T M S C A   H I G H   S C H O O L
M A T H E M A T I C S
T E S T # 5   ©
N O V E M B E R   1 9 , 2 0 1 6

Academic Excellence in Mathematics and Science through Competition

GENERAL DIRECTIONS

1. About this test:
 A. You will be given 40 minutes to take this test.
 B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate: $(0.272727...) + (0.222...) \div (1.1666...)$.

   (A) $1\dfrac{131}{198}$ (B) $\dfrac{107}{231}$ (C) $\dfrac{231}{500}$ (D) $\dfrac{131}{198}$ (E) $\dfrac{107}{231}$

2. Lindsey borrowed $2000 at 7.2% simple interest for 24 months. What will her monthly payments be?

   (A) $89.33   (B) $143.33   (C) $95.77   (D) $114.92   (E) $95.33

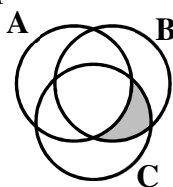3. The distances between the hash marks (|) are equal. Find $P + Q + R + S$
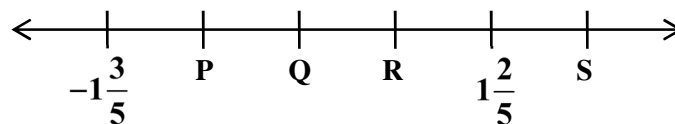
   (A) $1\dfrac{17}{20}$ (B) $1\dfrac{1}{10}$ (C) 3 (D) $1\dfrac{7}{20}$ (E) $1\dfrac{33}{100}$

4. The line $ax + 7y = 13$ is perpendicular to the line $25x + 21y = 32$. Find the value of a.

   (A) $-\dfrac{147}{25}$ (B) $\dfrac{25}{3}$ (C) $-\dfrac{25}{3}$ (D) $\dfrac{3}{25}$ (E) $-\dfrac{147}{3}$

5. Which of the following is a symbolic representation for the Venn diagram?

   (A) $(A' \cap B) \cup C$ (B) $(A \cap B) \cup C$ (C) $(A \cap B) \cap C$ (D) $(A' \cap B) \cap C$ (E) $(A' \cup B) \cup C$

6. Simplify: $\left(\dfrac{x^2 + 13x + 40}{3x^2 + 17x + 10} \right)\left(\dfrac{9x^2 - 4}{6x - 4} \right)$.

   (A) $\dfrac{x - 8}{2}$ (B) $\dfrac{x + 8}{2}$ (C) $\dfrac{x + 3}{3x - 2}$ (D) $\dfrac{3x - 2}{6x + 4}$ (E) $\dfrac{x + 8}{6x - 4}$

7. Find the greatest common divisor of $2^5 \times 3^5 \times 5^3$, $2^3 \times 3^3 \times 5^4$ and $2^2 \times 3^5 \times 5^2$.

   (A) 540   (B) 1350   (C) 270   (D) 2700   (E) 450

8. Susan plans on buying 4 shirts for $27.95 each, 2 skirts for $32.99 each and a pair of shoes for $42.95. If the local tax rate is 8.25%, how much money will Susan save by making her purchases on tax-free weekend.

   (A) $21.75   (B) $19.98   (C) $8.57   (D) $19.04   (E) $18.21

9. On the diagram, FC is the geometric mean of AB and ED. $AF = 7.5$ cm, $FE = 10.5$ cm and $EC = 19.8$ cm. Find FB. (nearest tenth)
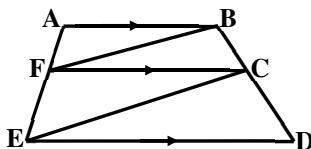
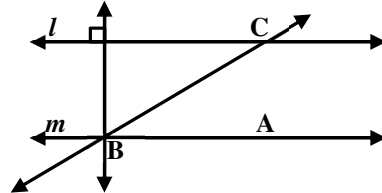   (A) 27.7   (B) 13.9   (C) 28.2   (D) 15.2   (E) 14.1

10. Events A and B are independent such that $P(A)=5P(B)$ and $P(A\cup B)=0.7875$. Find $P(B)$.

    (A)   0.75   (B)   0.20   (C)   0.15   (D)   0.125   (E)   0.375

11. $-2(8+12)=-16-24$ and $(15-6)\div 3=5-2$ are examples of _____property of equality.

    (A)   Distributive   (B)   Commutative   (C)   Associative   (D)   Transitive   (E)   Identity

12. The four lines in the figure are coplanar with $m\parallel l$. How many other angles have the same measure as $\angle ABC$?

