T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 1 2 ©

F E B R U A R Y 2 5 , 2 0 1 7

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016-2017 TMSCA Mathematics Test Twelve

1. **Evaluate: $0.58333... \div 0.75 + 1.125 - 0.8333... \times 1.2$.**

   (A) $\frac{65}{72}$ **(B)** $\frac{89}{72}$ **(C)** $\frac{7}{5}$ **(D)** $\frac{17}{72}$ **(E)** $\frac{3}{4}$

2. **Erin went to the grocery store to purchase supplies for a cake. She bought 3 bags of flour at \$3.29 per bag, 4 boxes of butter at \$3.99 each, 2 bags of powdered sugar for \$1.79 each, a gallon of milk for \$3.79, a flat of eggs for \$12.99 and a spatula for \$6.25. How much change did she receive if only the spatula was taxed at a rate of 8.25% and she paid with three twenty dollar bills?**

   (A) \$7.04 **(B)** \$3.23 **(C)** \$8.46 **(D)** \$11.06 **(E)** \$7.83

3. **Points A and B have coordinates $(3,-5)$ and $(-9,19)$ respectively. What is the *y*-coordinate of the *y*-intercept of the perpendicular bisector of the segment $\overline{AB}$?**

   (A) 7.5 **(B)** –8.5 **(C)** 15 **(D)** 8.5 **(E)** 1

4. **52 yards per second is the same speed as _____miles per hour. (nearest mph)**

   (A) 106 **(B)** 98 **(C)** 110 **(D)** 112 **(E)** 102

5. **If $17 = 3x - y$, $x - 4y = 13$ and $x + ay = 1$, then $a = ?$**

   (A) 2 **(B)** –2 **(C)** –1 **(D)** 0 **(E)** –3

6. **Which of the following is not a one-to-one function?**

   (A) $y = 7x^5$ **(B)** $y = e^{2x}$ **(C)** $\ln(x-5)$ **(D)** $y = 5x^2$ **(E)** all are one to one

7. **Dumas is 848 miles from Brownsville. Nancy drove the entire distance in 12 hours and 10 minutes. She averaged 72 mph except when she averaged 52 mph driving through the San Antonio metro area. How long did it take Nancy to drive through the San Antonio metro area? (nearest minute)**

   (A) 1 hr 24 min **(B)** 2 hr 6 min **(C)** 1 hr 50 min **(D)** 1 hr 36 min **(E)** 1 hr 20 min

8. **Simplify:** $\frac{1}{(n-2)!} \div \frac{(n+2)!}{(n+1)!(n-1)!}$.

   (A) $\frac{n^2-2n-3}{n-1}$ **(B)** $\frac{n^2+2n-3}{n+1}$ **(C)** $\frac{n^2-n-2}{n+1}$

   (D) $\frac{1}{n^2-4}$ **(E)** $\frac{n-1}{n+2}$

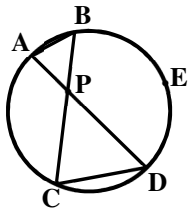
9. **Point $P(-5,2)$ lies on the x-y plane. Point P is reflected over the line $y = -x$ to point Q. Point Q is translated –6 units horizontally to point R. Point R is rotated 90° clockwise around the origin to point S. The coordinates of point S is (x,y). Find $x + y$.**

   (A) –11 **(B)** 13 **(C)** 1 **(D)** 11 **(E)** –3

10. If $15x^2 - 7x - 2 = (ax + b)(cx + d)$ then $ab + cd =$?

(A) 29 (B) 28 (C) 35 (D) –2 (E) –1

11. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$ and $\overline{CD}$ are chords of the circle shown. Find $m\overset{\frown}{BED}$ if $m\angle ADC = 54^\circ$ and $m\angle APB = 41^\circ$.

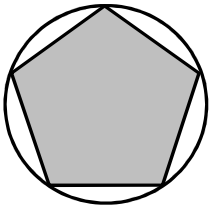


(A) 170° (B) 171° (C) 190° (D) 168° (E) 182°

12. Which of the following are the side lengths of a scalene, obtuse triangle?

(A) 13, 17, 21 (B) 9, 13,16 (C) 9, 11, 14 (D) 11, 15, 18 (E) 11, 11, 15

13. The regular pentagon in the illustration is inscribed in the circle. If a dart thrown at random strikes inside the circle, what are the odds that it will land in the shaded region? (nearest hundredth)



(A) 0.76 (B) 1.32 (C) 4.78 (D) 3.11 (E) 1.21

14. A hexagonal dipyramid has 18 edges, and the number of vertices is 4 less than the number of faces. How many vertices does it have?

