T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 3 ©

N O V E M B E R 7, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

2015-2016 TMSCA Mathematics Test Three

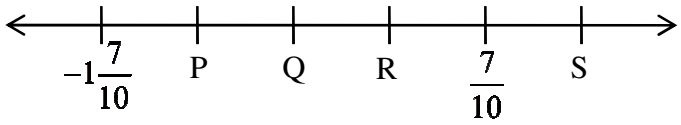
1. Evaluate: $5!+12(3-10)\div 20\times 15$.

   A. 183 B. 119 C. 120 D. 70 E. 57

2. Caroline had a rope that was 15 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:5:10 with 10 inches of string left over. How long was the longest piece?

   A. 8 ft. 4 in. B. 7 ft. 10 in. C. 11 ft. 6 in. D. 8 ft. 10 in. E. 10 ft. 8 in.

3. The distances between the hash marks (|) are equal. Find $P+Q+R+S$.



   A. 2.4 B. 4.9 C. -0.5 D. 0.3 E. -0.2

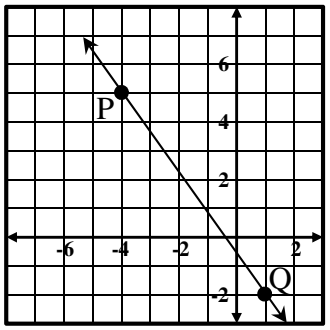
4. What is the median of the first five perfect numbers?

   A. 262 B. 4312 C. 6711798 D. 496 E. 8128

5. Evaluate: $\dfrac{(x+2)!}{(x-2)!}\div\dfrac{x!}{(x-1)!}$.

   A. $x^5+2x^4-x^3-2x^2$ B. x^3+2x^2-x-2 C. x^4-5x^2+4 D. x^6+4x^2 E. x^4-5x^2+5

6. Which of the following is the standard form of the equation of the perpendicular bisector of $\overline{PQ}$?



   A. $5x+7y=3$ C. $7x-5y=-10$ E. $7x+5y=-3$

   B. $7x-5y=-17$ D. $5x-7y=-18$

7. Which of the following properties, if any is not used in this example?

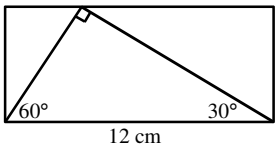
$$\begin{aligned}
5a-5a+7\times\left(3\times\frac{4}{7}\right) &= a(5-5)+7\times\left(3\times\frac{4}{7}\right)\\
&= a\times 0+7\times\left(3\times\frac{4}{7}\right)\\
&= a\times 0+7\times\left(3\times\frac{4}{7}\right)\\
&= 0+7\times\left(3\times\frac{4}{7}\right)\\
&= 0+7\times\left(\frac{4}{7}\times 3\right)\\
&= 0+\left(7\times\frac{4}{7}\right)\times 3\\
&= 0+4\times 3\\
&= 0+12\\
&= 12
\end{aligned}$$

   A. Distributive B. Commutative C. Associative D. Substitution E. All are used

8. The point of intersection of the three perpendicular bisectors of a triangle is called the_____.

   A. Center B. Circumcenter C. Centroid D. Incenter E. Orthocenter

9. Find the area of the rectangle shown.

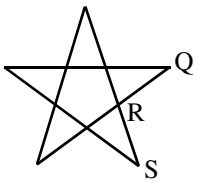


A. 36 cm^2 B. $36\sqrt{3}\text{ cm}^2$ C. 24 cm^2 D. $48\sqrt{3}\text{ cm}^2$ E. 72 cm^2

10. Two consecutive angles in a hexagon are supplementary. Another one of the angles is a right angle. The remaining three angles are congruent. What is the measure of one of the three congruent angles?

A. 60° B. 120° C. 135° D. 90° E. 150°

11. The angles at each point on the star shown are congruent. What is the measure of the angle QRS?



A. 144° B. 135° C. 120° D. 96° E. 108°

12. How many integral values of n exist such that $n > 2$ and $\frac{n!}{(n-2)!} \le 150$?

A. 10 B. 9 C. 11 D. 8 E. 7

13. The chess club consists of 6 girls and 4 boys. How many different 3-person teams can the coach form if any team must have at least one boy?

