



**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 4 ©
NOVEMBER 15, 2014**

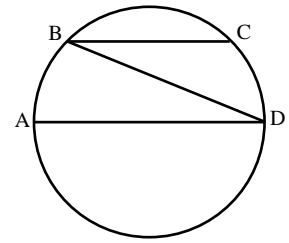
GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2014-2015 TMSCA Mathematics Test Four

- What is $\frac{13}{24} \div 0.555\dots + 0.65$?
 A) $\frac{65}{294}$ B) $\frac{13}{8}$ C) $\frac{1027}{1080}$ D) $\frac{195}{434}$ E) $\frac{2821}{4320}$
- The full cost of a bunch of flowers is \$15.99. Lesley bought 5 bunches with a 60% sale discount. The sales clerk then applied a 15% student discount to Lesley's total bill. How much did Lesley pay for the flowers including sales tax of 8.25%?
 A) \$27.18 B) \$29.43 C) \$44.14 D) \$15.57 E) \$22.07
- Find the number of positive integral divisors of 366.
 A) 6 B) 7 C) 9 D) 8 E) 4
- On a map of Texas, El Paso and Texarkana are $15\frac{1}{8}$ inches apart. The legend for the map shows that $\frac{3}{4}$ inches on the map represents 40 miles. Approximately how far is El Paso from Texarkana?
 A) 454 miles B) 600 miles C) 807 miles D) 630 miles E) 908 miles
- If $\frac{47-3x}{x^2-2x-15} = \frac{A}{x-5} + \frac{B}{x+3}$, then $A+B=$
 A) 5 B) 11 C) -3 D) 7 E) -4
- Let $X = \{m, a, s, c, o, t\}$, $Y = \{s, p, o, r, t\}$ and $Z = \{p, o, i, n, t, s\}$. How many elements are in $(X \cap Z) \cup (Z \cap Y) \cup (X \cap Y)$?
 A) 6 B) 4 C) 10 D) 5 E) 3
- Given $P(-3, 4)$ and $Q(6, -11)$ find an equation of the perpendicular bisector of \overline{PQ} .
 A) $3x-5y=22$ B) $3x+5y=7$ C) $3x-5y=13$ D) $3x+5y=-29$ E) $3x-5y=7$
- A smaller pulley with a radius of 12 cm is connected with a fan belt to a larger pulley with a diameter of 44 cm. Find the speed of the larger pulley if the speeds of the pulleys are in inverse proportion to their diameters and the smaller pulley runs at 3553 rpm.
 A) 6513 rpm B) 969 rpm C) 4225 rpm D) 1938 rpm E) 3081 rpm
- How many distinguishable arrangements can be made with the letters in the words "TIMBUKTU"?
 A) 20160 B) 40320 C) 5760 D) 10080 E) 6480
- A right cone has a radius of 18.3 inches and a vertex angle of 40° . What is the volume of the cone? (nearest cubic inch)
 A) 17633 in^3 B) 7648 in^3 C) 4128 in^3 D) 10516 in^3 E) 3076 in^3
- A box contains five rods that are 5'', 8'', 10'', 12'' and 14''. How many different obtuse triangles can be made using only three rods at a time?
 A) 3 B) 6 C) 4 D) 5 E) 7
- The intersection of the altitudes of a triangle is the _____.
 A) Orthocenter B) Median C) Centroid D) Circumcenter E) Incenter
- A _____ is the set of all points (x, y) the difference of whose distances from two distinct fixed points is constant.
 A) Ellipse B) Circle C) Hyperbola D) Cartoid E) Parabola

14. On the illustration of a circle, \overline{AD} is a diameter, $AB \cong CD$ and $m\angle C = 2m\angle B$. Calculate $m\angle D + m\angle B$.



- A) 90° B) 45° C) 60° D) 75° E) 30°

15. The quality control worker at the peppermint factory selected a sample of eight bags. The bags contained 57, 64, 55, 68, 59, 57, 63, and 60 peppermints respectively. What is the sum of the mean, median and mode of this data set?

