TMSCA HIGH SCHOOL

MATHEMATICS

TEST #3 ©

NOVEMBER 8, 2014

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate $30+13\left[(3-11)\div 4\right]-12$.

A) -8   B) 44   C) 18   D) 22   E) -5

2. Find the number of positive integral divisors of 288.

A) 24   B) 12   C) 15   D) 16   E) 18

3. Larry borrowed $2200 for a home improvement project at a compound interest rate of 4.25%. The loan was for 2 years and interest was compounded monthly. How much interest will he have paid at the end of two years? (nearest cent)

A) $187.00   B) $206.78   C) $192.43   D) $194.82   E) $196.89

4. The two lines intersect at the point (x, y). Find $x+y$.

A) 1.8   B) 1.6   C) -0.2   D) $0.\overline{5}$ E) 0.8
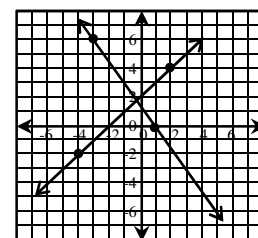
5. If $\dfrac{x^4-5x^2+4}{\left(x^2+4x+4\right)\left(x^2+2x+1\right)}=\dfrac{x^2+ax+2}{x^2+bx+2}$, find $\dfrac{a}{b}$.

A) 0.5   (B) -2   (C) 1   (D) -0.5   (E) -1

6. If $4\times(9+3)=36+12$ and $36+12=48$ then $4\times(9+3)=48$. This is an example of _____ property.

A) Reflexive   B) Associative   C) Substitution   D) Transitive   E) Symmetric

7. Levi and CW are riding a seesaw. Levi weighs 85 pounds and is sitting 5 feet from the center of the seesaw. CW weighs 91 pounds and is sitting on the other end of the seesaw. If the seesaw is balanced, how far is CW from the center? (nearest inch)

A) 4' 6''   B) 4' 8''   C) 4' 10''   D) 5'   E) 4' 9''

8. Mr. Harris finished teaching his history class at 3:12 pm. What was the smaller angle formed by the hour and minute hands of his clock at that time?

A) 18°   B) 24°   C) 27°   D) 32°   E) 15°

9. A set of positive integers has a mean of 18, a median of 15, a mode of 28 and a range of 19. If A, B, C, D and E are the integers arranged from least to greatest, the value of B is?

A) 9   B) 12   C) 10   D) 15   E) 11

10. The dots are 6 units apart vertically and horizontally. Find the area of the shaded region.

A) 288 units^2 B) 324 units^2 C) 306 units^2 D) 576 units^2 E) 352 units^2
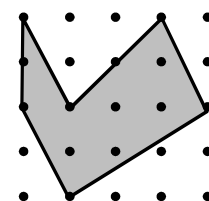
11. If $\begin{bmatrix} 2 & -3 \\ a & -11 \end{bmatrix}-\begin{bmatrix} 3 & b \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} -1 & 12 \\ 10 & -16 \end{bmatrix}$ then $a+b=$

A) 17   B) -15   C) 27   D) -7   E) 23

12. $\angle A$ and $\angle B$ are complementary and $m\angle B$ is 22° less than four times $m\angle A$. Find the measure of the supplement of $\angle A$. (nearest tenth)

A) 22.7°   B) 49.6°   C) 157.3°   D) 10.4°   E) 157.6°

13. Given $x-y=-5$ and $xy=32$ find the value of x^3-y^3.

A) -445   B) -355   C) -195   D) -605   E) -35

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14. Terry, Taylor and Tyler can mow Margie's lawn alone in 2 hours, 1 hour and 3.5 hours respectively. How long would it take them to mow Margie's lawn if they worked together? (nearest minute)
A) 32 minutes   B) 34 minutes   C) 33 minutes   D) 38 minutes   E) 36 minutes

15. What is the sum of all the numbers greater than one in the 10th row of Pascal's triangle?
A) 1022   B) 510   C) 2046   D) 548   E) 254

16. What is the shortest distance between the line $4x-3y=13$ and the point $(9,7)$?
A) 3   B) 0.8   C) 2.6   D) 1.2   E) 0.4