A. 100 B. 35 C. 60 D. 95 E. 72

14. What is the area of the region entirely bounded by the two functions $f(x) = 2x^2 - 3x + 8$ and $g(x) = -x + 20$?

A. $\frac{125}{6}$ B. 30 C. 40 D. $\frac{125}{3}$ E. $\frac{43}{2}$

15. If $x + y = -5$, and $xy = -8$, then $x^3 + y^3 =$

A. 5 B. -320 C. -85 D. 63 E. -245

16. $\tan\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) \div \cot\left(\frac{5\pi}{4}\right)\csc\left(\frac{\pi}{4}\right) \div \cos\left(\frac{5\pi}{4}\right)\csc\left(\frac{5\pi}{4}\right) =$

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 4 D. 2 E. 1

17. There are two values of k for which $\det\begin{bmatrix} -k & 6 \\ 3 & k+3 \end{bmatrix} = -58$. The sum of those two values is

A. -5 B. -3 C. 5 D. -13 E. 3

18. What are the odds that a factor of 144 is a multiple of 4?

A. 3 to 2 B. 9 to 7 C. 3 to 5 D. 9 to 16 E. 3 to 8

19. $(3-2i)^5 =$

A. $-597-122i$ B. $243-32i$ C. $-597+122i$ D. $-243-32i$ E. $-275+211i$

20. How many 3-digit numbers exist such that the sum of their digits equals 5?

A. 10 B. 9 C. 15 D. 12 E. 14

21. Find the digit in the millionths place of the sum of the series: $4+8+\frac{64}{3!}+\frac{256}{4!}+\frac{1024}{5!}...$

A. 5 B. 0 C. 2 D. 4 E. 8

22. How many distinct arrangements can be formed using all of the letters in the words "APPLE PIE"?
A. 6720 B. 1260 C. 3360 D. 20160 E. 4480

23. Which of the following statements is a false statement for $f(x)=\begin{cases}3x^2-3, & x\le 2\\ 5x-1, & x>2\end{cases}$
A. $f(2)$ exists C. $\lim_{x\to 2^-} f(x)$ exists E. None of them
B. $\lim_{x\to 2^+} f(x)$ exists D. f is continuous

24. If $g(x)=x-1$ and $f(x)=x^4$, find $f(g(x+2))$.
A. $x^4+3x^3+3x^2+x$ C. x^4 E. $x^4+4x^3+6x^2+4x-1$
B. $x^4+4x^3+6x^2+4x+1$ D. x^4+1

25. The Beanery would like to market a mix of two of its most popular coffees. The Good Morning coffee sells for \$6 per pound, while the Dark Rich coffee sells for \$8 per pound. How much Good Morning coffee should be used to create one pound of a mix that sells for \$6.50 per pound?
A. 2.5 oz. B. 12 oz. C. 8 oz. D. 4 oz. E. 9.5 oz.

26. Given $\begin{bmatrix}a & 2\\ 7 & b\end{bmatrix}\times\begin{bmatrix}2b & 4\\ -a & 1\end{bmatrix}=\begin{bmatrix}24 & 14\\ 55 & 33\end{bmatrix}$, find the value of *a*.
A. 5 B. 6 C. 3 D. 2 E. 10

27. A triangle with side lengths 12 cm, 12cm and 22 cm is a(n)_____triangle.
A. scalene acute B. scalene obtuse C. isosceles acute D. isosceles obtuse E. scalene right

28. A particle is moving along a straight line with a function $f(t)=6t^2-5t-11$, where $f(t)$ is the distance in in meters per second. Find the instantaneous rate of change at a time of 2 seconds.
A. 19 m/s B. 3 m/s C. 1 m/s D. $\frac{11}{6}$ m/s E. $\frac{5}{6}$ m/s

