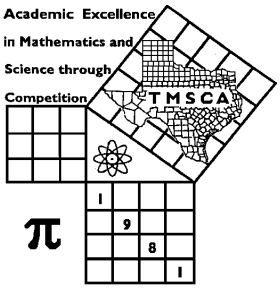


TMSCA HIGH SCHOOL MATHEMATICS TEST# 4 © NOVEMBER 12, 2016

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016-2017 TMSCA Mathematics Test Four

1. **Evaluate:** $\frac{9!}{5!}\times\frac{8!}{11!}\div\frac{7!}{6!}$.

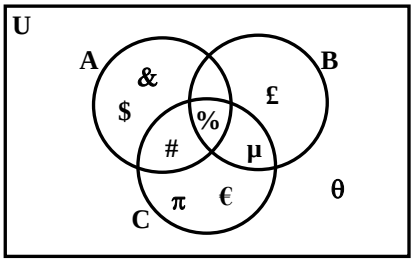
   (A) $\frac{1176}{55}$ **(B)** $\frac{24}{55}$ **(C)** $\frac{24}{11}$ **(D)** $\frac{4}{231}$ **(E)** $\frac{8}{231}$

2. **Karolyn invested $1000 for 4 years in a variable interest account. Her annual interest rates of return are shown below. What was the average interest rate for the four years? (nearest hundredth of a %)**

Year	1	2	3	4
Interest	+ 5%	–2.5%	–3%	+6.2%

   (A) +1.43% **(B)** +1.34% **(C)** +2.68% **(D)** +2.93% **(E)** +2.61%

3. **Use the Venn diagram to identify the set** $(A\cup B)'\cap C$.



   (A) {θ, π, €} **(B)** {#, %, μ} **(C)** {$, &, £} **(D)** {π, €} **(E)** {$, &, £, θ}

4. **The binomials** $(3x+5)$, $(x-8)$ **and** $(3x-5)$ **are all factors of**

   (A) $9x^3-72x^2-25x+100$ **(B)** $9x^3-97x^2+200$ **(C)** $9x^3-72x^2-25x+200$

   (D) $9x^3-72x^2-25x-200$ **(E)** **none of these**

5. **Jay's current age is one-third of his mother's age. In six years, Jay's age will be four years less than half of his mother's age. What is the sum of their current ages?**

   (A) 62 **(B)** 56 **(C)** 58 **(D)** 60 **(E)** 68

6. **The points P and Q have coordinates** $(-5,8)$ **and** $(7,-6)$ **respectively. Which of the following is an equation of the perpendicular bisector of** $\overline{PQ}$**?**

   (A) $7x+6y=13$ **(B)** $6x-7y=-86$ **(C)** $6x-7y=-1$

   (D) $7x+6y=-6$ **(E)** $6x-7y=0$

7. **What is the sum of the arithmetic sequence 11, 12.2, 13.4, …, 25.4?**

   (A) 218.4 **(B)** 254.8 **(C)** 244.4 **(D)** 236.6 **(E)** 252.2

8. **A box contains 5 black marbles, 8 red marbles and 11 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 green marbles?**

   (A) 15:184 **(B)** 55:768 **(C)** 15:169 **(D)** 55:713 **(E)** 121:983

9. $\left[(2+4+6+8+...+66+68)\div 70\right]\div\left[(72+74+76+78+...+136+138)\div 140\right]=$

   (A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{2}$ **(D)** 3 **(E)** 2

10. Simplify: $\left(\frac{x^2+x-56}{x^2+6x-16}\right)\div\left(\frac{x^2+4x-21}{x^2+12x+35}\right)$.

(A) $\frac{x^2-2x-35}{x^2-5x+6}$ **(B)** $\frac{x^2+2x-35}{x^2-x+6}$ **(C)** $\frac{x^2-2x-35}{x^2-x+6}$ **(D)** $\frac{x^2+2x-35}{x^2-5x+6}$ **(E)** $\frac{x^2-2x-35}{x^2+x+6}$

11. Which of the following relations describes a function?

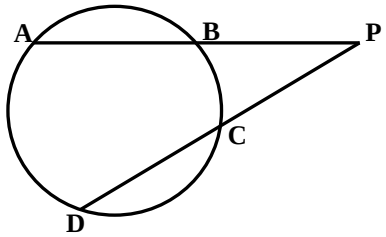
I. $\{(3,-1),(2,0),(1,1),(0,2)\}$

II. $\{(-1,0),(-1,2),(-1,4),(-1,6)\}$

III. $\{(6,-1),(4,-1),(2,-1),(0,-1)\}$

(A) I only **(B) I &III** **(C) II & III** **(D) I & II** **(E) II only**

12. Points A, B, C and D lie on the circle shown. Find x if AB = x, BP = 9', DC = (x + 6)', and CP = 7.5'. (drawing is not to scale)