    (A)   0   (B)   1   (C)   2   (D)   3   (E)   4

13. Let $7p+2q=-3$ and $2p-5q=12$. Find $p+q$.

    (A) $-1\dfrac{1}{13}$ (B) $\dfrac{3}{13}$ (C) $-2\dfrac{1}{13}$ (D) $1\dfrac{2}{13}$ (E) $-3\dfrac{1}{13}$

14. Find C if the remainder when $x^3-7x^2+Cx-12$ is divided by $x-2$ is -14.

    (A)   4   (B)   9   (C)   16   (D)   8   (E)   12

15. Let $f(x)=\dfrac{x}{3}$ and $g(x)=5x+2$. Find $f\big(g(x)\big)+g\big(f(-x)\big)$.

    (A) $\dfrac{8-10x}{3}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{2-10x}{3}$ (D) $\dfrac{8}{3}$ (E) $\dfrac{2}{3}$
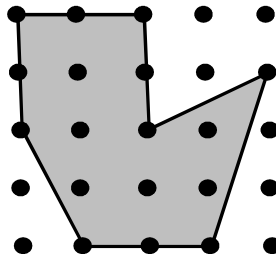
16. Given that $x-y=-7$ and $xy=28$, find x^3-y^3.

    (A) -735 (B) -931 (C) -539 (D)   245   (E)   49

17. The repeating decimal $0.242424...$ in base 7 can be written as which of the following reduced fractions in base 7?

    (A) $\dfrac{3}{11}$ (B) $\dfrac{3}{10}$ (C) $\dfrac{11}{45}$ (D) $\dfrac{11}{42}$ (E) $\dfrac{10}{42}$

18. The dots on the illustration are 3 units apart vertically and horizontally. Find the area of the shaded region.

    (A)   37.5 units²   (B)   94.5 units²   (C)   12.5 units²   (D)   90 units²   (E)   112.5 units²

19. If $5^y \cdot 25^x = 1$ and $3^{3x} \cdot 9^{2y} = \dfrac{1}{27}$ calculate $x + y$.

   (A) $-\dfrac{3}{2}$ (B) $\dfrac{3}{5}$ (C) $-\dfrac{3}{5}$ (D) $-\dfrac{5}{3}$ (E) $-\dfrac{3}{11}$

20. Let $\sin^2(x) = \dfrac{1}{3}$, where $\pi \le x \le \dfrac{3\pi}{2}$ and $\cos^2(y) = \dfrac{2}{3}$, where $\dfrac{3\pi}{2} \le y \le 2\pi$. Find

   $\sin(x)\cos(y) - \sin(y)\cos(x)$. (nearest hundredth)

   (A)   0   (B)   0.94   (C)   −0.94   (D)   0.47   (E)   −0.47

21. How many distinct 5-letter arrangements can be made from the letters in "ROUND ROCK"?

   (A)   5070   (B)   5040   (C)   3870   (D)   4920   (E)   4020

22. The diameter and length of the joint of PVC pipe is shown. What is the maximum number of complete fluid ounces of water the joint of pipe could hold if both ends were capped?
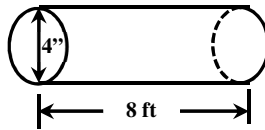
   (A)   668 fl. oz.   (B)   334 fl. oz.   (C)   481 fl. oz.   (D)   962 fl. oz.   (E)   835 fl. oz.

23. How many proper fractions in lowest terms have a denominator of 48?

   (A)   14   (B)   18   (C)   13   (D)   16   (E)   15

24. $2200220_3 - 20202_3 = $ _____ $_9$

   (A)   6404   (B)   2404   (C)   2704   (D)   2474   (E)   6474

25. If the pattern of the sequence 15, 55, 123, 225, 367, 555, 795, … continues, find the 20th term.

   (A)   12,975   (B)   9,915   (C)   11,377   (D)   6968   (E)   7704

26. If $\left[(3+5i)(2-3i)\right] \div (2-i) = a + bi$, then $a + b = $?

   (A)   21   (B)   −1.6   (C)   8.6   (D)   1   (E)   12.8

27. How many positive 3-digit numbers exist such that the sum of their digits equals 15?

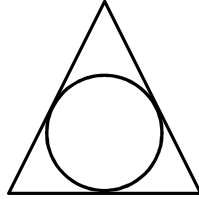   (A)   67   (B)   69   (C)   74   (D)   56   (E)   63

28. The population standard deviation of the set of numbers $\{4,6,6,7,9,9,14,17\}$ is:

   (A) $2\sqrt{5}$ (B) $\dfrac{\sqrt{952}}{7}$ (C) $\sqrt{17}$ (D) $\dfrac{\sqrt{17}}{7}$ (E) $\dfrac{2\sqrt{5}}{5}$

29. If $f'(x) = 6x^2 - 12x - 11$ and $f(2) = -21$, find $f(-2)$.

   (A)   −9   (B)   −18   (C)   −30   (D)   37   (E)   −11

30. The picture shown has a circle inscribed in an equilateral triangle. The area of the circle is 27π in^2. What is the area of the triangle?

