



**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 2 ©
OCTOBER 29, 2016**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016-2017 TMSCA Mathematics Test Two

1. Evaluate: $(22+7) \times 5 \div 4 + 7 \div (16-14)$.

- (A) 4.95 (B) 37.25 (C) 39.75 (D) 25.1 (E) 26.7

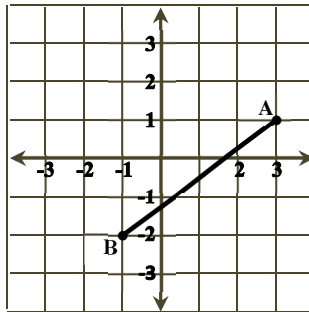
2. Mitch bought a set of 4 tires for his car. He got the first one for regular price, the second one for a 15% discount, the third for $\frac{1}{5}$ off, and 30% off the fourth one. What was his total cost if the regular price of a single tire was \$108, and the tax rate on his purchase was 8.25%?

- (A) \$379.96 (B) \$392.55 (C) \$380.84 (D) \$386.25 (E) \$391.65

3. 3000 revolutions per hour = _____ degrees per second.

- (A) 360 (B) 18000 (C) 300 (D) 3600 (E) 320

4. The coordinates of points A and B are integers. Which of the following is an equation of the perpendicular bisector of \overline{AB} ?



- (A) $8x - 6y = -5$ (B) $8x + 6y = 5$ (C) $4x + 3y = 15$
 (D) $3x - 4y = -1$ (E) $3x - 4y = 5$

5. If $m\angle A + m\angle B + m\angle C = 180^\circ$ and $m\angle C + m\angle D = 180^\circ$, then $m\angle A + m\angle B + m\angle C = m\angle C + m\angle D$ is an example of _____ property.

- (A) Distributive (B) Commutative (C) Associative (D) Transitive (E) Closure

6. Bob walked to school to pick up his bicycle at an average rate of 5 mph. He rode his bike home at an average rate of 15 mph. The total trip took 40 min. How far does Bob live from the school?

- (A) 1.5 mi (B) 2.4 mi (C) 1.8 mi (D) 2.5 mi (E) 2.6 mi

7. Use the table of values to create a function to find K.

X	1	2	3	4	...	16	...
Y	-3	3	13	27	...	K	...

- (A) 445 (B) 573 (C) 507 (D) 445 (E) 387

8. Solve for x in terms of y : $\frac{3y}{7} - \frac{4}{3x} = \frac{y}{6}$

- (A) $\frac{56}{11y}$ (B) $\frac{14}{3y}$ (C) $\frac{56}{15y}$ (D) $\frac{14}{15y}$ (E) $\frac{14}{3x}$

9. If p and q are the zeros of the function $f(x) = 21x^2 - 29x - 10$ then $pq^2 + p^2q =$

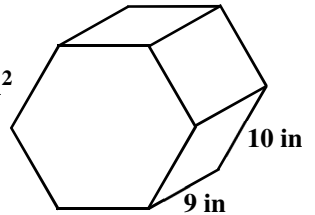
- (A) $-\frac{290}{441}$ (B) $\frac{290}{441}$ (C) $-\frac{145}{441}$ (D) $\frac{145}{441}$ (E) $\frac{293}{441}$

10. $(31315_8 + 41315_8) \times 4_8 = \text{_____}_8$

- (A) 353142 (B) 332155 (C) 347230 (D) 353150 (E) 325140

11. Find the total surface of the regular hexagonal prism shown. (nearest square inch)

- (A) 800 in^2 (B) 540 in^2 (C) 1060 in^2 (D) 520 in^2 (E) 790 in^2



12. Multiply $(2x - 7)(x - 2)(2x + 3)$.

- (A) $4x^3 - 16x^2 - 5x + 42$ (B) $4x^3 + 28x^2 - 5x + 42$ (C) $4x^3 - 16x^2 - 61x + 42$
 (D) $4x^3 - 28x^2 - 5x + 42$ (E) $4x^3 - 16x^2 - 5x - 42$

13. The graph of the function $f(x) = x^4 - 12x^3 + 48x^2 - 64x$ has points of inflection when $x = a$ and $x = b$. Find $a + b$.

