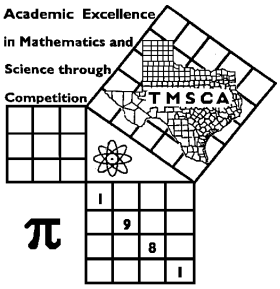


TMSCA HIGH SCHOOL MATHEMATICS TEST #2 ©

NOVEMBER 2, 2013

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2013-2014 TMSCA Mathematics Test Two

1. Evaluate $3 \div (5!) \bullet 8^{2/3} + \sqrt{17}$ to the nearest hundredth.

(A) 4.62 (B) 4.17 (C) 4.52 (D) 4.32 (E) 4.22

2. $\angle A$ and $\angle B$ are supplementary. If the $m\angle A : m\angle B$ is 5:7, find the measure of the complement of $\angle A$.

(A) 24° (B) 15° (C) 21° (D) 18° (E) 25°

3. Mr. Nelson's statistics class surveyed 95 students. Of these, 58 listen to country music, 69 listen to pop music and 18 don't listen to either. How many students listen to both?

(A) 32 (B) 46 (C) 50 (D) 27 (E) 36

4. Jon's mother is three times as old as Jon. Seven years from now he will be a year younger than half her age. What is the sum of their current ages?

(A) 27 (B) 50 (C) 43 (D) 36 (E) 45

5. Carla invested $1500 in a variable interest account. In the first year, she made 3.5% profit. In the second, she lost 2%. In the third and fourth years, she had 4% and 3.7% gains respectively. What was her average interest rate for the 4 years?

(A) 2.27% (B) 3.30% (C) 3.27% (D) 2.37% (E) 2.30%

6. $3x+7$, $2x-5$ and $5x+2$ are all factors of

(A) $30x^3 + 7x^2 - 177x - 70$ (C) $30x^3 - 133x^2 - 175x - 70$ (E) $30x^3 + 7x^2 - 177x + 70$

(B) $30x^3 - 133x^2 - 117x - 70$ (D) $30x^3 - 133x^2 - 233x - 70$

7. Which of the following is an equation of the line passing through $(-3.2, 1.5)$ parallel to $2x + 3y = 9$?

(A) $20x + 30y = -19$ (B) $20x + 30y = 141$ (C) $20x - 30y = -19$ (D) $20x - 30y = 19$ (E) $20x - 30y = -141$

8. What is the sum of the arithmetic series $11 + 8.5 + 6 + \ldots + (-26.5)$?

(A) -116 (B) 48.5 (C) -270 (D) -124 (E) -288

9. Bob's Ice Cream Counter serves ten flavors of ice cream and a choice of three types of cones. How many different ways can a customer order a cone with two scoops?

(A) 135 (B) 220 (C) 60 (D) 165 (E) 660

10. $\dfrac{2x^3 + 3x^2 - 18x - 27}{2x^2 - 5x - 3} \div \dfrac{x^2 - 9}{2x^2 + 15x + 7} =$

(A) $\dfrac{2x+3}{x+7}$ (B) $\dfrac{2x^2 - 3x + 6}{x+7}$ (C) $\dfrac{2x+3}{x-3}$ (D) $\dfrac{2x^2 - 3x - 9}{x+7}$ (E) $\dfrac{2x^2 + 17x + 21}{x-3}$

11. $a \bullet (b \bullet c) = (a \bullet b) \bullet c$ is an illustration of _____ property.

(A) Distributive (B) Associative (C) Transitive (D) Commutative (E) Reflexive

12. $\dfrac{d}{d\theta} \sin(3\theta^2) =$

(A) $6\theta \cos(3\theta^2)$ (B) $\cos(6\theta)$ (C) $-3\theta \cos(6\theta)$ (D) $3\theta \cos(3\theta^2)$ (E) $-3\theta \cos(3\theta^2)$

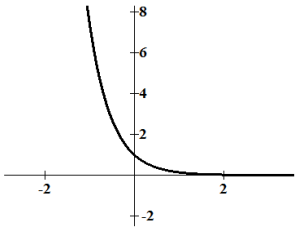
13. You have a piggy bank containing a total of 82 dimes and quarters. If the bank contains $16.45, how many dimes are there?