A) 6 (B) 18 (C) 16 (D) 8 (E) 12

15. If $\frac{A}{x+5} + \frac{B}{2x+3} = \frac{9x-4}{2x^2+13x+15}$, then $A + B =$?

(A) –6 (B) 2 (C) –2 (D) 9 (E) –5

16. Let $f(x) = 2x - 1$ and $g(x) = \sqrt[3]{-27x} - 1$. Calculate $f(g(8))$.

(A) –17 (B) 9 (C) 15 (D) –15 (E) 13

17. Using the following array, determine the value of the last number in the 27th row.

1					(row 1)
3	5				(row 2)
7	9	11			(row 3)
13	15	17	19		(row 4)
21	23	25	27	29	(row 5)
…					(…)

(A) 649 (B) 755 (C) 811 (D) 869 (E) 929

18. How many distinguishable arrangements can be made from the letters "AMARILLO"?

(A) 10,080 (B) 40,320 (C) 5040 (D) 5,760 (E) 6,720

19. If the pattern of the sequence 47250, 47248, 47244, 47238, 47230, 47220, …continues, find the smallest positive term.

(A) 56 (B) 1059 (C) 1144 (D) 378 (E) 948

20. If an integral factor of 255, not including 1 or 255 is chosen at random, what are the odds that it is a multiple of 3?

(A) 1:2 (B) 1:3 (C) 2:1 (D) 3:1 (E) 1:1

21. Given that the binomial $x+2$ is a factor of $3x^4 - Ax^3 + Ax^2 + 2x + 16$, calculate the value of A.

(A) -5 (B) -3 (C) 5 (D) -17 (E) 27

22. Let $f(x) = \dfrac{9x^3 - 5x}{x-3}$. Find $f'(-1)$.

(A) $-\dfrac{21}{16}$ (B) 1 (C) $\dfrac{9}{10}$ (D) 27 (E) $-\dfrac{21}{4}$

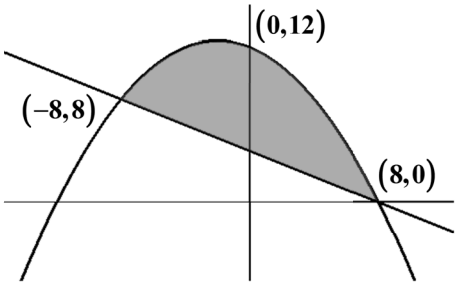
23. The bearing from town A to town B is 320°, and the bearing from town A to town C is 60°. If the distance from A to B is 5 km, and the distance from A to C is 3 km, how far town B from town C? (nearest tenth of a kilometer)

(A) 7.2 km (B) 4.7 km (C) 6.3 km (D) 7.9 km (E) 6.5 km

24. Find the sum of the solutions of $2\sin^2\theta - \cos\theta = 1$, where $0 < \theta < 3\pi$.

(A) $\dfrac{7\pi}{3}$ (B) 3π (C) 2π (D) $\dfrac{13\pi}{6}$ (E) $\dfrac{16\pi}{3}$

25. Find volume of the solid generated when the shaded region bounded by the parabola and the line in the illustration is rotated 360° around the line $y = 15$. (nearest cubic unit)



(A) 1,084 (B) 4,182 (C) 11,338 (D) 690 (E) 2,168

26. The point of intersection of all of the altitudes of a triangle is called the_____.

(A) center (B) incenter (C) centroid (D) circumcenter (E) orthocenter

27. If $z = a + bi$ is a complex number such that $z^5 = 316 + 12i$ and $z^4 = 28 + 96i$, then $a + b =$?

(A) 1 (B) -2 (C) 3 (D) -1 (E) 2

28. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$ be the terms of the Fibonacci sequence. Find f_{16}.