- (A) 8 (B) 18 (C) 6 (D) 9 (E) 24

14. The regular hexagon ABCDEF is inscribed in a circle with the vertices arranged clockwise alphabetically on the circle. What is $m\angle EAD$?

- (A) 30° (B) 36° (C) 45° (D) 54° (E) 60°

15. Carrie's 30-sided dice has sixty edges. How many vertices does it have?

- (A) 24 (B) 36 (C) 28 (D) 30 (E) 32

16. A carpenter cut a flat, narrow board into four pieces. The lengths of the pieces were 12", 9", 8" and 6". How many triangles could the carpenter make using only three pieces at a time?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

17. If $\frac{x-2}{x+2} - \frac{3x}{x+5} = \frac{ax^2+bx+c}{px^2+qx+r}$, then $(a+b+c) - (p+q+r)$ equals:

- (A) -21 (B) -33 (C) 1 (D) -31 (E) -19

18. Train A leaves New York at 8 am travelling at an average speed of 56 mph towards Chicago. Forty-five minutes later, Train B leaves Chicago travelling at an average speed of 59 mph towards New York. At what time will the trains pass each other on the 1100 mile trip? (nearest minute)

- (A) 5:34 pm (B) 5:57 pm (C) 5:09 pm (D) 5:18 pm (E) 5:29 pm

19. If $f_0 = -2, f_1 = 3, f_2 = 1, f_3 = 4, f_4 = 5, \dots, f_k = 7375, \dots$ are terms of a Fibonacci-type sequence. Find f_{k+1} .
- (A) 11993 (B) 19308 (C) 15621 (D) 15621 (E) 11933
20. Find the sum of the arithmetic mean, median, mode and range of 3, 19, 5, 2, 10, 35, & 3.
- (A) 49 (B) 48 (C) 55 (D) 53 (E) 52
21. The number of integers between 1 and 54 that are relatively prime to 54 is:
- (A) 15 (B) 20 (C) 18 (D) 17 (E) 24
22. Simplify: $(a) \left(\frac{(a^3)^3 (\sqrt{a})}{a^{-2}} \right)$
- (A) $(\sqrt{a})^{17}$ (B) $(\sqrt{a})^{21}$ (C) $(\sqrt{a})^{12}$ (D) $(\sqrt{a})^{13}$ (E) $(\sqrt{a})^{25}$
23. Mr. Smith has 13 students in his science club. He wants to select a 6-member team for district competition. How many distinct teams could he choose?
- (A) 1235520 (B) 279936 (C) 618618 (D) 6468 (E) 1716
24. Solve $\sin 2x = -\sin x$, where $0 \leq x < 2\pi$.
- (A) $\left\{ \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}, \frac{4\pi}{3} \right\}$ (B) $\left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$ (C) $\left\{ 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \right\}$
- (D) $\left\{ 0, \frac{7\pi}{6}, \pi, \frac{11\pi}{6} \right\}$ (E) $\left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$
25. Which of the following is a formula for the area of an equilateral triangle in terms of its height?
- (A) $\frac{h^2}{3}$ (B) $\frac{2h\sqrt{3}}{3}$ (C) $\frac{h^2\sqrt{3}}{4}$ (D) $\frac{h\sqrt{3}}{2}$ (E) $\frac{h^2\sqrt{3}}{3}$
26. $\frac{d}{d\theta} \cos(2\theta^2) = ?$
- (A) $-8\theta \sin(\theta^2)$ (B) $4\theta \sin(2\theta^2)$ (C) $8\theta \sin(\theta^2)$ (D) $-4\theta \sin(2\theta^2)$ (E) $-\sin(4\theta)$
27. Find the average rate of change for $f(x) = 3x^3 - 2x^2 + 5$ on the interval $[-2, 3]$.
- (A) 1 (B) $\frac{121}{12}$ (C) 19 (D) $\frac{4837}{60}$ (E) $\frac{97}{5}$
28. $753_8 = k_4$. Find the sum of the digits in k .
- (A) 11 (B) 9 (C) 8 (D) 10 (E) 7
29. How many distinguishable arrangements can be made from the letters "SWEETWATER"?
- (A) 302400 (B) 907200 (C) 5040 (D) 120960 (E) 151200

30. Allen, Barney and Chris can build a brick wall together in 12 hours. If each individual works at the same rate, how long would it take Chris to build a wall twice as long and twice as high by himself?

