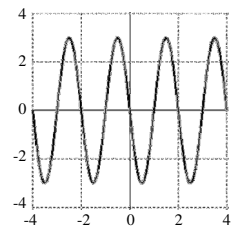
A) 26880   B) 21504   C) 215040   D) 15360   E) 276480

19. A particle's movement along the number line is defined by the function $f(t)=t^4-4t^3-26t^2+60t+25$. At
which of the following times is the particle moving to the right?

A) 2   B) 4   C) -4   D) 3   E) 0

20. The function $f(x)=$_____ will produce this graph.

(A) $3\sin(\pi x+1)$ (C) $\sin(\pi(x-1))+3$ (E) $3\sin(\pi(x-1))$

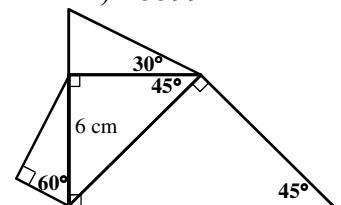(B) $3\sin(x+\pi)$ (D) $3\sin(\pi x-1)$

21. Solve $e^{2x}-9e^x=-20$ for x.

A) $2\ln 2$, $\ln 5$ B) $2\log 2$, 1   C) $\ln 2$, $\ln 10$ D) $\ln 5$, 1   E) e^4, e^5

22. If $A-B=13$ and $AB=37$, then $A^3-B^3=$

A) 3159   B) 4121   C) 2678   D) 3640   E) 3399

23. Find the perimeter of the figure shown right. (nearest tenth)

A) 39.1 cm   B) 36.9 cm   C) 41.8 cm   D) 37.3 cm   E) 39.4

24. There are two values of k for which $\det\begin{pmatrix} k+1 & -5 \\ -2 & -k \end{pmatrix}=-22$. The sum of those two values is

A) 0   B) 1   C) 10   D) -1   E) -10

25. Given $f(x)=\sin x$, evaluate $\displaystyle\lim_{h\to 0}\dfrac{f(\pi+h)-f(\pi)}{h}$.

(A) 0   (B) 1   (C) undefined   (D) -1   (E) $\sqrt{3}$

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26. Carrie's Ice Cream Shop has nine flavors of ice cream and three types of cones. How many distinct 2-scoop cones can be ordered?

A) 108   B) 165   C) 45   D) 55   E) 135

27. $\sum_{n=1}^{10}\left[n(n+1)\right]=$

A) 245   B) 110   C) 55   D) 440   E) 385

28. How many distinct arrangements can be made with the letters "SUNSHINE"?

A) 20160   B) 5040   C) 4320   D) 15120   E) 10080

29. If $\int_{0}^{a}\sin x\,dx=C$ then $\int_{-a}^{a}3\sin x\,dx=$

A) 0   B) $2C$ C) $3C$ D) $6C$ E) $12C$

30. The point $(2,-7)$ is reflected over the x-axis, reflected over the line $y=x$, rotated 270° clockwise around the origin, then shifted down three units to the point (a,b). Find $a+b$.

A) 2   B) -12   C) -7   D) -5   E) -9

31. Which of the following is not a solution to $f(x)\geq\left|25-\sqrt{x^2}\right|$?

A) $(-3,22)$ B) $(-8,18)$ C) $(9,14)$ D) $(0,30)$ E) $(4,21)$

32. The seven trapezoidal means are constructed as segments in a trapezoid. Which one is the shortest?

A) Heronian   B) Contraharmonic   C) Geometric   D) Harmonic   E) Root-mean square

33. What is the area of a triangle with side lengths 8 inches, 11 inches, and 17 inches? (nearest hundredth)

A) 8.37 in^2 B) 44.00 in^2 C) 35.50 in^2 D) 26.00 in^2 E) 39.75n^2

34. At which of the following x-values is the graph of $f(x)=x^4-12x^3+48x^2-64x$ concave down?

A) 3   B) 4   C) 2   D) 1   E) 5

35. If $\sin\theta=-\dfrac{7}{25}$ and $\dfrac{\pi}{2}\leq\theta\leq\dfrac{3\pi}{2}$, then $\tan\theta=$

A) $-\dfrac{24}{25}$ B) $\dfrac{24}{7}$ C) $-\dfrac{24}{7}$ D) $\dfrac{24}{25}$ E) $\dfrac{7}{24}$

36. Let $(a+4i)(b+i)=17-31i$, where $a,b\in\mathbb{Z}$, find the value of $a+b$.

A) 3   B) 4   C) -7   D) 7   E) -10

37. Find BD. (nearest quarter inch)
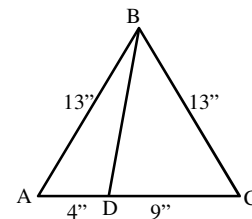
A) $9\dfrac{1}{2}$ B) 10   C) $10\dfrac{1}{4}$ D) $11\dfrac{3}{4}$ E) $11\dfrac{1}{2}$

38. Find the constant term in the expansion of $\left(2x^2-\dfrac{3}{x}\right)^9$.

A) 19683   B) -145152   C) 979776   D) 489888   E) -326592

39. In town A, during a one week period, the probability that it is cloudy on any particular day is 0.35. If it is cloudy, the chance of rain is 0.72. What is the probability that it will rain at least once during the week?

