

T M S C A H I G H S C H O O L
M A T H E M A T I C S
T E S T # 5 ©
N O V E M B E R 2 2 , 2 0 1 4

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2014-2015 TMSCA Mathematics Test Five

1. What is $\frac{(8+6) \cdot 3}{7 \cdot 2 + 7 \cdot 5} \cdot 2.3 \div 0.35$?

- A) $\frac{120}{7}$ B) $\frac{379}{12}$ C) $\frac{40}{7}$ D) $\frac{13}{30}$ E) $\frac{13}{10}$

2. If P is 224% more than Q and R is $\frac{3}{2}$ of Q, then R is what percent of P? (nearest tenth)?

- A) 67.0% B) 149.3% C) 46.3% D) 6.7% E) 56.7%

3. An electronics shop advertises a 20% off sale with an additional 10% off if purchases are made before noon. Carrie checked out at 10 am and used a coupon that gave her 15% off her total. What is the total percent discount on her items? (nearest %)

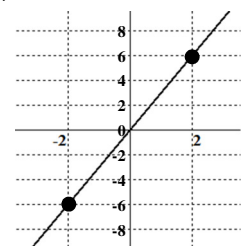
- A) 61% B) 30% C) 45% D) 53% E) 39%

4. If $\frac{x^3 + 6x^2 + 12x + 8}{x^4 - 8x^2 + 16} = \frac{ax + b}{x^2 - 4x + 4}$, find $a + b$.

- A) 2 B) 3 C) -2 D) 1 E) -3

5. Which of the following is an equation of the line shown right?

- A) $y - 6 = 3(x - 2)$ C) $y + 2 = 3(x + 6)$ E) $y - 2 = \frac{1}{3}(x - 6)$
 B) $y - 2 = 3(x - 6)$ D) $y - 2 = \frac{1}{3}(x - 6)$



6. Using the piecewise function shown calculate $f(1) + f(5) - f(-4)$.

- A) -2 B) 3 C) 4 D) 7 E) 1

$$f(x) = \begin{cases} x^2 + 2x - 3, & x \leq -3 \\ |x| + 1, & -3 < x < 2 \\ x + 2, & x \geq 2 \end{cases}$$

7. The point M is the midpoint of \overline{AB} . The coordinates of the three points are $A(x, 7)$, $B(-9, y)$ and $M(1, 14)$. Calculate $x + y$.

- A) 16 B) 6.5 C) 14.5 D) 21 E) 32

8. What is the lateral surface area of a cylinder if the diameter is 25π centimeters and the height is 40 centimeters? (nearest square centimeter)

- A) 3142 cm^2 B) 9870 cm^2 C) 9689 cm^2 D) 19559 cm^2 E) 19739 cm^2

9. A two-digit number exists such that the square of the number is nine times the sum of the number and ten. What is the sum of the two digits of the number?

- A) 9 B) 15 C) 6 D) 18 E) 12

10. Which of the following numbers appears in the 13th row of Pascal's triangle?

- A) 1287 B) 924 C) 3432 D) 330 E) 1001

11. The intersection of two distinct planes is a _____.

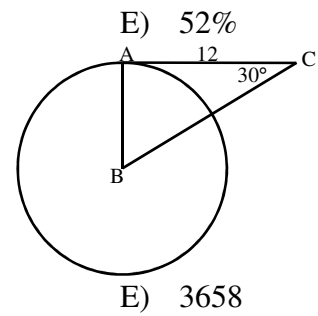
- A) Point B) Line C) Triangle D) Plane E) Parallelogram

12. On an academic UIL team of 45 students, 18 practice for mathematics, 20 practice for number sense and 11 students practice for both. How many students do not do either mathematics or number sense?

- A) 4 B) 8 C) 12 D) 18 E) 15

13. Events A and B are independent events. If $p(A \cap B) = 0.32$ and $p(A) = 2p(B)$. Calculate $p(A \cup B)$.
 A) 80% B) 88% C) 68% D) 82%
 E) 52%

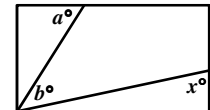
14. On the circle shown, \overline{AC} is tangent to the circle. Find the area of the circle.



- A) 144π B) 81π C) 48π D) 32π E) 36π

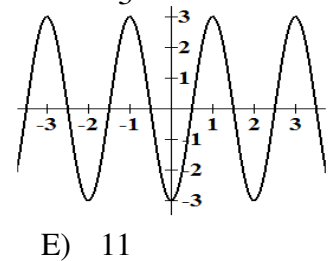
15. $x = 37 + y$ and $xy = 620$. Calculate $x^2 + y^2$.
 A) 2289 B) 2609 C) 1989 D) 3229
 E) 3658

16. In the rectangle shown right, what is x in terms of a and b ?
 A) $a + b$ B) $b - a$ C) $a - b$ D) $90 - a + b$ E) $90 + a - b$



17. A kayak can travel 48 miles downstream in 6 hours, while it would take 12 hours to make the same trip upstream. Find the speed of the kayak in still water?
 A) 4 mph B) 6 mph C) 2 mph D) 8 mph E) 12 mph

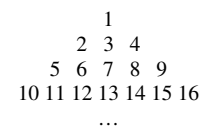
18. Determine the period of $y = \frac{3}{2} \sin\left(\frac{1}{3}x - 7\right) + 8$.
 A) $\frac{2\pi}{3}$ B) 2π C) $\frac{3\pi}{2}$ D) 6π E) $\frac{7\pi}{3}$



19. The function $f(x) = \underline{\hspace{2cm}}$ will produce this graph.
 A) $3 \cos(\pi x + 1)$ C) $3 \cos(\pi(x - 1))$ E) $3 \cos(x + \pi)$
 B) $3 \sin(\pi(x + 1))$ D) $3 \cos(\pi x - 1)$

20. Use the Fibonacci-type sequence 12, A, 7, B, C, 11, D... to find the value of D.
 A) 20 B) 31 C) -5 D) 9 E) 11

21. If the numbers $\{1, 2, 3, 4, 5, \dots\}$ continue in the triangular pattern shown below, the arithmetic mean of the 11th row would be?