(A) 987 (B) 1597 (C) 377 (D) 2584 (E) 4181

29. Find $\lim_{x\to-\infty}\dfrac{-4x^2+9x^2+7}{3x^3-9}$

(A) $-\dfrac{4}{3}$ (B) -1 (C) 0 (D) $\dfrac{4}{3}$ (E) does not exist

30. The number 567 in base 9 is equivalent to the number k in base 3. Find the sum of the digits in the number k.

(A) 6 (B) 7 (C) 4 (D) 8 (E) 5

31. The function $f(x)=2x^4-3x^2$ is decreasing over which of the following intervals?

(A) $(-\infty,0]$ (B) $\left[-\dfrac{\sqrt{3}}{2},\dfrac{\sqrt{3}}{2}\right]$ (C) $[0,\infty)$ (D) $\left[0,\dfrac{\sqrt{6}}{2}\right]$ (E) $\left[0,\dfrac{\sqrt{3}}{2}\right]$

32. If $a_0=-1$, $a_1=3$, $a_2=5$ and $a_n=(a_{n-3})(a_{n-1})+a_{n-2}$ for $n\geq 3$, then $a_6=$?

(A) -15 (B) 13 (C) 6 (D) -20 (E) -7

33. Find the equation of the tangent to the curve defined by $9x^2-y^2=56$ at the point $(-3,5)$.

(A) $27x+5y=-150$ (B) $27x+5y=-56$ (C) $5x-27y=-120$

(D) $27x+5y=-81$ (E) $5x+27y=120$

34. How many 3-digit numbers exist such that the sum of their digits equals 12?

(A) 69 (B) 66 (C) 71 (D) 67 (E) 68

35. A lightbulb company produces bulbs that are faulty on average 4.8% of the time. If 5 bulbs are packaged together, what is the probability that at least one of the bulbs is faulty? (nearest tenth)

(A) 16.9% (B) 21.8% (C) 15.2% (D) 78.2% (E) 83.1

36. There are two values of k for which $\det\begin{bmatrix}6 & 2 & 3\\ 4 & -k & 7\\ k & 0 & 5\end{bmatrix}=-60$. What is the smallest value of k?

(A) 3 (B) $\dfrac{10}{3}$ (C) 2 (D) $\dfrac{5}{2}$ (E) -3

37. The function $f(x)=\dfrac{6x^2+5x-25}{9x^2-25}$ has a vertical asymptote at $x=V$ and a horizontal asymptote at $y=H$. Find $V+H$

(A) $\dfrac{7}{3}$ (B) 1 (C) $\dfrac{2}{3}$ (D) -1 (E) $-\dfrac{1}{6}$

38. A and B are the roots of $f(x)=3x^2-8x+4$. Calculate the value of $A^4+4A^3B+6A^2B^2+4AB^3+B^4$.

(A) $\dfrac{4096}{81}$ (B) $\dfrac{256}{81}$ (C) $\dfrac{16}{81}$ (D) $-\dfrac{4096}{81}$ (E) $-\dfrac{256}{81}$

39. If both the sides of the square base of a rectangular prism are doubled in length, and the height of the prism is tripled, what is the ratio of volume of the new prism to the volume of the original?

(A) 6:1 **(B)** 3:2 **(C)** 3:1 **(D)** 2:1 **(E)** 12:1

40. Simplify: $\left(a^{-3}\right)\left(\dfrac{\left(a^{-2}\right)^4}{\sqrt[3]{a}}\right)^5$

(A) $\left(\sqrt[3]{a}\right)^{-98}$ **(B)** $\left(\sqrt[3]{a}\right)^{-130}$ **(C)** $\left(\sqrt[3]{a}\right)^{-100}$ **(D)** $\left(\sqrt[3]{a}\right)^{-128}$ **(E)** $\left(\sqrt[3]{a}\right)^{-110}$

41. How many integral values of *n* exist such that $n \geq -1$ **and** $\dfrac{(n+3)!}{(n+1)!} \leq 432$

(A) 19 **(B)** 18 **(C)** 22 **(D)** 20 **(E)** 21

42. $\left(-2-3\sqrt{-27}\right)\left(4\sqrt{-12}\right)$

(A) $-216-16\sqrt{3}\,i$ **(B)** $-72-16\sqrt{3}\,i$ **(C)** $216-16\sqrt{3}\,i$

(D) $72-16\sqrt{3}\,i$ **(E)** $-216+16\sqrt{3}\,i$

43. $\left(212121_3 + 121212_3\right) \times 2_3 =$ _____$_3$

(A) 200,000 **(B)** 1,000,000 **(C)** 100,000 **(D)** 2,000,000 **(E)** 2,222,220

44. Ten liters of 30% acid solution is obtained by mixing a 20% solution with a 50% solution. How much 50% solution is used in the final mixture?