A) 0.131   B) 0.869   C) 0.999   D) 0.855   E) 0.900

40. If a hiker travels 6 miles on a bearing of 12°, then another 5 miles on a bearing of 334°, what is the shortest distance back to his starting point?

A) 10.00 mi   B) 2.26 mi   C) 10.06 mi   D) 13.06 mi   E) 10.41 mi

41. Let $a_1 = 3$, $a_2 = -2$, and $a_n = 2(a_{n-1})(-a_{n-2})$. Find a_5.

A) 284   B) -576   C) -288   D) 1728   E) -1152

42. Express $\log(10\sqrt{ab})$ in terms of P and Q if $P = \log a$ and $Q = \log b$.

A) $5PQ$ B) $10P + \frac{1}{2}Q$ C) $5P + \frac{1}{2}Q$ D) $\frac{PQ}{2}$ E) $\frac{1}{2}(P+Q)+1$

43. A belt joins two pulleys. The larger pulley has a radius of 84 cm and revolves at a rate of 15 rpm. The smaller has a radius of 7 cm. How fast is the smaller pulley revolving?

(A) 70 rpm   (B) 90 rpm   (C) 140 rpm   (D) 180 rpm   (E) 270 rpm

44. The line $3x + 2y = 39$ forms a chord in circle $x^2 + y^2 + 4x - 6y - 156 = 0$. Find the length of the chord.

A) 13   B) $2\sqrt{39}$ C) $\sqrt{195}$ D) $4\sqrt{13}$ E) 12

45. An investment is made in a fund that pays an annual percentage rate of 6.5%, compounded monthly. How long (to the nearest tenth of a year) will it take for the investment to double?

A) 10.6   B) 10.7   C) 10.8   D) 10.9   E) 11.0

46. There are seven boys and five girls on the student council. Their sponsor is asked to select three boys and three girls to attend a community luncheon. How many different groups could the sponsor choose?

A) 350   B) 45   C) 12600   D) 315   E) 6300

47. The sum of the first ten terms of an arithmetic sequence is 27.5 and the sum of the first twenty terms is 205. Find the common difference of the sequence.

A) 2.5   B) 2.25   C) 1.25   D) 0.75   E) 1.5

48. If $\frac{x-6}{x+7} + \frac{x+7}{x-6} = A\frac{B}{C}$, then $B =$

A) 52   B) 1   C) -13   D) 169   E) 26

49. What is the area of the largest isosceles triangle that can be inscribed in the circle with the equation $x^2 + y^2 + 6x + 10y - 87 = 0$?

A) $\frac{121\sqrt{3}}{16}$ B) $\frac{363\sqrt{3}}{16}$ C) $\frac{121\sqrt{3}}{4}$ D) $\frac{363}{4}$ E) $\frac{363\sqrt{3}}{4}$

50. Two players, X and Y, play a game in which X throws 6 coins and Y throws a fair 6-sided die. Player X wins if the number of heads is greater than the number on the die, otherwise Y wins. If Y throws a 3, what is the probability of Y winning?

A) $\frac{21}{32}$ B) $\frac{5}{12}$ C) $\frac{11}{32}$ D) $\frac{1}{2}$ E) $\frac{7}{12}$

51. Quadrilateral ABCD has vertices $(-7,3)$, $(-4,6)$, $(2,1)$ and $(9,-2)$ respectively. What is the area of ABCD?

A) 46   B) 30   C) 15   D) 27   E) 23

52. If $f(x) = ax^4 + bx^2 + x$ and $f(2) = 30$ then $f(-2) =$

A) -30   B) 28   C) 30   D) 26   E) 32

53. What is the 10^{-8} digit in the expansion of $1+(x-2)+\dfrac{(x-2)^2}{2!}+\dfrac{(x-2)^3}{3!}+\dfrac{(x-2)^4}{4!}+....$ when $x=4$?

A) 9 B) 2 C) 3 D) 0 E) 8

54. The operation € is defined as $A€B = A^3 + 2A + B^3$. Compute $3€(1€2)$.

A) 756 B) 1764 C) 1364 D) 42876 E) 36

55. How many solutions are there to $11x+4y=153$ such that $x, y \in \mathbb{Z}^+$.

A) 8 B) 3 C) 7 D) 6 E) 2

56. The probability that Joy will get a multiple choice question right is 0.25 by randomly guessing. If she guesses on every question on a 20 question quiz, what is the probability that she will make above a 30%?

