T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 1 3 ©

M A R C H 2 , 2 0 1 3

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. **Evaluate the following to the nearest tenth:** $3 + 6! \div (9)^{\frac{3}{2}} - \sqrt{12} \times 15^{(-1)}$

   (A) 56.1 (B) 29.4 (C) 12.2 (D) 7.2 (E) 1.6

2. **The regular price for a Quik Math calculator is $105.75. If Al Geebrah buys four calculators for his math team, he gets a 20% discount and he can buy one for himself at one-third off of the regular price. How much will AL have to pay before taxes if he buys all five calculators?**

   (A) $408.90 (B) $472.35 (C) $493.50 (D) $582.05 (E) $620.40

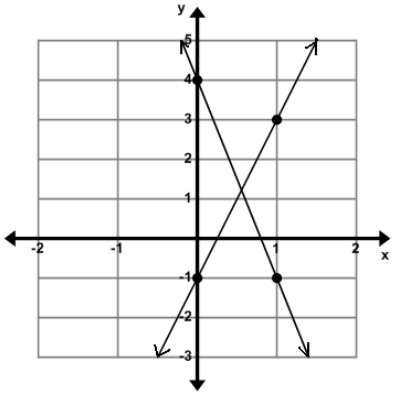
3. $\frac{4}{5} + (-\frac{4}{5}) = 0$ **and** $\frac{4}{5} \times \frac{5}{4} = 1$ **are examples of the _____ properties of equality.**

   (A) associative (B) commutative (C) distributive (D) identity (E) inverse

4. **If a nonnegative number has an even number of ones in its binary expansion then it is a(n) _____ number.**

   (A) odd (B) odious (C) primeval (D) even (E) evil

5. **The two lines shown intersect at point P(x,y). Find x + y.**



   (A) $1\frac{5}{6}$ **(B)** $1\frac{7}{9}$ **(C)** $1\frac{2}{3}$ **(D)** $1\frac{5}{9}$ **(E)** $1\frac{1}{3}$

6. **2x — 1, 3x — 2 and 4x — 3 are factors of which of the following?**

   (A) $24x^3 - 10x^2 + 29x - 6$ **(B)** $24x^3 - 46x^2 - 17x + 6$ **(C)** $24x^3 + 46x^2 - 17x - 6$
   (D) $24x^3 - 10x^2 + 13x + 6$ **(E)** $24x^3 - 46x^2 + 29x - 6$

7. **In how many ways can twelve coins be divided into four piles, so that two coins are in the first pile, three in the second pile, four in the third pile, and three in the fourth pile?**

   (A) 222 (B) 663 (C) 11,880 (D) 277,200 (E) 831,600

8. **Justin Time had to row his canoe 8 miles down the creek to the finish line in 4 hours. What was Justin's speed if the speed of the creek's current was 0.5 mph, assuming that he kept a constant speed relative to the water?**

   (A) 1.25 mph (B) 1.5 mph (C) 1.75 mph (D) 2.25 mph (E) 2.5 mph

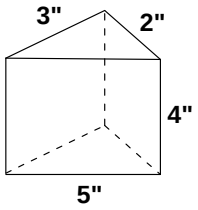
9. The _____ notation *dy/dx* used when working with derivatives is contributed to which of the following mathematicians?

(A) Descartes (B) Napier (C) Euclid (D) Agnesi (E) Leibniz

10. ∠A and ∠B are supplementary . The ratio of m∠A to m∠B is 4:5. Find the ratio of m∠A to its complement.

(A) 5:4 (B) 8:1 (C) 2:1 (D) 1:1 (E) 1:9

11. Find the lateral surface area of the trigonal prism shown.



(A) 14 in^2 (B) 20 in^2 (C) 32 in^2 (D) 40 in^2 (E) 46 in^2

12. The *measure of a minor arc* is _____ the measure of its central angle.

(A) less than (B) greater than (C) equal to (D) half of (E) double

13. If you connect the centers of all the faces of a cube you will form a(n):

(A) octahedron (B) cube (C) rectangular prism (D) square pyramid (E) decahedron

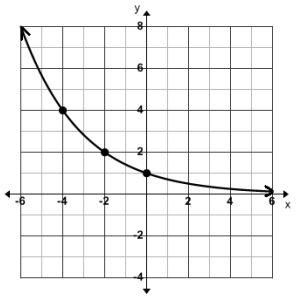
14. Two distinct numbers are selected randomly from the set { 1 , 3 , 6 , 10 , 15, 21}. What are the odds that their sum is divisible by 3?