- (A) 72 hours (B) 108 hours (C) 144 hours (D) 108 hours (E) 56 hours

31. Given that the set of even numbers continue in the triangular pattern shown below, find the median of the terms in the 18th row.

				2				(row 1)
			4	6	8			(row 2)
	10	12	14	16	18			(row 3)
20	22	24	26	28	30	32		(row 4)
			...					(...)

- (A) 614 (B) 546 (C) 686 (D) 136 (E) 144

32. Given $a_0 = 2$, $a_1 = 3$ and $a_{n+1} = a_{n-1} + 2(a_n)$ for $n \geq 1$. Find a_6 .

- (A) 111 (B) 268 (C) 458 (D) 647 (E) 239

33. Let $A = \begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -3 \\ 2 & 4 \end{bmatrix}$. Find $|(A+B)^T|$.

- (A) 1 (B) -1 (C) 2 (D) -2 (E) 0

34. Find $\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{x^2 - 7x + 12}$.

- (A) -10 (B) -2 (C) 5 (D) 7 (E) does not exist

35. If $x - y = 3$ and $xy = 7$ then $x^3 - y^3 = ?$

- (A) 48 (B) 90 (C) 69 (D) 63 (E) 6

36. Find C if the remainder of $2x^3 - 7x^2 + 5x + C$ divided by $x - 3$ is 16.

- (A) 148 (B) 90 (C) 69 (D) 10 (E) 6

37. The chords \overline{AD} and \overline{BC} intersect inside circle O at point P . $\overline{AD} = 23$, $\overline{AP} = 6$ and $\overline{BP} = 3$. Find \overline{CB} .

- (A) 19 (B) 34 (C) 37 (D) 51 (E) 54

38. The graph of $9x^2 - y^2 - 36x - 6y + 18 = 0$ is a

- (A) Circle (B) Parabola (C) Cartoid (D) Ellipse (E) Hyperbola

39. What is the constant term in the binomial expansion of $\left(\frac{2}{x^2} + x^4\right)^9$?

- (A) 672 (B) 5376 (C) 512 (D) 64 (E) 168

40. Two roots of $f(x) = x^3 + bx^2 + cx + d$ are $2+i$ and 5 . Find $b+c+d$.

- (A) 3 (B) 1 (C) -9 (D) -1 (E) 2

41. The graph of the parametric equations $x = 3\cos t$ and $y = 6\sin t$ is a(n)_____.

- (A) Ellipse (B) Circle (C) Parabola (D) Line (E) Hyperbola

42. Let $f(x) = 3x^2 - 2x + 4$ and $g(x) = 3x + 1$. Find $g(f'(x-2))$

- (A) $18x - 41$ (B) $18x - 28$ (C) $9x^2 - 42x + 61$
 (D) $18x - 5$ (E) $9x^2 - 6x + 12$

43. The repeating decimal $2.242424..._6$ can be written as which of the following fractions in base 6?

- (A) $\frac{222}{30}_6$ (B) $\frac{222}{35}_6$ (C) $\frac{222}{55}_6$ (D) $\frac{220}{35}_6$ (E) $\frac{220}{30}_6$

44. Which of the following are sides of an isosceles, obtuse triangle?

- (A) 11, 11, 15 (B) 48, 55, 73 (C) $14, 14\sqrt{2}, 14$ (D) 19, 11, 17 (E) 8, 14, 8

45. Triangle PQR is such that $m\angle R = 60^\circ$, $PR = 32$ and $PQ = 28$. There are two possible values for QR. Find the product of the two values.

- (A) 240 (B) 32 (C) 1808 (D) 55 (E) 896

46. $\det \begin{pmatrix} \cos A & \sin A \\ \cos B & \sin B \end{pmatrix} = ?$

- (A) $\sin(A+B)$ (B) $\cos(A+B)$ (C) $\cos(A-B)$ (D) $\sin(B-A)$ (E) $\cos(B-A)$

47. Leonard demonstrated the sample probability experiment using a spinner with four divisions once and rolling a fair die once. He then wrote down the sample space. How many outcomes were there in the sample space?