(A) $81\sqrt{3}$ (B) $108\sqrt{3}$ (C) 108 (D) 162 (E) $162\sqrt{3}$

31. The polynomial $x^2 - 2x - 15$ is a factor of $x^3 + (a-5)x^2 + (1-4a)x - 30$. Find the value of the constant a.

(A) -6 (B) 5 (C) 2 (D) 3 (E) -5

32. The sixth term of a geometric sequence is 25.6 and the twelfth term is 1638.4. Find the fourth term.

(A) 6.4 (B) 3.2 (C) 2.4 (D) 4.8 (E) 12.8

33. Blackbeard sailed from his hideout at bearing 275° for 80 miles to island A, then on to island B at a bearing of 122° for 144 miles. How far would Blackbeard sail to go directly back to his hideout? (nearest mile)

(A) 116 mi (B) 81 mi (C) 165 mi (D) 86 mi (E) 102 mi

34. In a survey of 75 students, 32 listened to country music, 43 listened to pop music and 18 listened to both. How many students listened to neither?

(A) 22 (B) 16 (C) 19 (D) 21 (E) 18

35. The illustration shows two congruent circles each with a radius of 28 cm and $\theta = 1.15$ radians. Find the area of the shaded region. (nearest square centimeter)
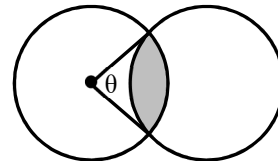
(A) 186 cm^2 (B) 378 cm^2 (C) 210 cm^2 (D) 93 cm^2 (E) 886 cm^2

36. If $a_1 = 2, a_2 = -1$ and $a_n = (a_{n-1})(a_{n-2}) - (a_{n-1})$ for $n \geq 3$, then $a_6 =$

(A) 20 (B) 0 (C) -3 (D) 2 (E) -4

37. Find the area of the ellipse $25x^2 + 4y^2 - 100x + 24y + 36 = 0$.

(A) 10π (B) 20π (C) 100π (D) 4π (E) 50π

38. If $P = \begin{bmatrix} -1 & 2 \\ -3 & 2 \end{bmatrix}$ then $P^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find $a + b + c + d$.

(A) 18 (B) -9 (C) -8 (D) 15 (E) -11

39. Let $f(x) = 3x^2 - 5x + 2$, $g(x) = x + 2$ and $s(x)$ be the slant asymptote of $\dfrac{f(x)}{g(x)}$. Find $s(-2)$.

(A)   −9   (B)   −17   (C)   24   (D)   −5   (E)   −7
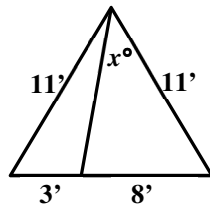
40. Find the area of the region bounded by $f(x) = x^2 - 9x$ and the x-axis.

(A)   18   (B) $\dfrac{63}{2}$ (C) $\dfrac{243}{2}$ (D) $\dfrac{117}{2}$ (E) $\dfrac{153}{2}$

41. Circle P has a radius of 7 cm. $\overline{AB}$ is a chord of circle P and AB = 5.6 cm. How far is $\overline{AB}$ from the center of circle P? (nearest tenth of a centimeter)

(A)   6.4 cm   (B)   4.2 cm   (C)   9.0 cm   (D)   5.3 cm   (E)   2.1 cm

42. Find the value of x in the triangle diagram. (nearest degree)

(A)   40.0°   (B)   43.6°   (C)   44.7°   (D)   48.0   (E)   44.2

43. Find the constant term in the binomial expansion of $\left(2x + \dfrac{4}{x^2}\right)^6$.

(A)   1280   (B)   960   (C)   7680   (D)   3840   (E)   10240

44. A plane flew 1740 km in 6 hours with a tail wind of a constant velocity. It then flew back 540 km in 3 hours with a head wind of the same velocity. Find the speed of the tail wind.