(A) 13.5' **(B) 30'** **(C) 16.5'** **(D) 4.5'** **(E) 10.5'**

13. A buried conical tank with a 10' diameter and 12' depth is leaking at a rate of 6 gallons per hour. If there is no incoming water, how long will it take for a completely full tank to completely empty? (nearest hour)

(A) 846 hr **(B) 1175 hr** **(C) 783 hr** **(D) 282 hr** **(E) 392 hr**

14. An enneahedron has 9 faces and 14 vertices. How many edges does it have?

(A) 14 **(B) 23** **(C) 21** **(D) 25** **(E) 9**

15. The graph of $y=6-5\sin(3x-75^\circ)$ **reaches a maximum value at:**

(A) $(-65^\circ,1)$ **(B)** $(55^\circ,11)$ **(C)** $(0^\circ,11)$ **(D)** $(-125^\circ,11)$ **(E)** $(55^\circ,1)$

16. Simplify: $\cos(\theta)+\sin(\theta)\tan(\theta)-\sec(\theta)$.

(A) 1 **(B) 0** **(C)** $\cos^2(\theta)$ **(D)** $2\sin(\theta)$ **(E)** $\sin(2\theta)$

17. Use the Fibonacci characteristic sequence ..., $p,-2,q,r,8,...$ **to find** $p+q+r$.

(A) 12 **(B) 9** **(C) 8** **(D) 6** **(E) 15**

18. Let $f(x)=\frac{x+4}{x-7}$, **where** $x\neq 7$. **Find** $f^{-1}(x)$.

(A) $\frac{7x+4}{x+1}$ **(B)** $\frac{x-7}{x+4}$ **(C)** $\frac{11}{x-1}$ **(D)** $\frac{7x+4}{x-1}$ **(E)** $\frac{4x-7}{x+1}$

19. Find $\lim_{x\to 1/2}\frac{2x^2+9x-5}{2x^2+15x-16}$

(A) 0 **(B)** $\frac{1}{2}$ **(C)** $\frac{11}{17}$ **(D)** $\frac{19}{17}$ **(E) does not exist**

20. There are 9 teams in a soccer league. How many in-league soccer games will be played during a season if the teams all play each other once?

(A) 45 (B) 32 (C) 36 (D) 28 (E) 30

21. Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \ldots$ be the terms of the Fibonacci sequence. Find f_{25}.

(A) 78125 (B) 121393 (C) 46368 (D) 46875 (E) 75025

22. The slope of the line going through the points $(-8, y), (-11, 9), (x, -3)$ is $-\frac{4}{3}$. Find $x + y$.

(A) –1 (B) 3 (C) 2 (D) 5 (E) –3

23. $(567_9 + 35_9) \times 5_9 =$ _____$_9$

(A) 1366 (B) 3065 (C) 4114 (D) 3366 (E) 1371

24. The ellipse $49x^2 + 16y^2 - 196x - 588 = 0$ has a center at (h, k). Find $h + k$.

(A) 4 (B) 2 (C) 6 (D) 7 (E) 5

25. Find the sample standard deviation of the set $\{78, 78, 87, 88, 93, 99\}$. (nearest tenth)

(A) 8.3 (B) 7.6 (C) 7.0 (D) 6.4 (E) 6.8

26. If f is continuous on the closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$. This is the _____.

(A) Rolle's Theorem (B) Sandwich Theorem (C) Fundamental Theorem of Calculus

(D) Intermediate Value Theorem (E) Fundamental Theorem of Algebra

27. If $6^x \cdot 36^{2y} = 1$ and $5^{5x} \cdot 25^y = \frac{1}{25}$, then $x + y =$

(A) $-\frac{1}{6}$ (B) $\frac{4}{9}$ (C) $-\frac{6}{25}$ (D) $-\frac{2}{3}$ (E) $-\frac{1}{3}$

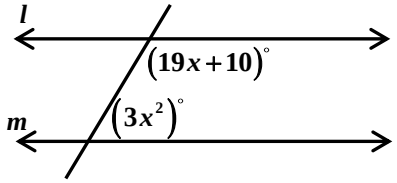
28. If $A + B = 21$ and $AB = 105$, then $A^2 + B^2 =$

(A) 221 (B) 336 (C) 546 (D) 126 (E) 231

29. A pizza restaurant has 3 crust options, 2 cheese options and 10 choices of toppings. On Saturday nights, the restaurant offers a special deal on 2-topping pizzas including pizzas with double portions of one topping. How many distinct special deal pizzas are possible?