- (A) 24 (B) 12 (C) 10 (D) 32 (E) 18

48. Let a, b and c be real numbers such that $c = a + b + 8$, $c^2 = a^2 + b^2$ and $ab = 8$. Find the value of $6c$.

- (A) 30 (B) -30 (C) 30 (D) -24 (E) 18

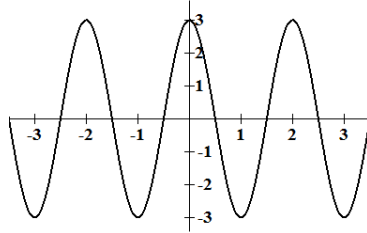
49. The real value solution for $2x^2 - 11x + 12 \geq 0$ is?

- (A) $\{x | \{x \leq 1.5\} \cup \{x \geq 4\}\}$ (B) $\{x | \{x \leq -1.5\} \cup \{x \geq 4\}\}$ (C) $\{x | 1.5 \leq x \leq 4\}$
 (D) $\{x | -1.5 \leq x \leq 4\}$ (E) $\{x | x \geq 4\}$

50. Let $f(x) = \frac{4x^2 - 3x + 8}{2x + 1}$ and $s(x)$ be the slant asymptote of $f(x)$. Find $s(4)$.

- (A) 5.5 (B) 3.1 (C) 10.5 (D) 7.5 (E) 4.5

51. Which of the following functions yields the graph shown?



- (A) $y = 3\sin(\pi(x - 2))$ (B) $y = 3\cos(\pi(x - 2))$ (C) $6\sin(x - 2)$
 (D) $3\sin(\pi x - 2)$ (E) $y = 6\cos(\pi(x - 2))$

52. The lengths of the sides of triangle PQR are the roots of $f(x) = x^3 - 19x^2 + 117x - 231$. Find the area of the triangle PQR. (nearest tenth square unit)

- (A) 11.2 (B) 6.4 (C) 14.8 (D) 25.2 (E) 5.7

53. Solve $e^{2x} - 10e^x + 21 = 0$.

- (A) 0, log 21 (B) log 3, log 7 (C) ln 3, ln 7 (D) 0, ln 21 (E) 3, 7

54. Let $f(x) = ax^7 - bx^3 - cx + 8$. If $f(3) = 15$ then $f(-3) = ?$

- (A) -1 (B) -15 (C) 8 (D) 1 (E) 7

55. How many solutions are there for $3x + 5y = 1138$ where x and y are both positive integers?

- (A) 68 (B) 77 (C) 76 (D) 75 (E) 70

56. $\tan\theta < 0$ and $\sin\theta > 0$. Where will θ terminate?

- (A) QI (B) QII (C) QIII (D) QIV (E) y-axis

57. The geometric mean of 2017 and 7102 is _____% of the arithmetic mean of 2017 and 7102.

- (A) 35 (B) 83 (C) 120 (D) 118 (E) 68

58. Find the value of $A + B + C$, where A , B and C are non-negative integers and $\frac{40}{9} = A + \left(\frac{1}{B + \frac{1}{C + 1}} \right)$.

- (A) 10 (B) 11 (C) 9 (D) 15 (E) 17

59. All the edges of a cube are expanding at a rate of 4.25 centimeters per second. The volume of the cube is changing at a rate of _____ cm^3s^{-1} when the length of one edge is 10 cm.

- (A) 541.875 (B) 4250 (C) 1275 (D) 114.75 (E) 767.656

60. The function f is such that $\int_{-9}^1 f(x)dx = 48$. What is the value of $\int_{-9}^1 (-3f(x) + 9)dx$.

- (A) -135 (B) 54 (C) -54 (D) -12 (E) -153

Test Two Answer Key

- | | | |
|--------------|--------------|--------------|
| 1. C | 21. D | 41. A |
| 2. E | 22. E | 42. A |
| 3. C | 23. E | 43. C |
| 4. B | 24. C | 44. E |
| 5. D | 25. E | 45. A |
| 6. D | 26. D | 46. D |
| 7. C | 27. C | 47. A |
| 8. A | 28. A | 48. E |
| 9. A | 29. E | 49. A |
| 10. D | 30. C | 50. A |
| 11. C | 31. A | 51. B |
| 12. A | 32. B | 52. C |
| 13. C | 33. D | 53. C |
| 14. A | 34. A | 54. D |
| 15. E | 35. B | 55. C |
| 16. D | 36. D | 56. B |
| 17. B | 37. C | 57. B |
| 18. B | 38. E | 58. C |
| 19. E | 39. B | 59. C |
| 20. E | 40. C | 60. C |