(A) 2:5 (B) 3:5 (C) 2:3 (D) 1:3 (E) 2:1

15. How many two-digit numbers exist such that the sum of the digits is a square number and reversing the digits results in a prime number.

(A) 2 (B) 4 (C) 9 (D) 13 (E) 17

16. Which of the following equations will produce the graph shown here?



(A) $y = e^{2x}$ (B) $y = 2\ln(x)$ (C) $y = \sqrt{2^x}$ (D) $y = e^{(-2x)}$ (E) $y = 2^{(-\frac{x}{2})}$

17. Using Blaise Pascal's triangle and letting the 1 at the top be row 0, determine the 3rd number in the 14th row.

(A) 78 (B) 84 (C) 88 (D) 91 (E) 105

18. Let $f(x) = ax^3 - bx + 2$ where a, b, and c are constants. If $f(4) = -5$ then $f(-4) = ?$

(A) 9 (B) 5 (C) 1 (D) −1 (E) −4

19. Captain Don sails his scow from his dock on Valley Lake 5 km on a bearing of 315°. Then he changes course and sails 3 km on a bearing of 75° to pick up materials at the mission. What bearing will Captain Don have to sail his scow from the mission straight back to his dock? (nearest degree)

(A) 83° (B) 98° (C) 105° (D) 172° (E) 187°

20. $(1 + i)^3 - (1 + i)^2 = a + bi$. Find $a + b$.

(A) −3 (B) −2 (C) −1 (D) 0 (E) 2

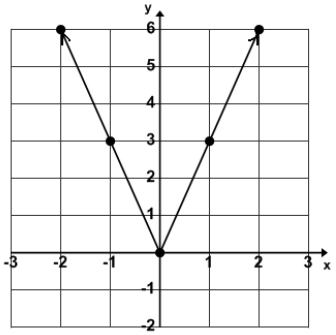
21. Determine the frequency of the function $y = 2 + 4\cos(2x + 2\pi)$

(A) $\frac{1}{2\pi}$ (B) π (C) 4π (D) $\frac{1}{\pi}$ (E) 2π

22. Betty Notsogood and Willie When bowl two games. Willie is one and one-half times as likely to win any game as is Betty. What is the probability that Willie will win both games?

(A) 25% (B) $36\frac{1}{9}\%$ (C) 50% (D) 52% (E) $63\frac{8}{9}\%$

23. Which of the following is true about the relation graphed below?



(A) It is an odd function. (B) It is an even function. (C) It is not a function.

(D) It is neither an even nor an odd function (E) It is a one-to-one function.

24. If $A = \begin{bmatrix} -1 & 5 \\ 3 & x \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}$ then $AB = \begin{bmatrix} 28 & -44 \\ 18 & -4 \end{bmatrix}$. What is the value of x?

(A) 28 (B) 8 (C) 7 (D) 3 (E) 2

25. A infinite geometric sequence has a common ratio of $-1\frac{2}{3}$ and a sum of 45. What is the first term of the sequence?

(A) $67\frac{1}{2}$ **(B)** $16\frac{7}{8}$ **(C)** 120 **(D)** 75 **(E)** $43\frac{1}{3}$

26. Let y = f(x) be a function that is continuous on [a,b]. If y_1 is between f(a) and f(b), then y_1 = f(c) for some c in [a,b]. This theorem is known as:

(A) Fundamental Theorem of Algebra **(B) Intermediate-value Theorem**

(C) Sandwich Theorem **(D) Max-Min Theorem** **(E) Fundamental Theorem of Calculus**

27. Let f(x) = $\sqrt[3]{3x}$. Find f′(9).

(A) $\frac{1}{9}$ **(B)** 3 **(C)** $\frac{1}{3}$ **(D)** 9 **(E)** $\frac{1}{27}$

28.