17. What is the Heronian mean of 24 and 96?
A) 56   B) 60   C) 48   D) 52   E) 64

18. Lila walks to a friend's house every day to carpool. Use the graph to determine the mean speed of her friend's car.
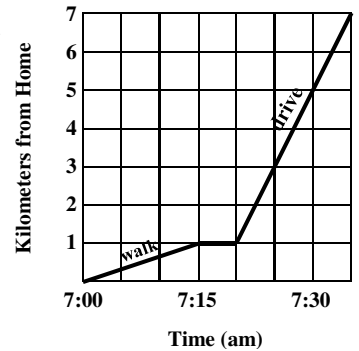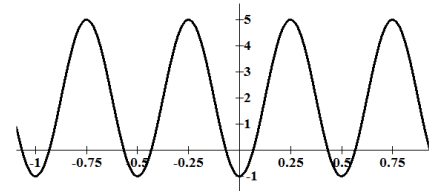

A) 30 kmph   B) 24 kmph   C) 36 kmph   D) 28 kmph   E) 32 kmph

19. If $f(x)=3^x$, $g(x)=\log_3 x$ and $a\geq 2$, then $g(f(a+1))=$
A) $a+1$ B) 3^{a+1} C) $3a+3$ D) $\log_3(a+1)$ E) $3a+1$

20. Solve $2\sin x = \tan x$, for $0 < x \leq \dfrac{\pi}{2}$.
A) $\dfrac{\pi}{6}$ B) $\dfrac{\pi}{3}$ C) $\dfrac{\pi}{4}$ D) $\dfrac{\pi}{2}$ E) no solution

21. The graph of $f(x)=A+B\cos(C\pi x)$. $ABC=$

A) -6   C) 12   E) -24
B) -12   D) 18

22. This year 82% of Miss Marple's class passes their final exam. What are the odds that a student chosen at random to critique the class did not pass?
A) 2:9   B) 9:41   C) 9:50   D) 41:50   E) 41:9

23. John would like to know the height of his apple tree before he buys a ladder. The angle of elevation to the top of the tree is 25° from where he is standing. From a point 45 feet closer, the angle of elevation is 41°. How tall is the tree? (nearest inch)
A) 52' 1''   B) 20' 5''   C) 45' 3''   D) 19' 6''   E) 59' 11''

24. A and B are the roots of $f(x)=2x^2-3x-20$. Calculate the value of $A^4-4A^3B+6A^2B^2-4AB^3+B^4$.
A) $\dfrac{81}{16}$ B) $\dfrac{2401}{16}$ C) $\dfrac{14641}{16}$ D) $\dfrac{28561}{16}$ E) $\dfrac{81}{256}$

25. $\displaystyle\int_{-n}^{n}\left(a-bx^5\right)dx=$
A) $2an$ B) 0 C) $\dfrac{bn^6}{3}$ D) $an-\dfrac{bn^6}{6}$ E) $2an-\dfrac{bn^6}{6}$

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26. $101_2 + 323_4 + 545_6 = \underline{\hspace{2cm}}_8$

A) 421   B) 969   C) 243   D) 303   E) 333

27. If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$. This is the $\underline{\hspace{2cm}}$,

A)   Sandwich Theorem   C)   Rolle's Theorem   E)   Fundamental Theorem of Calculus

B)   Intermediate Value Theorem   D)   Fundamental Theorem of Algebra

28. S_n is the sum of the first n terms of the arithmetic sequence $2+5+8+....$ For what value of n is $S_n = 260$?

A) 11   B) 15   C) 12   D) 13   E) 10

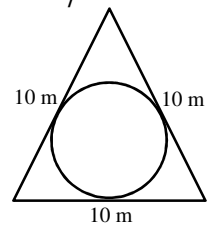29. Find the equation of the directrix of the parabola with the equation $2x^2 - 4x + y + 4 = 0$.

A) $y = \dfrac{8}{15}$ B) $x = -\dfrac{15}{8}$ C) $x = -\dfrac{8}{15}$ D) $x = \dfrac{8}{15}$ E) $y = -\dfrac{15}{8}$

30. If $f(x) = \sqrt[3]{x^4}$, $f'(x) =$

A) $\dfrac{4x\sqrt{x}}{3}$ B) $\dfrac{4\sqrt[3]{x}}{3}$ C) $\dfrac{3x^2\sqrt[3]{x}}{7}$ D) $\dfrac{7x\sqrt{x}}{3}$ E) $\dfrac{3x^3\sqrt{x}}{7}$