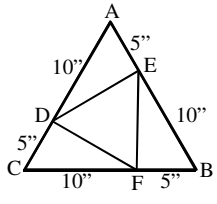
29. If $\log 9=P$, and $\log 5=Q$, then $\log 16.2=$
A. $2P-Q$ B. $\frac{2Q}{P}$ C. $\frac{Q^2}{P}$ D. $\frac{P}{Q^2}$ E. $\frac{P-Q}{5}$

30. If *P, Q* and *R* represent digits in $RPQ_4+QRP_3-PQR_2$ has a numeric value in base 10 of:
A. $9P+12Q+20R$ B. $7P-6Q+14R$ C. $21P+11Q+6R$ D. $P+8Q+18R$ E. $21P-11Q+6R$

31. If *P, Q* and *R* are real numbers such that $P+Q+R=10$, $R^2=P^2+Q^2$ and $PQ=6$, find the value of *R*.
A. 4.1 B. 5.2 C. 4.4 D. 5 E. 5.1

32. Which of the following equations in rectangular form can be written as $r-12\sin\theta=0$ in polar form?
A. $x^2+y^2=36$ C. $x^2+y^2=0$ E. $x^2+y^2-6y=0$
B. $x^2-6x+y^2=0$ D. $x^2+y^2-12y=0$

33. Find the area of $\triangle DEF$. (nearest tenth)
A. 21.7 in^2 B. 43.3 in^2 C. 28.9 in^2 D. 30.7 in^2 E. 32.5 in^2



34. Find the remainder when $f(x)=6x^3-x^2+7x+5$ is divided by $x-7$.
A. -2151 B. 2063 C. 2161 D. -2053 E. 1965

35. A sales clerk is packaging blue, red and black pens for a back-to-school sale. How many different packages of 5 pens can he make?

A. 21 B. 56 C. 28 D. 56 E. 35

36. Two roots of $f(x) = x^3 + bx^2 + cx + d$ are 5 and $3 + i$. Find $b + c + d$.

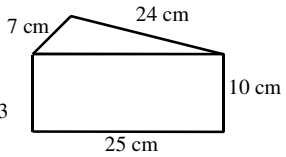
A. 15 B. -11 C. -21 D. -36 E. 26

37. Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a) = f(b)$, then there is at least one number c in (a,b) such that $f'(c) = 0$.

A. Sandwich Theorem C. Rolle's Theorem E. Fundamental Theorem of Calculus

B. Intermediate Value Theorem D. Fundamental Theorem of Algebra

38. Calculate the total volume of the triangular prism shown.



A. 728 cm^3 B. 644 cm^3 C. 924 cm^3 D. 840 cm^3 E. 560 cm^3

39. Line *m* has a slope of -5 and passes through the point $(-4,6)$. Line *n* passes through the points $(1,-1)$ and $(4,8)$. Line *m* intersects line *n* at (x, y). Find $x + y$.

A. 6.5 B. -6.5 C. -5.25 D. -10.25 E. -9

40. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 9.

			2				(row 1)
		4	6	8			(row 2)
	10	12	14	16	18		(row 3)
20	22	24	26	28	30	32	(row 4)
			…				(…)

A. 166 B. 134 C. 154 D. 146 E. 170

41. The ratio of length to width of a rectangle is 5:3 and the area is 43.35 in^2. What is the perimeter of the rectangle?

A. 13.6 in B. 26.3 in C. 21.7 in D. 13.2 in E. 27.2 in

42. The function $f(x) = \dfrac{2x^3}{x^2 - 8}$ is increasing at which of the following values of *x*?

A. -5 B. -4 C. -1 D. 0 E. 2

43. If $\dfrac{2x-5}{5x-2} - \dfrac{5x+2}{2x-5} = \dfrac{Ax^2 + Bx + C}{Px^2 + Qx + R}$, then $\dfrac{A+B+C}{P+Q+R} =$

A. $-57\dfrac{1}{10}$ B. $1\dfrac{1}{3}$ C. $\dfrac{2}{5}$ D. $-\dfrac{8}{9}$ E. $2\dfrac{2}{9}$

44. Carmen Cents has 75 nickels, dimes and quarters. She has three times as many nickels as dimes and two more nickels than quarters. How much money does she have?