- A) 176.875 B) 174.625 C) 175.375 D) 58.968 E) 177.500

16. The graph of the polar equation $r = 1 - 2\cos\theta$ is a _____.

- A) Rose Curve B) Lemniscate C) Circle D) Cardioid E) Limacon

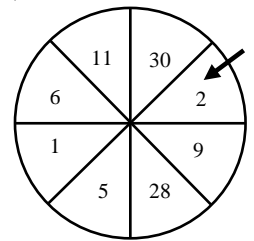
17. $2 + 9 + 22 + 41 + 66 \dots + 281 =$

- A) 1085 B) 355 C) 548 D) 829 E) 1055

18. Carla invited seven friends to a luncheon. In how many ways can Carla and her friends be seated around a round table?

- A) 64 B) 5040 C) 548 D) 40320 E) 1055

19. Eric spins the spinner shown, where each sector is congruent. If it lands on a perfect number he wins \$5. If it lands on an abundant number, he wins \$20. Otherwise, he loses \$6. Assuming that it can't land on a line, what is the mathematical expectation of one spin?



- A) \$3.88 B) \$2.75 C) \$3.25 D) 0 E) \$1.75

20. $(\sin x + \cos x)^2$

- A) $1 - \sin 2x$ B) $1 + \sin 2x$ C) $\sin 2x - 1$ D) $1 + 2\sin x$ E) $2\sin x - 1$

21. A cube is stretched so that the length is increased by 15%, the height is increased by 12% and the width is decreased by 9%. What is the percent change in the volume of the cube?

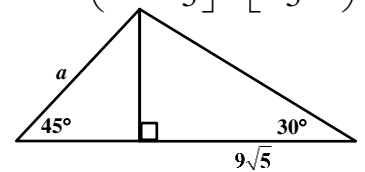
- A) 16.57% B) 17.21% C) 15.92% D) 16.20% E) 40.39%

22. The real number solution set for $3|4 - 5x| \leq 9$ is

- A) $\left(-\infty, -\frac{1}{5}\right] \cup \left[\frac{7}{5}, \infty\right)$ B) $\left[-\frac{7}{5}, \frac{1}{5}\right]$ C) $\left[\frac{1}{5}, \frac{7}{5}\right]$ D) $\left(-\infty, -\frac{7}{5}\right] \cup \left[-\frac{1}{5}, \infty\right)$ E) $\left(-\infty, -\frac{7}{5}\right] \cup \left[-\frac{1}{5}, \infty\right)$

23. What is the length of side a on the diagram shown right?

- A) $3\sqrt{15}$ B) $\frac{9\sqrt{10}}{2}$ C) $9\sqrt{15}$ D) $9\sqrt{10}$ E) $3\sqrt{30}$



24. A counselor surveyed students to determine where they study. Of the 126 students surveyed, 82 studied in the cafeteria, 64 studied in the student center, 26 studied in both the student union and the cafeteria. Of the students interviewed, how many studied only in the student center?

- A) 6 B) 56 C) 64 D) 18 E) 38

25. The distance between Los Angeles, CA and New York, NY is approximately 2700 miles. An airplane flying from L.A. to N.Y. can make the trip in 4 hours flying with the wind. The trip back takes 4.5 hours against the same wind. What would the speed of the airplane be without the wind?

- A) 568.4 mph B) 37.5 mph C) 637.5 mph D) 635.3 mph E) 600 mph

26. If $\begin{pmatrix} 2 & 3 & a \\ 0 & -1 & -8 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -11 \\ 6a-1 \end{pmatrix}$, find the value of a .

- A) -2 B) 1 C) 0 D) -1 E) 3

27. The points $P(-2,11)$, $Q(1,k)$ and $R(7,32)$ are collinear. Find the value of k .

- A) 6 B) 3 C) 18 D) 15 E) 9

28. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What length of pasture should he leave open to the river in order to minimize the length of fencing on the rest of the pasture?

- A) 300 m B) 424 m C) 848 m D) 600 m E) 900 m

29. Integrate: $\int \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)d\theta$.

- A) $\cos(2\theta)+C$ B) $-\frac{\cos\theta}{2}+C$ C) $2\sin\theta+C$ D) $-\frac{\sin\theta}{2}+C$ E) $2\cos\left(\frac{\theta}{2}\right)+C$

30. Two soccer teams, A and B play a series of three games. The probability that team A wins any given match is 0.5, while the probability that team B wins any given match is 0.3. What is the probability that the series is a tie?