(A) 26 (B) 54 (C) 27 (D) 41 (E) 55

14. The height and diameter of the base of a right cone are both 8 in. What is the lateral surface area?

(A) $4\pi\sqrt{5}$ in^2 (B) $\frac{128}{3}\pi$ in^2 (C) $16\pi\left(1+\sqrt{5}\right)$ in^2 (D) $32\pi\sqrt{5}$ in^2 (E) $16\pi\sqrt{5}$ in^2

15. Which of the following will produce the graph shown?



(A) $y=e^{2x}$ (B) $y=2\ln x$ (C) $y=\sqrt{2^x}$ (D) $y=e^{(-2x)}$ (E) $y=2^{\left(-\frac{1}{2}x\right)}$

16. If 1 is the sum of the first 0th row, determine the sum of the numbers in the 16th row of Pascal's triangle.

(A) 12870 (B) 32768 (C) 65536 (D) 11440 (E) 131072

17. On an icosahedron, there are F faces, E edges and V vertices. Calculate F + E + V.

(A) 26 (B) 50 (C) 68 (D) 62 (E) 52

18. If f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists a number c in (a,b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$. This is the

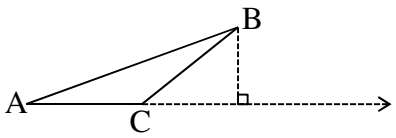
(A) Fundamental Theorem of Algebra (C) Fundamental Theorem of Calculus (E) Mean Value Theorem

(B) Intermediate Value Theorem (D) Sandwich Theorem

19. If $A+B=16$ and $A\times B=24$, then $|A-B|=$

(A) 12 (B) $4+\sqrt{10}$ (C) $2\sqrt{10}$ (D) $4\sqrt{10}$ (E) $16-\sqrt{10}$

20. On triangle ABC shown, $m\angle BAC=\frac{\pi}{6}$ radians, $AB=12$ and $AC=8$.



Find the area of triangle ABC.

(A) 24 (B) $24\sqrt{3}$ (C) 36 (D) 48 (E) $48\sqrt{3}$

21. $A=\begin{bmatrix}2 & 3\\ 5 & 6\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 2\\ 1 & 0\end{bmatrix}$. Find $\det AB$.

(A) -66 (B) -30 (C) 6 (D) 2 (E) 30

22. $2\log a^2b^2+\frac{1}{2}\log a^3b-3\log ab=$

(A) $\log ab^2\sqrt{ab}$ (B) $-\log\frac{a^4b^4}{2}$ (C) $\log a^2b\sqrt{ab}$ (D) $\log\frac{1}{a^2b^2}$ (E) $\log a^8b^7\sqrt{ab}$

23. If $f'(x)=9x^2-4x+5$ and $f(-1)=-21$, then $f(2)=$

(A) 33 (B) 15 (C) 26 (D) 31 (E) 29

24. $\sum_{k=1}^{12}2k(k-2)=$

(A) 748 (B) 1144 (C) 1440 (D) 988 (E) 846

25. How many distinct arrangements can be made using all the letters in MISSISSIPPI?

(A) 34650 (B) 3150 (C) 415800 (D) 6652800 (E) 69300

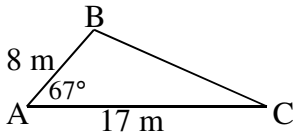
26. $\int_{-a}^{a}\left(5x^3-7x\right)dx=$

(A) 0 (B) $\frac{5a^4-14a}{4}$ (C) $10a^3-14a$ (D) $2a$ (E) $\frac{5a^4-14a}{2}$

27. How many positive perfect cubes are factors of $(3!)(7!)$?

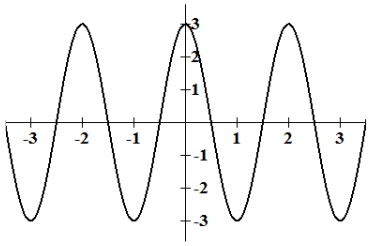
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