31. Find the area of the inscribed circle. (nearest tenth)

A) 26.2 m^2 B) 43.3 m^2 C) 31.4 m^2 D) 56.2 m^2 E) 78.5 m^2

10 m   10 m

10 m

32. Find the Real number solution set of $|6 - 2x| \geq 4$.

A) $(-\infty, \infty)$ B) $(-\infty, -5] \cup [-1, \infty)$ C) $[1, 5]$ D) $(-\infty, 1] \cup [5, \infty)$ E) $[-5, -1]$

33. Susan plans on buying 5 shirts for \$27.95 each, 2 skirts for \$32.99 each and a pair of school shoes for \$54.95. The local tax rate is 8.5%. How much money will she save if she makes her purchases during tax-free weekend?
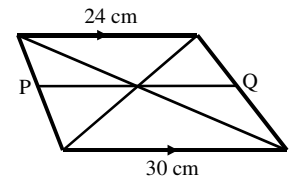
A) \$22.16   B) \$21.51   C) \$19.35   D) \$20.76   E) \$23.56

34. A group agrees to share equally in the cost of a \$48,000 piece of machinery. If they can find two more group members, each member's share will decrease by \$4000, How many are presently in the group?

A) 3   B) 6   C) 4   D) 5   E) 2

35. Find PQ.

24 cm

A) $\dfrac{80}{3}$ cm   B) 27 cm   C) $\dfrac{82}{3}$ cm   D) $12\sqrt{5}$ cm   E) $18 + 4\sqrt{5}$ cm

P   Q

30 cm

36. Let $a_1 = 7$, $a_2 = -2$ and $a_n = a_{n-2} - 2a_{n-1}$. Find a_6.

A) 59   B) -24   C) -66   D) -142   E) 25
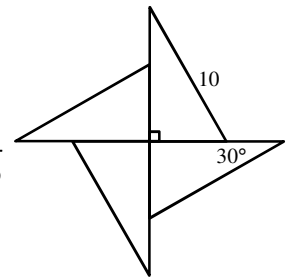
37. What is the mean value of the function $f(x) = 3x^2 - 2x$ on the interval $[1,4]$?

A) 20.5   B) 19.5   C) 16   D) 29   E) 18

38. The chords $\overline{AC}$ and $\overline{BD}$ intersect inside $\odot O$ at P. If $AP = 9$, $CP = x^2$, $BP = x+1$ and $DP = 6x$, find the positive value of x.

A) 4   B) 6   C) 3   D) 2   E) 5

39. The octagon shown is made up of four congruent triangles. What is the perimeter
    of the octagon?

    A) $20\sqrt{3}-20$ B) $40\sqrt{3}$ C) $40+20\sqrt{3}$ D) 60   E) $20+20\sqrt{3}$

40. The letters in the world TUESDAY are arranged in a line. How many of distinct
    arrangements are possible that begin with D and end with S?
A) 21   B) 35   C) 120   D) 24   E) 720

41. Which of the following is not a triangular number?
A) 1225   B) 728   C) 630   D) 903   E) 406

42. If $(2-i)-(3-2i)\times(5-3i)=a+bi$ then $b=?$
A) -6   B) 18   C) 20   D) -7   E) -2

43. Find the sum of the infinite series: $-1.2-0.9-0.675-0.50625....$
A) -3.28125   B) -1.6   C) -2.4   D) -4.8   E) 3.6

44. Given the Fibonacci-type sequence 12, A, B, -58, C, -139… . Calculate the value of A+B+C.
A) -81   B) -35   C) -220   D) -58   E) -139

45. How many 3-digit numbers can be made with the digits 0, 0, 2, 4 and 6?
A) 21   B) 11   C) 15   D) 18   E) 12

46. A regular octagon with vertices A, B, C, D, E, F, G and H in order clockwise is inscribed in a circle. What
    is $m\angle BHF$?
A) 67.5°   B) 45°   C) 90°   D) 75°   E) 22.5°

47. Which of the following is an equation of the line normal to the curve defined by $2x^2+3xy-y^2=1$ at the
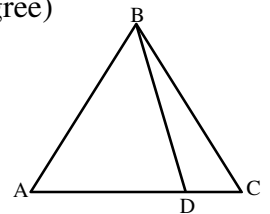   point $(2,-1)$?
A) $5x-8y=18$ B) $x+2y=0$ C) $5x+8y=2$ D) $5x+8y=2$ E) $7x+5y=9$