(A) $2\dfrac{1}{2}$ liters **(B)** 5 liters **(C)** $6\dfrac{2}{3}$ liters **(D)** $3\dfrac{1}{3}$ liters **(E)** $7\dfrac{1}{2}$ liters

45. $5^3 + 6^3 + \ldots + 21^3 =$

(A) 53,262 **(B)** 53,288 **(C)** 53,297 **(D)** 53,331 **(E)** 53,261

46. If $f(x) = \cos x$**, then** $\lim\limits_{h \to 0} \dfrac{f(\pi + h) - f(\pi)}{h}$ **is**

(A) 0 **(B)** -1 **(C)** 1 **(D)** $\sqrt{3}$ **(E)** undefined

47. Sweet Tooth Candy Store has bins of cherry, grape, strawberry, orange and lime lollipops. How many distinct sets of 6 lollipops can they package?

(A) 252 **(B)** 126 **(C)** 210 **(D)** 125 **(E)** 120

48. If $f(x) = 3^x$**,** $g(x) = \log_3 x$ **and** $a \geq 2$**, then** $g\left(f\left(a+2\right)\right) =$ **?**

(A) $a+2$ **(B)** 3^{a+2} **(C)** $3a+6$ **(D)** $\log_3(a+2)$ **(E)** $3a+2$

49. Which of the following statements about $f(x)=|5x-1|+7$?

I. $f'(x)$ **exists for all** x **in the domain of** $f(x)$

II. $f^{-1}(x)$ **is a function**

III. $\lim_{x\to a} f(x)$ **exists for all** a **in the domain of** $f(x)$

(A) II only (B) I & II (C) III only (D) II & III (E) none of these

50. The rolls of a biased 6-sided dice have the probability distribution shown. What is the expected value of a single roll?

Roll	1	2	3	4	5	6
Probability	$1/3$	$1/12$	$1/48$	$5/48$	$5/24$	x

(A) 1 **(B)** $\frac{145}{48}$ **(C)** $\frac{25}{8}$ **(D)** $\frac{51}{16}$ **(E)** $\frac{169}{48}$

51. The lengths of the sides of triangle PQR are the roots of $f(x)=2x^3-29x^2+134x-198$**. The perimeter of triangle PQR is 14.5. Find the area of triangle PQR. (nearest tenth)**

(A) 6.2 (B) 6.4 (C) 7.3 (D) 9.1 (E) 7.8

52. Find the area of a convex quadrilateral with vertices at $(-1,9)$, $(3,11)$, $(7,2)$ **and** $(2,-5)$ **.**

(A) 71.5 (B) 69.5 (C) 56.5 (D) 67.5 (E) 73

53. Change the base 10 proper fraction $\frac{4}{15}$ **to a repeating decimal in base 6.**

(A) 0.2444… (B) 0.151515… (C) 0.1434343… (D) 0.5333… (E) 0.1333…

54. If $\sin\alpha=-\frac{1}{2}$ **, where** $\pi<\alpha<\frac{3\pi}{2}$ **and** $\cos\beta=\frac{\sqrt{3}}{2}$ **, where** $\frac{3\pi}{2}<\beta<2\pi$ **, then** $\tan(\beta-\alpha)=$ **?**

(A) $\sqrt{3}$ **(B)** $-\frac{\sqrt{3}}{3}$ **(C)** -1 **(D)** $\frac{\sqrt{3}}{3}$ **(E)** $-\sqrt{3}$

55. If $x+\frac{1}{x}=17$ **then** $x^3+\frac{1}{x^3}=$ **?**

(A) 4862 (B) 4845 (C) 4896 (D) 4879 (E) 4913

56. If $h(x)\le f(x)\le g(x)$ **for all** x **in an open interval containing** c**, except possibly at** c **itself, and if** $\lim_{x\to c} h(x)=L=\lim_{x\to c} g(x)$ **then** $\lim_{x\to c} f(x)$ **exists and is equal to** L**. This theorem is known as:**

(A) Rolle's Theorem (B) Sandwich Theorem (C) Fundamental Theorem of Calculus

(D) Intermediate Value Theorem (E) Fundamental Theorem of Algebra

57. Find the shortest distance between the x-intercept of the line $2x+7y=14$ to the line $5x-6y=-48$. (nearest tenth)

(A) 4.6 **(B) 10.6** **(C) 7.7** **(D) 4.5** **(E) 8.1**

58. Find the sum of the lengths of all the diagonals of a regular pentagon if the length of each side is 5 ft. (nearest tenth of a foot)

(A) 40.5 ft **(B) 43.3 ft** **(C) 20.2 ft** **(D) 21.7 ft** **(E) 35.4 ft**

59. $\det\begin{bmatrix} \sin A & \cos A \\ -\sin A & \cos A \end{bmatrix} =$

(A) $\cos 2A$ **(B)** $\cos^2 A - \sin^2 A$ **(C) 0** **(D)** $\sin 2A$ **(E) 1**