28. On triangle ABC shown, find BC to the nearest hundredth.



(A) 22.22 m (B) 17.32 m (C) 15.71 m (D) 20.58 m (E) 15 m

29. Which of the following functions yields the graph shown?



(A) $y=3\cos\left(\pi\left(x-2\right)\right)$ (B) $y=3\sin\left(\pi\left(x-2\right)\right)$ (C) $y=6\sin\left(\pi x-2\right)$ (D) $y=3\sin\left(\pi x-2\right)$ (E) $y=6\cos\left(\pi\left(x-2\right)\right)$

30. Given $a_{n+1}=a_{n-1}+\left(a_n\right)^2$, $a_0=2$ and $a_1=3$, find a_4.

(A) 2608 (B) 1611 (C) 124 (D) 6898 (E) 15387

31. The chords $\overline{AD}$ and $\overline{BC}$ intersect inside circle *O* at point *P*. If $AD=20$, $AP=2$ and $BP=3$, find *CB*.

(A) 12 (B) $\frac{40}{3}$ (C) 18 (D) $\frac{49}{3}$ (E) 15

32. The perpendicular bisectors of the sides of a triangle intersect at the

(A) Orthocenter (B) Incenter (C) Circumcenter (D) Median (E) Centroid

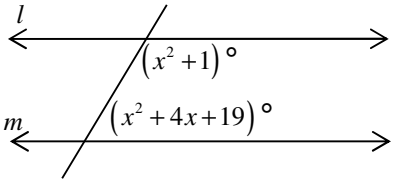
33. The graph of $9x^2-y^2-36x-6y+18=0$ is a

(A) Circle (B) Hyperbola (C) Parabola (D) Ellipse (E) Cartoid

34. Solve $\frac{2}{x}+\frac{3}{y}=\frac{3}{xy}$ for *y*.

(A) $y=3-3x^2$ (B) $y=\frac{3-3x}{2}$ (C) $y=1-3x^2$ (D) $y=\frac{3-3x^2}{2}$ (E) $y=1-x^2$

35. In the diagram, $l\,//\,m$. Find the value of *x*.



(A) 10 (B) 12 (C) 9 (D) 8 (E) 6

36. Given $f\left(x\right)=2x^2+5$ and $g\left(x\right)=2x-1$, find $g\left(f'\left(3\right)\right)$.

(A) 25 (B) 47 (C) 45 (D) 13 (E) 23

37. If *P* and *Q* are the roots of $f\left(x\right)=6x^2+13x-28$, then $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4=$

(A) $\frac{28561}{1296}$ (B) $\frac{190385}{1296}$ (C) $\frac{198577}{1296}$ (D) $\frac{707281}{1296}$ (E) $\frac{367921}{1296}$

38. $\dfrac{\cos\theta}{1-\sin\theta} =$

(A) $\sec\theta-\cot\theta$ (B) $\csc\theta-\cot\theta$ (C) $\sec\theta+\tan\theta$ (D) $\csc\theta+\tan\theta$ (E) $\sec\theta+\cot\theta$

39. The length of one edge of a regular tetrahedron is $7\sqrt{2}$ cm. Find the volume.

(A) $\dfrac{343\sqrt{2}}{3}\text{cm}^3$ (B) $98\sqrt{3}\ \text{cm}^3$ (C) $\dfrac{343\sqrt{2}}{6}\ \text{cm}^3$ (D) $98\sqrt{6}\ \text{cm}^3$ (E) $\dfrac{343}{3}\ \text{cm}^3$

40. If x varies inversely with y and $x=\dfrac{2}{3}$ when $y=\dfrac{7}{5}$, find x when $y=\dfrac{3}{2}$.

(A) $\dfrac{5}{7}$ (B) $\dfrac{28}{45}$ (C) $\dfrac{23}{30}$ (D) $\dfrac{315}{421}$ (E) $\dfrac{17}{30}$

41. Find the slope of $2x^2+3y^2=29$ at the point $(-1,3)$.

(A) $-\dfrac{2}{9}$ (B) $\dfrac{23}{18}$ (C) $\dfrac{4}{9}$ (D) $-\dfrac{23}{18}$ (E) $\dfrac{2}{9}$

42. Find the angle between the vectors v_1 and v_2 given $v_1=\langle -3,2\rangle$ and $v_2=\langle 5,-11\rangle$. (nearest degree)

(A) 103° (B) 148° (C) 77° (D) 122° (E) 32°

43. Simplify $(2-5i)^2-(3+2i)^3$.

(A) $-12-26i$ (B) $-12+66i$ (C) $-12+26i$ (D) $-12-66i$ (E) $12-66i$

44. There are 84 students in the senior class in Texas HS senior class. The ratio of boys to girls is 7:5. If the number of boys remains constant, how many new girls would have to enroll to change the ratio to 1:1?