- A) 0.110 B) 0.458 C) 0.118 D) 0.008 E) 0.188

31. Determine the number of non-negative integer solutions to $p+q+r=10$.

- A) 14 B) 231 C) 210 D) 66 E) 55

32. What is the sum of all 2-digit numbers whose tens digits are divisible by 3 or 9?

- A) 1935 B) 1741 C) 1953 D) 1759 E) 1290

33. Mr. Data gives a ten question quiz to his class. When he is done grading, he gives the following frequency table to his class and offers extra credit to the first student to find the mean. What is the mean number of questions the students got right on the quiz?

Questions Right	2	3	4	5	6	7	8	9	10
Number of Students	1	4	2	5	4	8	4	2	1

- A) 6.00 B) 6.03 C) 6.50 D) 3.44 E) 7

34. $323_4 + 545_6 + 767_8 = \underline{\hspace{2cm}}_{10}$.

- A) 1635 B) 771 C) 1076 D) 1759 E) 1290

35. Mrs. Cook has chocolate, cinnamon, butterscotch, and peanut butter chips in her pantry. She also bought four colors of sprinkles to use as decoration. If Mrs. Cook always puts two types of chips and two colors of sprinkles on each batch of cookies, how many distinct types of cookies can she make with her supplies?

- A) 9 B) 18 C) 6 D) 36 E) 24

36. Quadrilateral ABCD has vertices $(-9,3)$, $(-4,6)$, $(2,1)$ and $(8,-2)$ respectively. What is the area of ABCD?

- A) 9 B) 18 C) 32 D) 36 E) 24

37. If $\frac{x-9}{x+4} + \frac{x+4}{x-9}$ is equal to the mixed number $A + \frac{B}{(x+4)(x-9)}$, then $B =$

- A) 25 B) 13 C) 169 D) 36 E) 5

38. Given the vectors $u = 6i - 7j$ and $v = -13i + 5j$, find the measure of the angle between the vectors. (nearest degree)

- A) 62° B) 41° C) 110° D) 152° E) 20°

39. On triangle ABC , $AB = 8$ cm, $BC = 7$ cm, and $m\angle A = 60^\circ$. Let x and y be the two possible lengths of \overline{AC} . Find $x + y$

- A) 15 cm B) 11 cm C) 9 cm D) 12 cm E) 8 cm

40. What is the coefficient of the constant term in the expansion of $\left(x^3 - \frac{2}{x}\right)^8$?

- A) 1120 B) 1456 C) 28 D) 1792 E) 448

41. Solve $5 + 2\ln x = 4$ for x .

- A) $\frac{1}{e^2}$ B) $\frac{5}{e^4}$ C) $\ln\left(\frac{1}{2}\right)$ D) $\frac{1}{e^2}$ E) $\ln\left(\frac{\sqrt{5}}{2}\right)$

42. A fair coin is tossed six times. What is the probability of at least four consecutive heads?

- A) $\frac{15}{64}$ B) $\frac{1}{8}$ C) $\frac{11}{32}$ D) $\frac{11}{21}$ E) $\frac{3}{32}$

43. John is 6' 2'' tall. At 2:05 pm, his shadow is 4' 11'' long. If his daughter's shadow is 3' 7'' long at the same time, how tall is his daughter? (nearest inch)

- A) 3' 9'' B) 4' 2'' C) 4' 6'' D) 3' 11'' E) 4' 8''

44. Grandpa needs to bring home four cartons of ice cream to share with all of his grandkids. If the corner store carries 22 flavors of ice cream, how many distinct orders could Grandpa bring home?

- A) 7315 B) 14950 C) 12650 D) 11132 E) 2600

45. Which "trapezoidal mean" is used for standard deviation and is considered to be a measure of the magnitude of a set of numbers?