A. \$10.90 B. \$10.50 C. \$11.60 D. \$11.90 E. \$7.60

45. The triangles ABC and PQR exist such that $\angle ABC \cong \angle QRP$, $\angle ACB \cong \angle QPR$, $AB = 12$, $BC = 27$, $QP = 48$ and $RP = 36$. Find $AC + QR$.

A. 36 B. 42 C. 48 D. 52 E. 54

46. The point $(3,-11)$ lies on a circle with center $(8,-5)$. Which of the following points lies inside the circle?

A. $(15,2)$ B. $(13,3)$ C. $(16,-5)$ D. $(16,0)$ E. $(15,-8)$

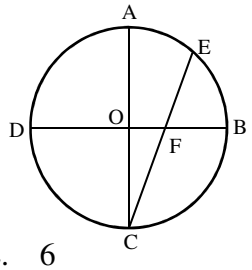
47. Find the area of the triangle with vertices $(-3,5)$, $(6,12)$ and $(4,-1)$.

A. 51.5 B. 46.5 C. 49 D. 47.5 E. 48

48. Meredith set out to row on a lake. She rowed 750 m on a bearing of 75°, then 250 m on a bearing of 25°, then 800 m on a bearing of 52°. How far is she from her original starting point? (nearest meter)

A. 1800 m B. 933 m C. 1723 m D. 1328 m E. 1524

49. Circle O has perpendicular diameters and a chord, find AE if CF =11 inches and EF = 9 inches. (nearest tenth)



A. 7 in B. 9.9 in C. 6.3 in D. 8.7 in E. 5.6 in

50. If $9^{x+y} = 6561$ and $27^{x-y} = 729$, then $x^2 - y^2 =$

A. 20 B. 8 C. 12 D. 4 E. 6

51. Cornelius has a bag that contains 6 blue chips, 5 red chips and 2 green chips. If he selects two chips without replacement, what is the probability that he will draw two of the same color?

A. $\frac{65}{132}$ B. $\frac{5}{13}$ C. $\frac{4}{13}$ D. $\frac{13}{36}$ E. $\frac{1}{3}$

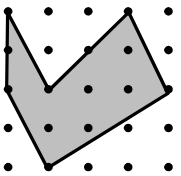
52. What is the constant term in the binomial expansion of $\left(3x^3 - \frac{1}{x}\right)^8$?

A. 5670 B. 70 C. 28 D. 6561 E. 252

53. Classify the graph of $3x^2 - y^2 + 18x + 6y = 9$

A. Parabola B. Ellipse C. Hyperbola D. Circle E. None of These

54. The dots are 3 units apart vertically and horizontally. Find the area of the shaded region.



A. 36 units^2 B. 72 units^2 C. 34 units^2 D. 64 units^2 E. 39 units^2

55. How many solutions are there for the equation $3x + 5y = 125$ where both x and y are non-negative integers?

A. 9 B. 10 C. 8 D. 7 E. 6

56. If $\frac{4x+13}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$, then $AB =$

A. 4 B. $-\frac{17}{16}$ C. $-\frac{17}{4}$ D. $\frac{17}{4}$ E. $\frac{17}{16}$

57. The square root of 207 in base 9 is:

A. 15_9 B. 17_9 C. 16_9 D. 14_9 E. 13_9

58. If $y^2 = -5 - 12i$ and $y^3 = 46 + 9i$ where $y = a + bi$ then $a + b =$

A. 1 B. -38 C. 5 D. -62 E. 6

59. $4^3 + 5^3 + \ldots + 12^3 + 13^3 + 14^3 + 15^3$

A. 14400 B. 10989 C. 11025 D. 14364 E. 2744

60. What is the area of a regular hexagon in terms of the length of the apothem, a?

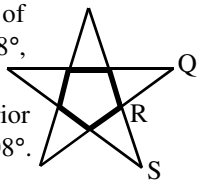
A. $2a^2\sqrt{3}$ B. $\frac{4a^2\sqrt{3}}{3}$ C. $\frac{3a^2\sqrt{3}}{4}$ D. $\frac{3a^2\sqrt{3}}{2}$ E. $3a^2\sqrt{3}$