(A)   42 mph   (B)   50 mph   (C)   45 mph   (D)   48 mph   (E)   55 mph

45. If $x + \dfrac{1}{x} = 18$ then $x^3 + \dfrac{1}{x^3} =$

(A)   5886   (B)   5850   (C)   5868   (D)   5778   (E)   5832

46. Given that the set of even numbers continues in the triangular pattern shown below, find the median of the 30th row.

			2				(row 1)
		4	6	8			(row 2)
	10	12	14	16	18		(row 3)
20	22	24	26	28	30	32	(row 4)
			...				(...)
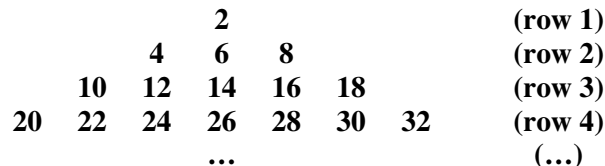
(A)   1626   (B)   1862   (C)   1744   (D)   1742   (E)   1685

47. The function $f(x) = 8x^3 - 28x^2 - 18x + 63$ is concave up at which of the following values of x.

(A)   −1   (B)   −1.5   (C)   0.9   (D)   1.75   (E)   1

48. If $\sin x \cos x = -\dfrac{1}{4}$, then $\cos 2x =$

    (A) $-\sqrt{3}$ (B) $\dfrac{\sqrt{2}}{2}$ (C) $\dfrac{1}{2}$ (D) $\sqrt{2}$ (E) $\dfrac{\sqrt{3}}{2}$

49. Solve $\log_a (x) + \log_a (x+4) = \log_a (32)$.

    (A) 4   (B) $-4, 8$ (C) -4 (D) -8 (E) $4, -8$

50. $f(x) = ax^5 + bx^3 + cx + 10$. If $f(-5) = 37$ then $f(5) =$

    (A) -17 (B) -37 (C) -27 (D) 27   (E) 17

51. The polar coordinates of point P are $\left(-8, \dfrac{5\pi}{6}\right)$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian plane?

    (A) QI   (B) QII   (C) QIII   (D) QIV   (E) *y*-axis

52. What is the digit in the ten-thousandth place of sum $9 - \dfrac{3^6}{3!} + \dfrac{3^{10}}{5!} - \dfrac{3^{14}}{7!} + \dots$.

    (A) 3   (B) 4   (C) 1   (D) 6   (E) 9

53. A particular model of car has an advertised gas mileage of 33 mpg for in-town driving. Upon investigation, a consumer group discovers that the gas mileage is actually normally distributed with a mean of 33 mpg and a standard deviation of 1.2 mpg. What is the probability that a driver will get over 35 mpg for in-town driving? (nearest tenth of a percent)

    (A) 4.5%   (B) 4.8%   (C) 11.5%   (D) 3.8%   (E) 11.2%

54. In a triangle ABC, the three medians intersect at point M. If $AM = 18$ in and point P is the midpoint of $\overline{BC}$ then the length of the median from point P is

    (A) 36   (B) 54   (C) 9   (D) 24   (E) 27

55. Five professional drivers drove a course with speeds of 95 mph, 102 mph, 98 mph, 107 mph and 110 mph. What was the average speed for all the trips? (nearest hundredth mile per hour)

    (A) 102.40 mph   (B) 102.25 mph   (C) 102.10 mph   (D) 101.92 mph   (E) 102.03 mph

56. Find the equation of the directrix of the parabola with the equation $2x^2 - 4x + y + 4 = 0$.

    (A) $y = \dfrac{8}{15}$ (B) $y = -\dfrac{15}{8}$ (C) $x = -\dfrac{15}{8}$ (D) $x = -\dfrac{8}{15}$ (E) $x = \dfrac{8}{15}$

57. Clarence is organizing movies into an ordered playlist for a trip. He has 12 movies to choose from, but only has enough memory to include 7. In how many ways can he organize a set of 7 movies?
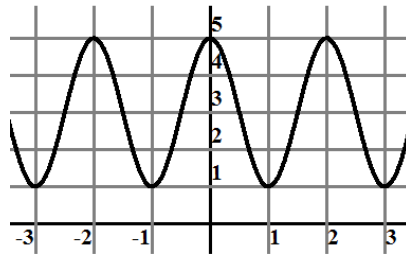
(A) 792   (B) 665,280   (C) 1716   (D) 570,240   (E) 3,991,680

58. Which of the following statements about $f(x) = 3x^2 - x$ is/are true?

I. $f(x)$ is a function

II. $f(x)$ is a one-to-one function on its domain

III. $f(x)$ has an inverse function on its domain

(A) II only   (B) I only   (C) I & II   (D) II & III   (E) I & III

59. The equation $y =$ _____ will produce this graph.

(A) $2\cos(\pi(x-5)) + 3$ (B) $-2\sin(x-5) + 3$ (C) $-2\sin(\pi(x-5)) + 3$

(D) $-2\cos(\pi(x-5)) + 3$ (E) $2\sin(x-5) + 3$

60. The length of the sides of triangle PQR are the roots of $f(x) = x^3 - 19x^2 + 110x - 180$. Find the area of triangle PQR. (nearest tenth unit)