- A) Heronian B) Root-mean-square C) Harmonic D) Centroidal E) Contraharmonic

46. Find the area of the ellipse defined by the equation $49x^2 - 294x + 16y^2 + 160y = -57$.

- A) 49π B) 36π C) 28π D) 16π E) 35π

47. Peter can peel a peck of potatoes in 2 hours. The same job takes Paula 1 hour and 42 minutes. Peter and Paula can peel a peck of potatoes in _____ minutes together? (nearest minute)

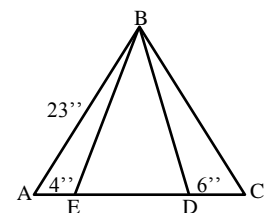
- A) 61 B) 55 C) 58 D) 53 E) 56

48. If $y = \frac{2x-5}{x+4}$, what is the sum of all real values of x for which $\frac{dy}{dx} = \frac{dx}{dy}$?

- A) 16 B) -8 C) 4 D) -16 E) 0

49. Triangle ABC is an equilateral triangle. Find the area of triangle BDE . (nearest tenth)

- A) 259 in^2 B) 100 in^2 C) 179 in^2 D) 194 in^2 E) 129 in^2



50. The repeating decimal $0.363636\dots$ in base 7 can be written as which of the following fractions in base 7?

- A) $\frac{12}{22_7}$ B) $\frac{36}{343_7}$ C) $\frac{6}{11_7}$ D) $\frac{12}{343_7}$ E) $\frac{6}{66_7}$

51. Classify the graph of $3x^2 + 8xy + 4y^2 - 7 = 0$.

- A) Ellipse B) Hyperbola C) Circle D) Parabola E) None of these

52. The radius of a spherical balloon is decreasing at a rate of 0.2 centimeters per second. How fast is the surface area of the balloon changing when the radius is 19.3 cm? (nearest tenth)

- A) $-32.3 \text{ cm}^3/\text{sec}$ B) $-97.0 \text{ cm}^3/\text{sec}$ C) $-92.6 \text{ cm}^3/\text{sec}$ D) $-94.8 \text{ cm}^3/\text{sec}$ E) $-93.7 \text{ cm}^3/\text{sec}$

53. The table below shows the effective interest rates on an investment of \$2700 for each of 5 years. What was the average annual interest rate for the 5-year period? (nearest hundredth)

Year	1	2	3	4	5
Percent	2.5%	-4%	4.7%	1.1%	-2%

- A) 0.21% B) 2.07% C) 2.26% D) 0.41% E) 0.46%

54. If $f(x) = ax^4 + bx^2 + x$ and $f(5) = 15$ then $f(-5) =$

- A) 10 B) 30 C) -10 D) -5 E) 5

55. The areas of the bases of a frustum are A_1 and A_2 and the height is 12 inches. The Heronian mean of A_1 and A_2 is $217\pi \text{ in}^2$. Find the volume of the frustum.

- A) $1302\pi \text{ in}^3$ B) $868\pi \text{ in}^3$ C) 868 in^3 D) $1555\pi \text{ in}^3$ E) $2604\pi \text{ in}^3$

56. What is the 10^{-8} digit in the expansion of $1 + (x-2) + \frac{(x-2)^2}{2!} + \frac{(x-2)^3}{3!} + \frac{(x-2)^4}{4!} + \dots$ when $x = 3$?

- A) 9 B) 3 C) 0 D) 2 E) 8

57. The function f is such that $\int_{-1}^3 f(x) dx = 12$. What is the value of $\int_{-1}^3 (2f(x) + 2) dx$?

- A) 26 B) 14 C) 32 D) 30 E) 24

58. Point P has polar coordinates $\left(11, \frac{11\pi}{6}\right)$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian plane?

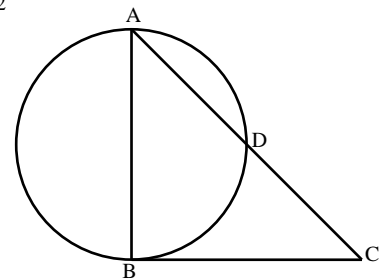
- A) QI B) QII C) QIII D) QIV E) x -axis

59. Two positive integers are in a ratio of 7 to 12. If the smaller number is increased by 3 and the larger number is increased by 18, the resulting ratio is 1 to 2. What is the sum of the original two integers?