(A) 396 (B) 275 (C) 1320 (D) 600 (E) 330

30. In the diagram, $l \parallel m$. Find the value of x.



(A) 5 (B) 7 (C) 9 (D) 4 (E) 3

31. P and Q are the roots of $f(x)=3x^2-x-14$. Calculate $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$.

(A) $\frac{28561}{81}$ **(B)** $\frac{1}{81}$ **(C)** $\frac{83521}{2401}$ **(D)** $\frac{14641}{2401}$ **(E)** $\frac{28561}{1296}$

32. The length of one edge of a regular tetrahedron is $7\sqrt{3}$ cm. The surface area of the tetrahedron is _____cm^2. (nearest tenth)

(A) 191.0 **(B)** 254.6 **(C)** 63.7 **(D)** 147.0 **(E)** 110.3

33. Let $\begin{bmatrix} a & 5 \\ 1 & b \end{bmatrix}\times\begin{bmatrix} 2 \\ 8 \end{bmatrix}=\begin{bmatrix} 54 \\ -22 \end{bmatrix}$. **$a+b=$**

(A) 10 **(B)** –4 **(C)** 7 **(D)** –3 **(E)** 4

34. What is the probability that a factor of 216 is a multiple of 3 greater than 0?

(A) $\frac{1}{4}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{5}{7}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{5}{6}$

35. The function $f(x)=2x^3+9x^2-60x+18$ is decreasing over which of the following intervals?

(A) $[-1,4]$ **(B)** $[-2,5]$ **(C)** $[-7,-2]$ **(D)** $[-5,5]$ **(E)** $[-4,1]$

36. In the expansion of $(ax+3y)^6$, the coefficients of the 3rd and 4th terms are equal. Find the value of *a*.

(A) 3 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

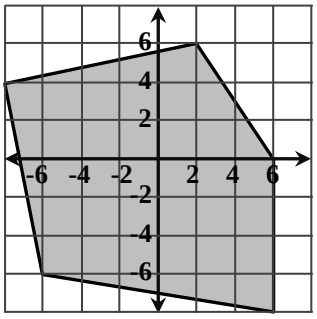
37. How many 3-digit numbers exist such that the sum of their digits is 12?

(A) 62 **(B)** 66 **(C)** 65 **(D)** 54 **(E)** 60

38. If the pattern of the sequence 5, 9, 16, 26, 39, 55, …continues, find the 30th term.

(A) 1339 **(B)** 1251 **(C)** 1430 **(D)** 1166 **(E)** 1298

39. The coordinates of the vertices of the figure on the coordinate plane below are all integers. What is the area of the figure?



(A) 96 units^2 **(B)** 122 units^2 **(C)** 52 units ^{2} **(D)** 72 units^2 **(E)** 148 units^2

40. Given $(2-3i)^3+(1+2i)^4=a+bi$. Calculate $a+b$.

(A) –20 **(B)** –86 **(C)** 20 **(D)** –53 **(E)** 53

41. A principal's student advisory committee consists of 6 seniors, 4 juniors, 2 sophomores and 2 freshmen. In how many ways can the principal pick a 5-member fundraiser subcommittee if any subcommittee must contain at least 3 seniors?

(A) 680 **(B)** 596 **(C)** 560 **(D)** 686 **(E)** 566

42. The Real value solution set of $|2x+4|+3<9$ is

(A) $\{x|\{x<-5\}\cup\{x>1\}\}$ **(B)** $\{x|x<1\}$ **(C)** $\{x|-1<x<5\}$

(D) $\{x|\{x<-1\}\cup\{x>5\}\}$ **(E)** $\{x|-5<x<1\}$

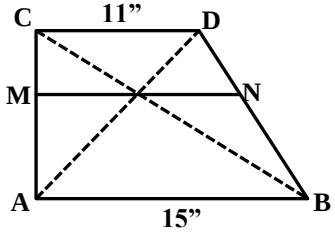
43. Given that $f(x)=\dfrac{3x^3+8x^2-4x+9}{x^2+8}$ and $s(x)$ is a slant asymptote of $f(x)$ find $s(2)$.