The distances between the hash marks (|) are equal. Find P + Q + R.

(A) 2 **(B) 1.7** **(C) 1.6** **(D) 1.1** **(E) 0.5**

29. The probability that statement P is true is 3/7, and the probability that statement Q is true is 5/12. Determine the probability that P → Q is true.

(A) $\frac{3}{4}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{28}$

30. Simplify: $b^{-4} \div a^{6} \times b^{8} \div a^{-10} \times b^{0} \div a^{2}$

(A) 0 **(B)** $a^{14}b^{4}$ **(C)** $a^{6}b^{-12}$ **(D)** $a^{2}b^{4}$ **(E)** $a^{-2}b^{4}$

31. $77_8 + 333_4 + 1111_2$ = _____ **16**

(A) 8D **(B) 2AB** **(C) E1** **(D) 4C5** **(E) F**

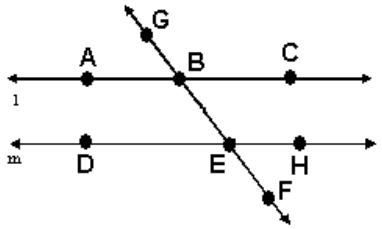
32. Simplify: $\left(\frac{6x^2 - 7x - 3}{6x^2 + x - 2}\right) \div \left(\frac{4x^2 - 4x - 3}{6x^2 + 7x + 2}\right)$

(A) $6x^2 - x - 1$ **(B)** $\frac{2x-3}{3x+2}$ **(C)** $3x + 1$ **(D)** $6x^2 - 5x - 6$ **(E)** $\frac{3x+1}{2x-1}$

33. If y = x − 3 and xy = 4 then $x^3 - y^3$ = ?

(A) 9 **(B) 12** **(C) 31** **(D) 63** **(E) 91**

34. The three lines in the figure are coplanar with m // l.. How many of the following are false statements?



1. ∠ABG & ∠FBA are vertical angles **2. m∠BED = $\frac{1}{2}$m∠CBE**

3. ∠HEF ≅ ∠ABG **4. ∠DEG and ∠DEF are complementary**

(A) 0 **(B) 1** **(C) 2** **(D) 3** **(E) 4**

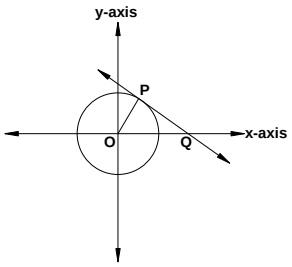
35. Find the circumference of the circle, $x^2 + y^2 - 4x + 2y - 31 = 0$. (nearest tenth)

(A) 113.1 **(B) 40.8** **(C) 37.7** **(D) 32.0** **(E) 18.8**

36. A _____ is a point in the domain of a function for which there is no corresponding point in the range.

(A) focus **(B) centroid** **(C) directrix** **(D) discontinuity** **(E) conjugate**

37. A circle with a radius of 4 cm and the a center at the origin O, in the xy-plane has a tangent line at point P in the first quadrant. The tangent line intersects the x-axis at Q and the angle of inclination of the radius is 60°. Find the area of △OPQ. (nearest tenth)



(A) 10.7 cm^2 **(B) 11.3 cm^2** **(C) 13.9 cm^2** **(D) 14.9 cm^2** **(E) 18.9 cm^2**

38. Which of the following is equivalent to $\frac{\sec\theta}{\sin(90° - \theta)} - \frac{\cos\theta\tan\theta}{\cos(90° - \theta)}$?