A) 0.786 B) 0.898 C) 0.190 D) 0.214 E) 0.101

57. If $y = x^{x^2}$, then $\dfrac{dy}{dx} =$

A) $x^2 \cdot x^{x^2-1}$ B) $(x+2x\ln x)x^{x^2}$ C) $x+2x\ln x$ D) $2x^{x-1}$ E) $x^{2x}\ln x$

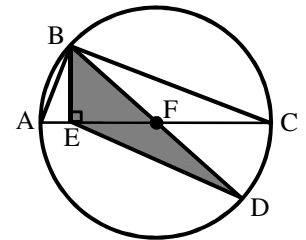58. If $\dfrac{43-2x}{x^2+9x-22} = \dfrac{A}{x-2}+\dfrac{B}{x+11}$, then $A+B =$

A) -5 B) 2 C) -3 D) -2 E) 5

59. Let P and Q be the roots of $3x^2+9x-25$. Find $P^5+5P^4Q+10P^3Q^2+10P^2Q^3+5PQ^4+Q^5$.

A) 54 B) -243 C) -1776 D) -32 E) -4096

60. On the circle with center F, EC = 7AE. The area of triangle ABC is 144 cm². Find the area of the shaded region.

A) 72 cm² B) 54 cm² C) 90 cm² D) 84 cm² E) 108 cm²

2013-2014 TMSCA Mathematics Test Eleven Answers

1. C	21. A	41. E
2. E	22. D	42. E
3. B	23. A	43. D
4. D	24. D	44. D
5. A	25. D	45. B
6. E	26. E	46. A
7. A	27. D	47. E
8. B	28. E	48. D
9. A	29. A	49. E
10. E	30. B	50. A
11. C	31. C	51. E
12. D	32. D	52. D
13. B	33. C	53. A
14. E	34. A	54. C
15. E	35. E	55. B
16. C	36. E	56. D
17. C	37. E	57. B
18. C	38. D	58. D
19. E	39. B	59. B
20. E	40. E	60. E

2013-2014 TMSCA Mathematics Test Eleven Answers

8. Using summation formulas the sum is

$$3\left(\frac{7(7+1)(2\cdot7+1)}{6}\right)+2\left(\frac{7(7+1)}{2}\right)-1(7)$$

which equals 469.

15. $\dfrac{\cot^2 t}{\csc t}=\dfrac{\csc^2 t-1}{\csc t}=\csc t-\sin t$

16. The geometric mean divides the trapezoid into two similar trapezoids, so $\dfrac{29.6}{37}=\dfrac{FB}{55.5}$ and $FB=44.4$.

17. Let A be the 10 digit in base 11.

$0.2222..._{11}=\dfrac{2}{A_{11}}=\dfrac{1}{5_{11}}$

18. $8(10)(12)(14)(16)=215040$

28. There are 8 letters with the S and the N each appearing twice, so the number of possible arrangements is $\dfrac{8!}{2!\cdot2!}=10080$.

29. $f(x)=\sin x$ is an odd function, so $\int_{-a}^{a}3\sin x\,dx=0$.

32. The smallest of the trapezoidal means is the harmonic.

37. $BD=\sqrt{13^2-4(9)}\approx11.5$

38. The constant term of the binomial expansion is $_9C_3\left(2x^2\right)^3\left(-\dfrac{3}{x}\right)^6=489888$

39. The probability of rain on a single given day is $0.35(0.72)=0.252$, so the probability of no rain on a single given day is 0.748. Instead of directly calculating the probability of at least one rainy day, do $1-(0.748)^7\approx0.869$.

44. Completing squares on the circle gives a center of $(2,-3)$ and radius of 13.

The distance between the point $(2,-3)$ and the line $3x+2y-39=0$ is

$$d=\frac{|2(3)+(-3)(2)-39|}{\sqrt{4+9}}=3\sqrt{13},$$

Pythagorean theorem with the radius and distance will give half the length of the chord $2\sqrt{13}$, so the full length is $4\sqrt{3}$.

49. The circle has a radius of 11 and the largest inscribed triangle is always an equilateral triangle. Using special triangles, one side of the triangle is $\dfrac{11}{2}\sqrt{3}$, so one side of the triangle is $11\sqrt{3}$.

The area will be $\dfrac{\left(11\sqrt{3}\right)^2\sqrt{3}}{4}=\dfrac{363\sqrt{3}}{4}$

57. Use logarithmic differentiation.

$\ln y=\ln x^{x^2}=x^2\ln x$

$\dfrac{1}{y}\dfrac{dy}{dx}=x^2\cdot\dfrac{1}{x}+\ln x\cdot2x$

Multiply both sides by y.

$\dfrac{dy}{dx}=(x+\ln x\cdot2x)y=(x+2x\ln x)x^{x^2}$

60. The heights of triangles ABC, EBF and EFD are all the same and the bases of EBF and EFD are each $\dfrac{3}{8}$ of the base of ABC, so shaded area $=2\left(\dfrac{3}{8}\right)144=108$.