48. Ten liters of 30% acid solution is obtained by mixing 20% solution with 50% solution. How much of the
    50% solution is used in the final mixture?
A) 5 liters   B) $2\frac{1}{2}$ liters   C) $3\frac{1}{3}$ liters   D) $6\frac{2}{3}$ liters   E) $7\frac{1}{2}$ liters

49. The triangle ABC is equilateral and AD is triple CD. Calculate $m\angle BDC$. (nearest degree)

A) 112°   B) 101°   C) 105°   D) 108°   E) 106°

50. $\det \begin{bmatrix} 2 & -1 & 7 \\ 5 & -3 & 4 \\ 2 & 0 & -2 \end{bmatrix} =$

A) 54   B) 36   C) 72   D) 12   E) 18

51. $\left(2cis\left(\dfrac{\pi}{6}\right)\right)^4 =$

A) $-4+4i\sqrt{3}$ B) $8-8i\sqrt{3}$ C) $-8\sqrt{3}+8i$ D) $4+4i\sqrt{3}$ E) $-8+8i\sqrt{3}$

52. Blackbeard sailed from his hideout at bearing 275° for 80 miles to island A, then on to island B at a bearing of 122° for 72 miles. How far will Blackbeard sail to go directly back to his hideout? (nearest mile)

A)   69 mi   B)   44 mi   C)   36 mi   D)   41 mi   E)   52 mi

53. If $f(x) = \tan x$ then $\lim\limits_{h \to 0} \dfrac{f(\pi + h) - f(\pi)}{h}$ is

A)   0   B)   -1   C)   1   D)   Undefined   E) $\sqrt{3}$

54. Solve $\log_3 x + \log_3(x^2 - 8) = \log_3(8x)$ for x.

A)   -4, 0 and 4   B)   4   C)   2   D)   -2, 0 and 2   E)   No solution

55. Set $S = \{0, 1, 2 \ldots 7\}$. How many 4-element subsets of set S are there?

A)   16   B)   35   C)   128   D)   70   E)   49

56. Which of the following series converges?

A) $\displaystyle\sum_{n=1}^{\infty} \left(\dfrac{3}{2}\right)^n$ B) $\displaystyle\sum_{n=0}^{\infty} \dfrac{n+1}{2n+1}$ C) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{1000(n+1)}$ D) $\displaystyle\sum_{n=0}^{\infty} \dfrac{3}{2^n}$ E) $\displaystyle\sum_{n=1}^{\infty} \log n$

57. A light bulb is placed 22 feet above a straight horizontal path. A man is walking away from the light at a rate of 5 feet per second. If the man is 6 feet tall, at what rate is the tip of the man's shadow moving? (nearest tenth)

A)   1.6 ft/sec   B)   5.5 ft/sec   C)   5.4 ft/sec   D)   7.7 ft/sec   E)   6.9 ft/sec

58. What are the odds of randomly choosing the letter T from a bag containing the letters in the words TEXAS MATH?

A) $\dfrac{2}{9}$ B) $\dfrac{1}{7}$ C) $\dfrac{2}{7}$ D) $\dfrac{1}{5}$ E) $\dfrac{5}{9}$

59. A belt joins two pulleys shown. If the smaller pulley rotates at 72 rpm, then the larger pulley is rotating at _____rpm.

A)   12.22   B)   39.60   C)   10.28   D)   130.91   E)   32.29
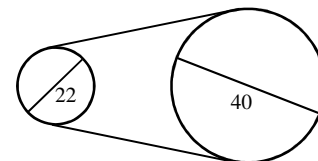
60. Change the base 10 proper fraction $\dfrac{3}{7}$ to a repeating decimal in base 6.

A)   0.244444…   B)   0.151515…   C)   0.232323…   D)   0.1434343…   E)   0.533333…

2014-2015 TMSCA Mathematics Test Three Answers

1. A	21. E	41. B
2. E	22. B	42. B
3. D	23. C	43. D
4. B	24. D	44. E
5. E	25. A	45. A
6. D	26. A	46. C
7. B	27. B	47. C
8. B	28. D	48. C
9. C	29. E	49. E
10. A	30. B	50. B
11. D	31. A	51. E
12. E	32. D	52. C
13. D	33. A	53. C
14. B	34. C	54. B
15. A	35. A	55. D
16. E	36. D	56. D
17. A	37. C	57. E
18. B	38. D	58. C
19. A	39. E	59. B
20. B	40. C	60. C

2013-2014 TMSCA Mathematics Test Three Select Solutions

9. The numbers must be: 9, B, 15, 28, 28 to account for the mode, median and range. The sum of the numbers must be 90 for the mean to be 18 so B = 10.