(A) 24 (B) 35 (C) 14 (D) 21 (E) 7

45. In how many ways can a class of 12 students be split into three groups of 6, 4 and 2?

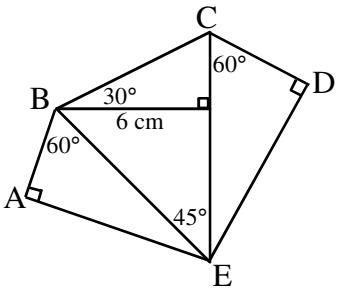
(A) 457380 (B) 30187080 (C) 13860 (D) 60984 (E) 48

46. Given $f(x)=\dfrac{2x-3}{5x+9}$ find $f'(3)$.

(A) $\dfrac{1}{8}$ (B) $\dfrac{33}{576}$ (C) $\dfrac{43}{24}$ (D) $\dfrac{2}{5}$ (E) $\dfrac{43}{576}$

47. Find the perimeter of pentagon ABCDE to the nearest tenth of a centimeter.

(A) 31.4 cm (B) 40.1 cm (C) 34.1 cm (D) 35.5 cm (E) 37.4 cm



48. The Real value solution for $x^2-3x-4<0$ is?

(A) $\{x|\{x<-1\}\cup\{x>4\}\}$ (B) $\{x|-1<x<4\}$ (C) $\{x|\{x<-4\}\cup\{x>1\}\}$ (D) $\{x|-4<x<1\}$ (E) $\{x|x>-1\}$

49. Which of the following points is in the solution set for the system $\begin{matrix} x+y<3 \\ x+2y>3 \end{matrix}$?

(A) $(1,2)$ (B) $\left(2,\frac{1}{2}\right)$ (C) $\left(-\frac{3}{2},\frac{9}{4}\right)$ (D) $\left(-\frac{1}{2},3\right)$ (E) $(3,0)$

50. What is the angle between the hour and minute hands on a clock at 6:23 pm?

(A) 53.5° (B) 35.5° (C) 42° (D) 56.5° (E) 47.5°

51. The polynomial function $f(x)=x^3+ax^2+bx+c$ has roots -7, -2 and 3. Find the value of $a+b+c$.

(A) -6 (B) -23 (C) -61 (D) -49 (E) 23

52. Quadrilateral ABCD is inscribed in a circle so that the vertices are on the circle in an alphabetical and clockwise arrangement. If $m\angle B=103^\circ$ and $m\angle D=(2x-7)^\circ$, find the value of *x*.

(A) 42 (B) 35 (C) 63 (D) 77 (E) 10

53. Evaluate the infinite series $-\frac{9}{2}+\frac{81}{24}-\frac{729}{720}+\frac{6561}{40320}-...$ to the nearest ten-thousandth.

(A) -0.8589 (B) -0.9900 (C) -1.9900 (D) 0.1411 (E) 1.0986

54. If $\sec\theta=-3$ and $\sin\theta>0$ then $\tan\theta=$

(A) $-2\sqrt{2}$ (B) 2 (C) $\frac{8}{3}$ (D) $2\sqrt{2}$ (E) -2

55. If $f(x)=ax^5+bx^3-10$ and $f(3)=-40$, then $f(-3)=$

(A) -30 (B) 20 (C) -20 (D) -50 (E) 30

56. Find the remainder when $9x^3-16x-18x^2+32$ is divided by $x-2$.

(A) -80 (B) 32 (C) -16 (D) 2 (E) 0

57. How many solutions (x,y) are there to $5x+3y=900$ where *x* and *y* are both positive integers*?*

(A) 61 (B) 58 (C) 60 (D) 62 (E) 59

58. $\frac{d}{dx}2^{\sin x}=$

(A) $2^{\sin x}\cos x$ (B) $\ln 2\left(2^{\sin x}\cos x\right)$ (C) $\sin x\left(2^{\sin x-1}\right)$ (D) $\ln 2(\cos 2x)$ (E) $-2^{\sin x}\cos x$

59. To the nearest degree, what is the dihedral angle of a regular octahedron?

(A) 71° (B) 19° (C) 161° (D) 109° (E) 146°

60. $444_5+777_8-222_3=$ _____$_{10}$.