- A) 114 B) 19 C) 40 D) 57 E) 154

60. Given that \overline{AB} is a diameter of the circle shown, \overline{BC} is tangent to the circle and $m\angle BAC = 45^\circ$, and $CD = 8 \text{ in}$. Find the area of triangle ABC. (nearest tenth)

- A) 45.3 in^2 B) 64.0 in^2 C) 128.0 in^2 D) 90.5 in^2 E) 32.0 in^2



2014-2015 TMSCA Mathematics Test Four Answers

1. B	21. B	41. A
2. B	22. C	42. B
3. D	23. E	43. C
4. C	24. E	44. C
5. C	25. C	45. B
6. B	26. A	46. C
7. A	27. C	47. B
8. B	28. D	48. B
9. D	29. B	49. E
10. A	30. E	50. A
11. D	31. D	51. B
12. A	32. A	52. B
13. C	33. B	53. D
14. B	34. B	54. E
15. A	35. D	55. E
16. E	36. C	56. D
17. E	37. C	57. C
18. B	38. D	58. D
19. D	39. E	59. A
20. B	40. D	60. B

2013-2014 TMSCA Mathematics Test Three Select Solutions

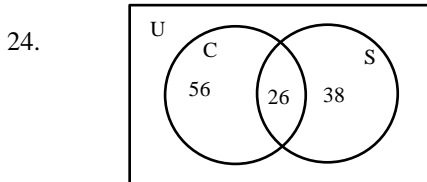
5. $47 - 3x = A(x+3) + B(x-5)$ so
 $A + B = -3$

9. T and U each repeat once. The of arrangements is $\frac{8!}{(2!)(2!)} = 10080$.

17. The terms are given by the function $3x^2 - 2x + 1$. The sum of the first 10 terms is 1055.

18. $8!$ would give the number of arrangements in a line, but to eliminate rotations of the same arrangements, $(n-1)! = 7! = 5040$.

21. $(1.15)(1.12)(0.91) - 1 \approx 17.21\%$



27. $\frac{k-11}{1-(-2)} = \frac{32-k}{7-1}$ using slope formula and $k = 18$.

30. The two ways to tie are:
 TTT
 TAB x 6 arrangements
 $(0.2)^3 + 6(0.2)(0.3)(0.5) = 0.188$

31. ${}_{10+3-1}C_{10} = 66$

32. $(30+31\dots+39) + (60+61\dots+69) + (90+91\dots+99) =$
 $3(1+2+3\dots+9) + 10(30+60+90) =$
 $135+1800=1935$

35. ${}_4C_2 \square {}_4C_2 = 36$

37. $((x-9)-(x+4))^2 = (-13)^2 = 169$

40. ${}_8C_6 (x^3)^2 \left(-\frac{2}{x}\right)^6 = 1792$

44. ${}_{22+4-1}C_4 = 12650$

48. $\left(\frac{dy}{dx}\right)^2 = 1$, so $\left[\frac{13}{(x+4)^2}\right]^2 = 1$. Solve

$(x+4)^2 = 13$ or $(x+4)^2 = -1$. The second equation has no real solutions and the sum of the solutions in the first is -8.

49. The triangle BDE has the same height as BAC, but the base is $\frac{13}{23}$ of the base of BAC. So the area is $\frac{13}{23} \left(\frac{23^2 \sqrt{3}}{4}\right) \approx 129$

$100n = 36.\overline{36}$
 50. $\frac{n}{66n} = \frac{0.\overline{36}}{36}$ so $\frac{36}{667} = \frac{12}{227}$.

54. Let $X + 5 = 15$, so $X = 10$ and $f(-5) = 10 - 5 = 5$.

56. Taylor series for $f(x) = e^{x-2}$ where $x = 3$. $e^1 \approx 2.718281828\dots$ the digit is 2.

57. $\int_{-1}^3 2f(x)dx = 24$ and $\int_{-1}^3 2dx = 8$ so together they are 32.

60 Draw in \overline{BD} which forms two special right triangles ABD and CDB each of which have base and heights of 8