10. Let I = dots interior to the polygon and P = dots on the perimeter.
$A = \dfrac{2I + P - 2}{2} = \dfrac{10 + 8 - 2}{2} = 8$, but each square on the grid represents 36 square units, so the actual area is 288 u^2.

13. $x^3 - y^3 = (x - y)\left((x - y)^2 + 3xy\right) =$
$(-5)(25 + 3(32)) = -605$

15. $2^{10} - 2 = 1022$

17. $\dfrac{24 + 96 + \sqrt{24(96)}}{3} = 56$

18. $\dfrac{6 km}{15 \min} \cdot \dfrac{60 \min}{1 hr} = 24 \dfrac{km}{hr}$

24. This is the binomial expansion of $(A - B)^4$. The roots are $-\dfrac{5}{2}$ and 4,
$\left(\dfrac{13}{2}\right)^4 = \dfrac{28561}{16}$

25. Disregard the term with the odd power. $\int_{-n}^{n} a \, dx = [ax]_{-n}^{n} = 2an$

26. $5_8 + 73_8 + 321_8 = 421_8$

28. $S_n = \dfrac{n}{2}(2u_1 + d(n-1))$ so
$260 = \dfrac{n}{2}(4 + 3(n-1))$ and $n = 13$

30. $\dfrac{d}{dx}\left(x^{4/3}\right) = \dfrac{4}{3} x^{1/3} = \dfrac{4\sqrt[3]{x}}{3}$

31. Using special triangles, the radius of the circle is $\dfrac{5}{\sqrt{3}}$ and the area of the circle is $\dfrac{25}{3}\pi \approx 26.2$

35. Harmonic mean
$PQ = \dfrac{2(24)(30)}{24 + 30} = \dfrac{80}{3}$

38. $9x^2 = 6x(x + 1)$ so $3x(x - 2) = 0$ and $x = 2$

43. Infinite geometric series with first term -1.2 and common ratio of 0.75.
$S = \dfrac{-1.2}{1 - .75} = -4.8$

46. The measure of the arc between any two consecutive points is 45°. $\angle BHF$ is an inscribed angle with an intercepted arc of 180°, so $m\angle BHF = 90°$.

49. Let AB = 4.
$(BD)^2 = 4^2 + 1^2 - 2(4)(1)\cos 60 = 13$
Use law of cosines again
$4^2 = 13 + 1 - 2\sqrt{13}\cos\theta$ and $\theta \approx 106°$

51. $16cis\left(\dfrac{2\pi}{6}\right) = 16\cos\dfrac{2\pi}{6} + 16i\sin\dfrac{2\pi}{6} =$
$16\left(-\dfrac{1}{2}\right) + 16i\left(\dfrac{\sqrt{3}}{2}\right) = -8 + 8i\sqrt{3}$

54. $\log_3\left(x(x^2 + 8)\right) = \log_3(8x)$ so
$x^3 - 8x = 8x$, $x^3 - 16x = 0$ and $x(x - 4)(x + 4) = 0$. Of the three solutions, 4 is the only one that is valid for the original equation.

57. Let x be the distance from the man to the pole an y be the length of his shadow.
Using similar triangles, $\dfrac{6}{22} = \dfrac{y}{x + y}$ which can be manipulated to be $3x = 8y$ and
$3\dfrac{dx}{dt} = 8\dfrac{dy}{dt}$. Replace $\dfrac{dx}{dt}$ with 5 and
$\dfrac{dy}{dt} = \dfrac{15}{8}$. The question is not how fast the shadow is expanding but rather how fast the tip is moving, so the actual answer is
$5 + \dfrac{15}{8} = \dfrac{55}{8} \approx 6.9$ feet/second.

60. $\dfrac{3}{7_{10}} = \dfrac{15}{35_{10}} = 0.\overline{23}_6$ because the 35 indicates 2 repeating places and $15_{10} = 23_6$