(A) -15 **(B)** -111 **(C)** 15 **(D)** 116 **(E)** 14

44. $\int_{-11}^{-1} f(x)dx=12$. Calculate $\int_{-11}^{-1}[-2f(x)+x]dx$.

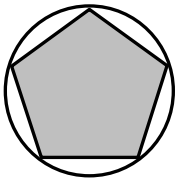
(A) -36 **(B)** -14 **(C)** -24 **(D)** -84 **(E)** -48

45. Given the trapezoid shown where $\overline{AB}\cong\overline{MN}\cong\overline{CD}$, find MN. (nearest hundredth)



(A) 12.85” **(B)** 13” **(C)** 12.95” **(D)** 13.31” **(E)** 12.69”

46. The regular pentagon shown is inscribed in the circle. What is the probability that a point chosen at random within the circle will be inside the shaded area? (nearest hundredth)



(A) 0.76 **(B)** 0.63 **(C)** 0.68 **(D)** 0.83 **(E)** 0.48

47. Calculate the angle between the vectors $v_1=\langle -27,11\rangle$ and $v_2=\langle 18,3\rangle$. (nearest degree)

(A) 58° **(B)** 42° **(C)** 148° **(D)** 122° **(E)** 36°

48. Use the function below to find $f(2)+f(-4)+f(5)$.

$$f(x)=\begin{cases} x-3, & x<0 \\ (2x)^2, & 0\le x\le 4 \\ \dfrac{x}{2}, & x>4 \end{cases}$$

(A) 25.5 **(B)** 11.5 **(C)** 3.5 **(D)** 65.5 **(E)** 33.5

49. The function $f(x)=\dfrac{2x^3+7x^2-8x-28}{x^3+2x^2-16x-32}$ has a removable discontinuity at the point

(A) $(2,1)$ **(B)** $(2,-1)$ **(C)** $(-2,1)$ **(D)** $(-2,-1)$ **(E)** $(2,0)$

50. $\csc\theta < 0$ **and** $\tan\theta < 0$**. Where will** θ **terminate?**

(A) QI (B) QII (C) QIII (D) QIV (E) x**-axis**

51. A scalene triangle is inscribed in a circle. The center of the circle is the____ of the triangle.

(A) Circumcenter (B) Incenter (C) Centroid (D) Vertex (E) Orthocenter

52. $(-8,-20)$ **and** $(8,12)$ **are opposite vertices of a parallelogram. If** $(4,-3)$ **is the third vertex, then the fourth vertex is:**

(A) $(-4,-3)$ **(B)** $(0,-8)$ **(C)** $(-4,-5)$ **(D)** $(0,-4)$ **(E)** $(-3,-4)$

53. Which of the following statements about $f(x)=\sqrt{x+12}$ **is/are true?**

I. $f(x)$ **is a one-to-one function on its domain**

II. $f(x)$ **has an inverse function on its domain**

III. $f(x)$ **has a one-to-one inverse function on its domain**

(A) I only (B) I, II & III (C) I & II (D) I & III (E) none of these

54. The graph of $f(x)=x^4-12x^2-9x+11$ **is concave down for which of the following values of** x**?**

(A) 1.5 (B) –1.5 (C) 1.75 (D) –1.25 (E) –1.75

55. The repeating fraction 0.4555… in base 6 can be written as which of the following fractions in base 6 simplified form?

(A) $\frac{41}{230}$ **(B)** $\frac{41}{50}$ **(C)** $\frac{5}{10}$ **(D)** $\frac{5}{11}$ **(E)** $\frac{41}{132}$

56. The perimeter of a square is 60 inches. The length of a rectangle is four times its width. The square and the rectangle have the same perimeter. What is the difference in the two areas?

(A) 56.25 in^2 **(B)** 144 in^2 **(C)** 87.75 in^2 **(D) 137.25 (E)** 81 in^2

57. $\sum_{k=0}^{12} 2k(k+3) =$

(A) 2184 (B) 1768 (C) 1807 (D) 1416 (E) 884

58. What is the distance between the point $(-3,8)$ **and the line** $12x+5y=27$ **? (nearest tenth)**

(A) 3.8 (B) 2.7 (C) 1.4 (D) 1.8 (E) 2.4

59. Meredith has twelve school books to fit on her shelves. Four of these books are math books and six of the books are language arts. In how many distinct ways can Meredith arrange the books in a single row on her shelf if she keeps all the math books together and all of the language arts books together?