2015-2016 TMSCA Mathematics Test Three Answers

1. E	21. B	41. E
2. A	22. C	42. A
3. E	23. E	43. B
4. D	24. B	44. B
5. B	25. B	45. D
6. D	26. C	46. E
7. E	27. D	47. A
8. B	28. A	48. C
9. B	29. A	49. C
10. E	30. D	50. B
11. E	31. C	51. E
12. A	32. D	52. E
13. A	33. E	53. C
14. D	34. B	54. B
15. E	35. A	55. A
16. D	36. C	56. B
17. B	37. C	57. D
18. A	38. D	58. A
19. A	39. E	59. D
20. C	40. D	60. A

2015-2016 TMSCA Mathematics Select Solutions

11. Each interior angle of the bold pentagon is 108°, and $\angle QRS$ is a vertical angle to one of the interior angles so $m\angle QRS = 108°$.



12. The left of the equation simplifies to $n(n-1)$, so estimate using $\sqrt{150} \approx 12.25$. $12(11) = 132$ while $13(12) = 156$, so 12 is the largest possible value of n. All the values are 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 or ten total values.

15. $x^3 + y^3 = (x+y)\left((x+y)^2 - 3xy\right)$ or $(-5)(25-3(-8)) = -245$.

21. This is the power series for the function $f(x) = e^x - 1$ when $x = 4$. $f(4) \approx 53.59815003$. The digit in the millionths place is 0.

22. There are 8 letters total with "P" repeated thrice and "E" repeated twice, so the number of distinct arrangements is: $\frac{8!}{(3!)(2!)} = 3360$.

24. $f\left(g(x+2)\right) = f(x+1) = (x+1)^4$, use coefficients from the binomial theorem or from Pascal's triangle to get $x^4 + 4x^3 + 6x^2 + 4x + 1$.

30. Re-written in base 10 the number will be $(16R + 4P + Q) + (9Q + 3R + P) - (4P + 2Q + R)$ or $P + 8Q + 18R$.

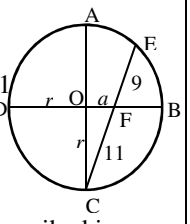
31. $(P+Q)^2 = (10-R)^2$, so $P^2 + 2PQ + Q^2 = 100 - 20R + R^2$ then using substitution, $R^2 + 2PQ = 100 - 20R + R^2$ which simplifies to $2(6) = 100 - 20R$. $R = 4.4$.

35. There are 3 types of pens and the clerk is making packages of 5, so the total number of distinct packages is ${}_{3+5-1}C_5 = 21$.

40. The median will always be the middle number in the row. Using the pattern:

2+4=6
6+8=14
14+12=26…..114+32=146.

49. To find *r*, use the two relationships $a^2 + r^2 = 121$ and $(r+a)(r-a) = 99$. Adding the two together, $2r^2 = 220$ and. Then, because triangle AEC is inscribed in a semi-circle, it has a right angle at E and $20^2 + (AE)^2 = (2r)^2$ and $AE = \sqrt{4r^2 - 400}$ or $AE = \sqrt{2(220) - 400} \approx 6.3$ inches.



52. The constant term will be ${}_8C_2\left(3x^3\right)^2\left(-\frac{1}{x}\right)^6 = 252$.

54. I = # of interior points
P = # of perimeter points
$A = \frac{2I+P}{2} - 1 = \frac{2(5)+8}{2} - 1 = 8$, but each square unit on the graph represents 9 square units because of the scale given, so the actual area is 72

57. $207_9 = 169_{10}$. The square root of 169 is 13 which is written as 14_9.

59. The sum of the first *n* cubes is given by the formula $\left(\frac{n(n+1)}{2}\right)^2$, so the sum of the series given will be $\left(\frac{15(16)}{2}\right)^2 - 1 - 8 - 27 = 14364$.