(A) 8.5   (B) 9.5   (C) 3.7   (D) 6.0   (E) 4.8

Test Five Answer Key

1. E	21. A	41. A
2. E	22. A	42. C
3. A	23. D	43. D
4. A	24. B	44. E
5. D	25. C	45. D
6. B	26. E	46. D
7. D	27. B	47. D
8. E	28. C	48. E
9. E	29. A	49. A
10. C	30. A	50. A
11. A	31. B	51. D
12. D	32. A	52. C
13. C	33. B	53. B
14. B	34. E	54. E
15. D	35. A	55. C
16. B	36. E	56. B
17. A	37. A	57. E
18. B	38. C	58. B
19. C	39. B	59. D
20. C	40. C	60. A

Test Five Select Solutions

9. Since FC equals the geometric mean of AB and ED, $\overline{FC}$ divides the trapezoid into two similar trapezoids so:
$$\frac{7.5}{10.5} = \frac{FB}{19.8} \text{ and } FB \approx 14.1 .$$

10. For independent events
$$P(A \cup B) = P(A) + P(B) - P(A)P(B) \text{ or}$$
$$0.7875 = 5P(B) + P(B) - 5P(B)^2, \ P(B) = 0.15 \text{ and}$$
$$P(A) = 0.75 .$$

16. , $(x-y)^2 = x^2 - 2xy + y^2$ and, so
$$(x-y)^3 = (x-y)(x^2 + xy + y^2) = -7\left((-7)^2 + 3xy\right) = -931$$

17.
$$\begin{array}{rcl} 100_7 n & = & 24.2424..._7 \\ - \quad n & = & 0.2424..._7 \\ \hline 66_7 n & = & 24_7 \end{array}, \text{ so } n = \frac{24}{66_7} = \frac{18}{48} = \frac{3}{8} = \frac{3}{11_7} .$$

18. $A = \frac{2I + P}{2} - 1 = \frac{12 + 11}{2} - 1 = 10.5$ square units in the diagram, but each square in the diagram represents 9 square units for a total of 94.5 square units.

21. There are 7 distinct letters in "ROUND ROCK", so the arrangements with
No repeats = $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$

One letter repeating = $2 \cdot \binom{6}{3} \cdot \frac{5!}{2} = 2400$

Two letters repeating = $\binom{5}{1} \cdot \frac{5!}{2 \cdot 2} = 150$

And the total is 5070.

35. The area will be $2\left(\frac{28^2(1.15)}{2} - \frac{28^2 \sin 1.15}{2}\right)$

37. Rearrange to $25(x^2 - 4x) + 4(y^2 + 6) = -36$, then complete each square for
$25(x-2)^2 + 4(y+3)^2 = -36 + 36 + 100$ then divide by 100
for $\frac{(x-2)^2}{4} + \frac{(x+3)}{25} = 1$, so half the lengths of each axis are 2 and 5 for an area of $2 \cdot 5\pi = 10\pi$.

41. The radius of the circle and half the length of the chord form the hypotenuse and one leg of a right triangle. The other leg is the distance between the center of the circle and the chord: $7^2 - 2.8^2 = d^2 = 41.16$ and $d \approx 6.4$.

43. $\binom{6}{2}(2x)^4\left(\frac{4}{x^2}\right)^2 = 3840$

45. $\left(x + \frac{1}{x}\right)^2 = \left(x^2 + 2 + \frac{1}{x^2}\right)$ and
$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right) = 18(18^2 - 3) = 5778 .$$

50. Let $f(-5) = C + 10$, then $C = 27$ and
$$f(5) = -27 + 10 = -17$$

52. This is the McClaurin series for $\cos(9) = -0.9111302... .$

53. The z-score for 35 mpg is $\frac{35 - 33}{1.2} = \frac{5}{3}$ and the area under the normal curve above $\frac{5}{3}$ is about 0.0478 or 4.8%.

55. $\dfrac{5}{\dfrac{1}{95} + \dfrac{1}{102} + \dfrac{1}{98} + \dfrac{1}{107} + \dfrac{1}{110}} \approx 102.10$ mph

57. $\binom{12}{7}(7!) = 3,991,680$

60. The sum of the roots is 19 and the semi-perimeter of the triangle is 9.5 and the area is $\sqrt{9.5 \cdot f(9.5)} \approx 8.5$