(A) $\sec^2\theta$ **(B) $\tan\theta\sec\theta$** **(C) $\tan^2\theta$** **(D) 1** **(E) 0**

39. In the expansion of $(x - 2)^5$, the sum of the coefficients of the 2nd and the 5th term is:

(A) 90 **(B) 70** **(C) 14** **(D) − 16** **(E) − 32**

40. Let f(x) = 3 − 4x, and g(x) = 5x + 2. Find the value of $f\big(g(-x)\big) + g\big(f(x)\big)$.

(A) 12 **(B) 10** **(C) 10 − 40x** **(D) − 20x** **(E) 5x − 9x**

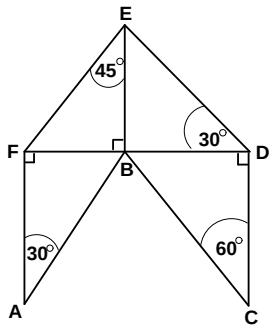
41. Let $f(x) = \begin{cases} 3x + 7 & \text{if } x \leq 4 \\ kx - 1 & \text{if } 4 < x \end{cases}$.
Find the value of k that makes f(x) continuous on $(-\infty, +\infty)$.

(A) -3 **(B)** -4 **(C)** 4 **(D)** 5 **(E)** 6

42. Determine the concavity of the graph of $f(x) = 3\sin(x) + 4(\cos(x))^2$ at $x = \pi$.

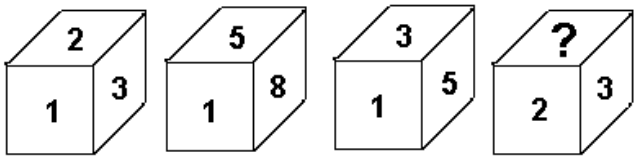
(A) -10 **(B)** -8 **(C)** -6 **(D)** 4 **(E)** 8

43. Find the perimeter of the quadrilateral ABEF if DE = 4 cm. (nearest tenth).



(A) 12.3 cm **(B)** 12.6 cm **(C)** 13.7 cm **(D)** 14.0 cm **(E)** 15.7 cm

44. Each face of a cube is labeled with the digits, 1, 1, 2, 3, 5, 8. Four views of the cube are shown. Which of the following digits should appear on the face where the "?" is? Ignore orientation.



(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

45. The TMSCA state math meet had exactly 37.5% girls registered. What is the smallest number of girls that the Young Ladies Academy can register late to raise the percentage to exactly 45%? Registration is closed for the boys.

(A) 8 **(B)** 11 **(C)** 12 **(D)** 18 **(E)** 29

46. The Real value solution set for $7 < |4 + 3x| - 2$ is?

(A) $\{x \mid \{-4\frac{1}{3} > x\} \cup \{x > 1\frac{2}{3}\}\}$ **(B)** $\{x \mid -1\frac{2}{3} < x < 4\frac{1}{3}\}$ **(C)** $\{x \mid -4\frac{1}{3} < x < 1\frac{2}{3}\}$

(D) $\{x \mid \{-1\frac{1}{3} < x\} \cup \{x > 4\frac{1}{3}\}\}$ **(E)** $\{x \mid \{-4\frac{1}{3} < x\} \cup \{x > 1\frac{2}{3}\}\}$

47. Which of the following points is not part of the solution set of $2 > 3x + y$ and $x - y < 6$?

(A) (0 , — 5) **(B)** (— 3, — 3) **(C)** (— 2, — 8) **(D)** (— 5, 0) **(E)** (1, — 4)

48. Let △JKL be an obtuse triangle. Which of the following are in the exterior of △JKL.
(1) centroid (2) circumcenter (3) incenter (4) orthocenter

(A) 1 only (B) 4 only (C) 1 & 3 (D) 1, 2, & 3 (E) 2 & 4

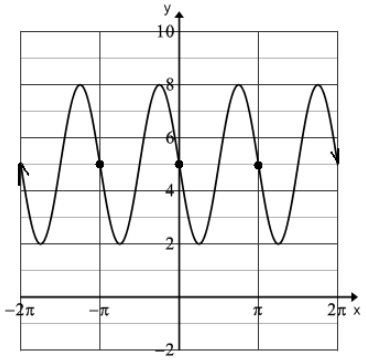
49. Find the slope of the tangent line to $4x^2 - 3y^2 = 12$ at point P($\sqrt{6}$, 2). (nearest hundredth)

(A) 1.63 (B) 1.84 (C) 2.45 (D) 3.27 (E) 3.67

50. Noah Sense gave his daughter, Lotta Sense, her allowance of 25¢ on the first Friday in January. He told her he would increase her allowance 10¢ each Friday after that if she promised to save all of her allowances for the first 20 Fridays of the year. How much will Lotta have saved?