(A) 103,680 (B) 1,244,160 (C) 414,720 (D) 207,360 (E) 34,560

60. $1331_b =$ _____$_{10}$ **when** $b>3$ **.**

(A) $b^2+3b+3+\frac{1}{b}$ **(B)** $(b+1)^3$ **(C)** $(b^2+1)(b+1)$ **(D)** $b(b+1)(b+2)$ **(E)** $(b^2+1)(b-1)$

Test Four Answer Key

1. B	21. E	41. D
2. B	22. B	42. E
3. D	23. D	43. E
4. C	24. B	44. D
5. B	25. A	45. E
6. C	26. D	46. A
7. D	27. E	47. C
8. C	28. E	48. B
9. A	29. E	49. C
10. A	30. A	50. D
11. B	31. A	51. A
12. A	32. B	52. C
13. E	33. E	53. B
14. C	34. D	54. D
15. D	35. E	55. C
16. B	36. D	56. E
17. E	37. B	57. B
18. D	38. A	58. D
19. A	39. E	59. C
20. C	40. B	60. B

Test Four Selected Solutions

14. $F+V-2=E$, so $9+14-2=21$

20. Team 9 will play 8 games. Team 8 will play 7 games that have not already been counted. Team 7 will play 6 games that have not already been counted and so on, so the number of games will be $8+7+6+5+4+3+2+1=36$.

23. $\begin{array}{r} 567_6 \\ +\ \ 35_6 \\ \hline 613_6 \end{array}$ **then** $\begin{array}{r} 613_6 \\ \times\ \ \ 5_6 \\ \hline 3366_6 \end{array}$.

24. $49x^2-196x+16y^2=588$, **then factor and complete the squares** $49\left(x^2-4x+4\right)+16y^2=588+196$. **Finally, divide** $\frac{(x-2)^2}{16}+\frac{y^2}{49}=1$ **with a center of** $(2,0)$ **and** $h+k=2$.

28. $A^2+B^2=(A+B)^2-2AB=21^2-2(105)=231$

29. $3\cdot 2\cdot\binom{10+2-1}{2}=330$

31. $\left(3x^2-x-14\right)=(3x-7)(x+2)=0$ **with roots of** $\frac{7}{3}$ **and** -2. **The expression shown is** $(P-Q)^4=\left(\frac{7}{3}+2\right)^4=\frac{28561}{81}$

36. The third and fourth terms are $\binom{6}{4}(ax)^4(3y)^2$ **and** $\binom{6}{3}(ax)^3(3y)^3$ **respectively so,** $15a^4\cdot 9=20a^3\cdot 27$ **and** $a=4$

41. $\binom{6}{3}\cdot\binom{8}{2}+\binom{6}{4}\cdot\binom{8}{1}+\binom{6}{5}=560+120+6=686$

44. $\int_{-11}^{-1}\left[-2f(x)+x\right]dx=-2\int_{-1}^{-11}f(x)dx+\int_{-1}^{-11}xdx=$
$-2(12)-60=-84$

45. The third parallel segment that passes through the intersection of the two diagonals of a trapezoid has a length equal to the harmonic mean of the lengths of the two bases, or $\frac{2ab}{a+b}=\frac{2\cdot 11\cdot 15}{11+15}\approx 12.69$ **inches.**

47. $\cos\theta=\frac{-27(18)+11(13)}{\sqrt{(-27)^2+11^2}\cdot\sqrt{18^2+3^2}}$ **for** $\theta\approx 148^\circ$

$\begin{array}{rcl} 10_6 n & = & 4.555..._6 \\ -\ \ n & = & 0.4555..._6 \\ \hline 5_6 n & = & 4.1_6 \end{array}$

55. then $\frac{41}{50_6}=\frac{25}{30}=\frac{5}{6}=\frac{5}{11_6}$

57. Use $\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$ **and** $\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$.

The $k=0$ **term is 0 then** $\sum_{k=1}^{12}2k(k+3)=\sum_{k=1}^{12}2k^2+6k=$

$2\cdot\frac{12(13)(25)}{6}+6\cdot\frac{12(13)}{2}=1768$.

59. Treat the two extra books as separate objects, then treat each group of books as a single object for 4! arrangements of groups. Within each group, there are 4! Possible arrangements of just math books and 6! arrangements of language arts books for a total of $4!\cdot 4!\cdot 6!=414{,}720$.