(A) 400 (B) 609 (C) 630 (D) 398 (E) 611

2013-2014 TMSCA Mathematics Test Two Answers

1. E	21. C	41. E
2. B	22. C	42. B
3. C	23. B	43. D
4. D	24. D	44. C
5. A	25. A	45. C
6. A	26. A	46. B
7. A	27. C	47. A
8. D	28. C	48. B
9. D	29. A	49. D
10. E	30. E	50. A
11. B	31. E	51. D
12. A	32. C	52. A
13. C	33. B	53. C
14. E	34. B	54. A
15. D	35. D	55. B
16. C	36. E	56. E
17. D	37. D	57. E
18. E	38. C	58. B
19. D	39. E	59. D
20. A	40. B	60. B

2013-2014 TMSCA Mathematics Test Two Select Solutions

3. $n(C \cup P) = n(C) + n(P) - n(C \cap P)$
$77 = 58 + 69 - n(C \cap P) \therefore n(C \cap P) = 50$

5. $\sqrt[4]{(1+0.035)(1-0.02)(1+0.04)(1+0.037)} \approx 2.27\%$

9. $\#\,flavors + \#\,scoops = 12$
${}_{12-1}C_2 \bullet 3 = 165$

16. First row $n = 0$ then sum of nth row is 2^n.
$2^{16} = 65536$

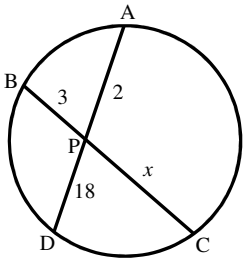
19. $(A-B)^2 = (A+B)^2 - 4AB$
$|A-B| = \sqrt{16^2 - 4(24)} = 4\sqrt{10}$

24. $\sum_{k=1}^{12}(2k^2 - 4k) = 2 \bullet \frac{12(13)(25)}{6} - 4 \bullet \frac{12(13)}{2} = 988$

25. 11 letters 4-S's, 4-I's and 2-P's
distinct arrangements = $\frac{11!}{(4!)(4!)(2!)} = 34650$

26. For all odd functions $\int_{-a}^{a} f(x)\,dx = 0$

31. $2 \bullet 18 = 3x$
$x = 12$
$BC = 15$



37. $(P-Q)^4 = \left(\frac{\sqrt{b^2-4ac}}{a}\right)^4 = \left(\frac{\sqrt{169-4(6)(-28)}}{6}\right)^4$
$= \frac{841^2}{6^4} = \frac{707281}{1296}$

38. $\frac{\cos\theta}{1-\sin\theta} \bullet \frac{1+\sin\theta}{1+\sin\theta}$
$= \frac{\cos\theta + \sin\theta\cos\theta}{\cos^2\theta} = \sec\theta + \tan\theta$

52. Opposite angles of a quadrilateral inscribed in a circle are supplementary.
$2x - 7 + 103 = 180 \ \therefore x = 42$

53. $= -1 + 1 - \frac{3^2}{2!} + \frac{3^4}{4!} - \frac{3^6}{6!} + \ldots = -1 + \cos 3 \approx -1.9900$

58. $\ln y = \ln 2^{\sin x} \rightarrow \ln y = \ln 2 \sin x$
$\frac{1}{y}\frac{dy}{dx} = \ln 2 \cos x$
$\frac{dy}{dx} = \ln 2\left(\cos x \ln 2^{\sin x}\right)$

60. $bbb_{b+1} = (b+1)^3 - 1$, where $b \in \mathbb{Z}^+$
$444_5 + 777_8 - 222_3 =$
$(5^3 - 1) + (8^3 - 1) - (3^3 - 1) = 609$