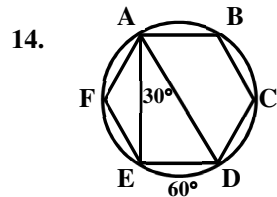
Test Two Select Solutions

9. $pq^2 + p^2q = pq(q + p)$ the sum of the roots is $-\frac{29}{21}$ and

the product of the roots is $\frac{10}{21}$, so

$$pq(q + p) = -\frac{29}{21}\left(\frac{10}{21}\right) = -\frac{290}{441}$$

$$10. \frac{31315_8}{72632_8} \text{ and } \frac{72632_8}{353150_8}$$



15. $F + V - 2 = E$ so $30 + V - 2 = 60$, so $V = 32$.

16. For three lengths to form a triangle $a + b > c$ which is true for 6,8,9 and 6,8,12 and 6,9,12 and 8,9,12.

24. $2\sin x \cos x = -\sin x$ then $\sin x(2\cos x + 1) = 0$,
 $\sin x = 0$ when $x = 0, \pi$ and $\cos x = -\frac{1}{2}$ when $x = \frac{2\pi}{3}, \frac{4\pi}{3}$.

26. $\frac{d}{d\theta} \cos(2\theta^2) = -\sin(2\theta^2)(4\theta) = -4\theta \sin(2\theta^2)$.

27. The average rate of change is the slope of the secant between the endpoints of the interval or $\frac{f(3) - f(-2)}{3 - (-2)} = 19$

29. "SWEETWATER" has 10 letters with 3-E's, 2-W's and 2-T's, so the number of possible arrangements

$$\frac{10!}{(3!)(2!)(2!)} = 151,200$$

30. Since all three men work at the same rate, Carl could finish the job himself in 36 hours. If he completes a wall that is four times as big, it will take him $4(36) = 144$ hours.

34. $\lim_{x \rightarrow 3} \frac{2x + 4}{2x - 7} = \frac{10}{-1} = -10$

35. $(x - y)^2 = x^2 - xy + y^2$, $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
 so $x^3 - y^3 = (x - y)[(x - y^2) + 3xy] = 3(3^2 + 3 \cdot 7) = 90$

37. $3x = 6(17)$, $x = 34$ and $BC = 34 + 3 = 37$.

39. $\binom{9}{3} \left(\frac{2}{x^2}\right)^6 (x^4)^3 = 84 \left(\frac{64}{x^{12}}\right) (x^{12}) = 5376$

43. $\frac{100_6 n}{55_6 n} = \frac{224.2424..._6}{222_6}$ and $n = \frac{35}{86_6}$

44. For an obtuse triangle, $a^2 + b^2 < c^2$ which of the values shown only $8^2 + 8^2 < 14^2$.

45. Use law of cosines $28^2 = 32^2 + QR^2 - 2 \cdot 32(QR) \cos 60^\circ$,
 $784 = 1024 + (PQ)^2 - 32(PQ)$, $0 = (PQ)^2 - 32(PQ) + 240$
 and the product of the roots is 240.

48. $c - 8 = a + b$, $c^2 - 16c + 64 = a^2 + 2ab + b^2$,
 $-16c + 64 = 2(8)$, $c = 3$ and $6c = 18$.

52. Sum of the roots is 19, semiperimeter is 9.5 and the area is $\sqrt{9.5 f(9.5)} \approx 14.8$.

58. $A = 4$, then $\frac{4}{9} = \frac{1}{B + \frac{1}{C+1}}$, $\frac{9}{4} = B + \frac{1}{C+1}$, $B = 2$,

$\frac{1}{4} = \frac{1}{C+1}$ and $C = 3$ so $A + B + C = 9$.

60. $-3 \int_{-9}^1 f(x) dx + \int_{-9}^{-1} 9 dx = -3(48) + 9(10) = -54$.