(A) $26.25 (B) $24.00 (C) $21.50 (D) $7.00 (E) $6.90

51. The equation $y = D + A\sin(Bx + C)$ will produce this graph. Find $A + B + C + D$, where $0 \le x \le \pi$.



(A) 10 (B) 9 (C) 6 (D) 5 (E) 4

52. △ABC exists such that $m\angle ACB = 90°$ and point P is the midpoint of segment AB. If CP = 12.5 cm, BC = 7 cm, and $m\angle CPB = \theta$, find $\sin(\theta)$. (nearest hundreth)

(A) 0.54 (B) 0.82 (C) 0.33 (D) 0.29 (E) 0.94

53. Simplify to the form $a + bi$: $(1 + 2i)(3 - 4i) \div (5i)$

(A) $0 + 1.8i$ (B) $0.4 - 11i$ (C) $2 - 11i$ (D) $0.4 - 2.2i$ (E) $0.2 + 2.2i$

54. Let $f(x) = \frac{2x^3 + 4x^2 - 9}{x - 3}$ and s(x) be the slant asymptote of f. Find the value of s(− 2).

(A) − 81 (B) 8.1 (C) 18 (D) 1.8 (E) 81

55. $\int \csc^2(x)\, dx$ = _____ + C, where C is some arbitrary constant.

(A) $-\sin^2(x)$ (B) $-\frac{\cos(x)}{\sin(x)}$ (C) $1 - \tan^2(x)$ (D) $\left(\frac{\sin(x)}{\cos(x)}\right)^2$ (E) $\cot(x)$

56. The curve $y = \frac{1}{3}x^3 - 2x^2 + 3x - 4$ has a horizontal tangent at:

(A) $(-1, -\frac{28}{3})$ (B) $(0, -4)$ (C) $(2, -\frac{10}{3})$ (D) $(\frac{8}{3}, -1)$ (E) $(3, -4)$

57. Lotta Scents plays a game at the Unlucky Number Casino. She tosses a quarter onto a large checker board. If it lands completely on a black square she gets 3 quarters. If it lands completely on the red square she gets 2 quarters. If it touches any line she loses her quarter. The probability of landing on black is 30%, on red is 30%, and on a line is 40%. What is the mathematical expectation on any one quarter tossed?

(A) $+65¢$ (B) $+60¢$ (C) $+47.5¢$ (D) $+40¢$ (E) $+27.5¢$

58. The dihedral angle (nearest degree) of a dodecahedron is:

(A) 87° (B) 117° (C) 120° (D) 129° (E) 138°

59. $A1B3_{12} \div 9_{12} =$ _____ 12

(A) 1115 (B) 1167 (C) A12 (D) 1B2 (E) B67

60. P, Q, and R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, and R is $1\frac{5}{7}$ and and PQR = −4. Find C.

(A) −7 (B) −5 (C) $\frac{7}{12}$ (D) $2\frac{1}{2}$ (E) 12

2012-13 TMSCA HS Math Test #13
Answer Key

1. B	**21. D**	**41. D**
2. A	**22. D**	**42. B**
3. E	**23. B**	**43. A**
4. E	**24. E**	**44. A**
5. B	**25. C**	**45. C**
6. E	**26. B**	**46. A**
7. D	**27. A**	**47. C**
8. B	**28. D**	**48. E**
9. E	**29. A**	**49. A**
10. B	**30. D**	**50. B**
11. D	**31. A**	**51. E**
12. C	**32. E**	**52. A**
13. A	**33. D**	**53. D**
14. C	**34. D**	**54. C**
15. B	**35. C**	**55. B**
16. E	**36. D**	**56. E**
17. A	**37. C**	**57. E**
18. A	**38. C**	**58. B**
19. D	**39. B**	**59. B**
20. B	**40. A**	**60. A**