60. Which of the following statements about $f(x)=\begin{cases} 15 & \text{if } x \le 3 \\ 2x^2 - x & \text{if } x > 3 \end{cases}$ **is/are true?**

I. $f(x)$ **is defined at 3** **II.** $\lim_{x\to 3} f(x)$ **exists**

III. $f(x)$ **is continuous at 3** **IV.** $f(x)$ **is differentiable at 3**

(A) I, II & IV **(B) I, II & III** **(C) I & II** **(D) I only** **(E) none of these**

Test Twelve Answer Key

1. A	21. A	41. D
2. A	22. E	42. C
3. D	23. C	43. E
4. A	24. E	44. D
5. A	25. B	45. E
6. D	26. E	46. A
7. A	27. B	47. C
8. E	28. A	48. A
9. B	29. C	49. C
10. E	30. D	50. E
11. A	31. E	51. D
12. B	32. B	52. D
13. D	33. B	53. E
14. D	34. B	54. E
15. B	35. B	55. A
16. D	36. C	56. B
17. B	37. D	57. B
18. A	38. A	58. A
19. D	39. E	59. D
20. E	40. B	60. B

Test Twelve Select Solutions

4. $\frac{52\,yd}{s}\cdot\frac{3600\,s}{hr}\cdot\frac{1\,mi}{1760\,yd}\approx 106\frac{mi}{hr}$

11. $m\overset{\frown}{AB}+m\overset{\frown}{CD}=2(41^\circ)=82^\circ$ and $m\overset{\frown}{AC}=108^\circ$, so $m\overset{\frown}{BED}=360-108-82=130^\circ$.

13. Let the radius of the circle be 1, then the area of circle is π and the area of the pentagon is $5\left(\frac{1}{2}\right)\sin 72^\circ\approx 2.38$ and the odds of landing in the shaded region are $\frac{2.38}{\pi-2.38}\approx 3.11$.

15. Let $A(2x+3)+B(x+5)=9x-4$ to set up the system of equations: $2A+B=9$ and $3A+5B=-4$ for $A=7$, $B=-5$ and $A+B=2$

16. The last numbers on each row form a quadratic sequence. Use quadratic regression, then find the 27th term.

18. The "A" and "L" both repeat once, so the total number of arrangements is $\frac{8!}{2\cdot 2}=10{,}080$.

21. $f(-2)=0=48+8A+4A-4+16$ for $A=5$.

25. $y_1=-\frac{1}{2}x+4$ and $y_2=-\frac{1}{8}x^2-\frac{1}{2}x+12$. The volume of the solid of revolution will be:
$\int_{-8}^{8}\pi(15-y_1)^2\,dx-\int_{-8}^{8}\pi(15-y_2)^2\,dx\approx 4182$

27. $\frac{316+12i}{28+96i}=1-3i$ for $a+b=-2$

33. $18x-2y\frac{dy}{dx}=0$ for $\frac{dy}{dx}=-\frac{27}{5}$ at $(-3,5)$. Use the slope and point to find the equation of the tangent line.

35. Use $p(\text{at least one fault})=1-p(\text{no fault})$ or $1-(1-0.048)^5\approx 0.218$ or 21.8%.

38. The pattern with A and B is the sum of the roots to the fourth power, so $\left(\frac{8}{3}\right)^4=\frac{4096}{81}$.

44. Solve $10(0.3)=0.5x+0.2(10-x)$ for $x=3\frac{1}{3}$.

45. $\left(\frac{21(22)}{2}\right)^2-1-8-27-64=53261$

46. This is the definition of the derivative of $f(x)=\cos x$ at $x=\pi$ or 0.

47. ${}_{6+5-1}C_6=210$.

50. Find x by $1-\left(\frac{1}{3}+\frac{1}{12}+\frac{1}{48}+\frac{5}{48}+\frac{5}{24}\right)=\frac{1}{4}$, then the expected value is $\frac{1}{3}+\frac{2}{12}+\frac{3}{48}+\frac{20}{48}+\frac{25}{24}+\frac{6}{4}$.

51. The area of the triangle is $\sqrt{7.25f(7.25)}\approx 9.1$.

57. The x-intercept is $(7,0)$ and the distance from this point to $5x-6y+48=0$ is $\frac{|7(5)+0(-6)+48|}{\sqrt{25+36}}\approx 10.6$