- A) 111 C) 57 E) 73
 B) 133 D) 157
22. At a concession stand, five hot dogs and four hamburgers cost \$14.50; four hot dogs and five burgers cost \$14.75. Find the cost of an order of six hamburgers and three hot dogs.
 A) \$14.25 B) \$15.25 C) \$14.50 D) \$15.00 E) \$15.50

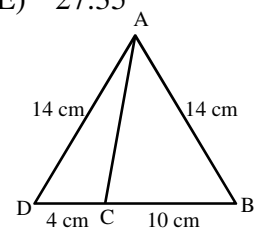
23. Find k when $4 + 5(k + 8) \div 3 = 8 - 3k$.
 A) 7 B) -4 C) -2 D) 5 E) -3

24. Which of the following series converges?
 A) $\sum_{n=0}^{\infty} \frac{n+1}{2n+1}$ B) $\sum_{n=1}^{\infty} \frac{n}{1000(n+1)}$ C) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$ D) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$ E) $\sum_{n=1}^{\infty} \log n$

25. A bowl with colored chocolate candies contains 33% blue candies, one-fifth yellow candies and 15% green candies. The remaining candies are orange. If one candy is drawn at random, what are the odds that it will not be orange?
 A) 17:8 B) 8:25 C) 33:67 D) 8:17 E) 25:8

26. When $3x^3 + 2x^2 - 7x + k$ is divided by $(x - 2)$ the remainder is m . Find the value of m in terms of k .
 A) $m = k - 2$ B) $m = k + 18$ C) $m = k - 30$ D) $m = k + 46$ E) $m = k + 30$
27. Mr. Meredith set up a scavenger hunt for his pre-calculus class. Group one travelled 200 yards on a bearing of 212° , then 350 yards on a bearing of 97° , then 275 yards on a bearing of 325° to retrieve their clues. How far were they from their point of origin? (nearest yard)
 A) 716 yd B) 85 yd C) 306 yd D) 97 yd E) 132 yd
28. If $y = \tan \theta$, for what value of θ does $\frac{dy}{dx} = \frac{dx}{dy}$?
 A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) π D) $\frac{\pi}{3}$ E) $\frac{\pi}{6}$
29. The point P has coordinates $(2, 1)$. P is reflected over the line $y = -x$, translated 2 units horizontally in the positive direction, then 5 units vertically in the positive direction to point S. Find the coordinates of S.
 A) $(4, 4)$ B) $(4, 0)$ C) $(3, 7)$ D) $(-1, -2)$ E) $(1, 3)$
30. A student council consists of eight boys and nine girls. How many ways can a four person delegation be formed for the state conference if the delegation must consist of two girls and two boys?
 A) 784 B) 1296 C) 1008 D) 1152 E) 1224
31. If $\frac{x+2}{x-7} + \frac{x-7}{x+2} = A + \frac{B}{(x+2)(x-7)}$, then $B =$
 A) 25 B) 81 C) 5 D) -5 E) -9
32. The Real value solution set of $|3x - 1| + 3 < 11$ is
 A) $\left\{x \mid -3 < x < -\frac{7}{3}\right\}$ B) $\left\{x \mid \left\{x < -3\right\} \cup \left\{x > \frac{7}{3}\right\}\right\}$ C) $\left\{x \mid -3 < x < \frac{7}{3}\right\}$ D) $\left\{x \mid \left\{x < -\frac{7}{3}\right\} \cup \left\{x > 3\right\}\right\}$ E) $\left\{x \mid -\frac{7}{3} < x < 3\right\}$
33. Find the sum of all 2-digit numbers whose digits differ by 5 and reversing the digits results in a square number.
 A) 220 B) 94 C) 155 D) 125 E) 172
34. Two workers can paint a fence in three hours. How long would it take six workers to paint a fence twice as long and twice as high if they each paint at the same rate?
 A) 4 hr 48 min B) 4 hr C) 1 hr 12 min D) 2 hr 24 min E) 4 hr 8 min
35. Find the sum of the infinite series: $-1.2 + 0.9 - 0.675 + 0.50625 \dots$
 A) $-\frac{11}{35}$ B) $\frac{8}{5}$ C) $-\frac{12}{5}$ D) $-\frac{24}{5}$ E) $-\frac{24}{35}$
36. A regular decagon has vertices A, B, C, D, ..., I and J respectively. What is the measure of $\angle BHF$?
 A) 90° B) 72° C) 108° D) 144° E) 126°
37. The harmonic mean of A and B is 26.8 and the contraharmonic mean is 27.9. What is the arithmetic mean of A and B?
 A) 27.34 B) 27.25 C) 27.33 D) 27.37 E) 27.35
38. Find AC. (nearest tenth)

- A) 9.9 cm B) 11.8 cm C) 13.3 cm D) 12.5 cm E) 12.4 cm



39. If $f(x) = \sin x$ then $\lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{3} + h\right) - f\left(\frac{\pi}{3}\right)}{h} =$

- A) $\frac{1}{2}$ B) 1 C) $\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) $-\frac{1}{2}$

40. Simplify: $(3 - 3\sqrt{-8})(1 + 2\sqrt{-50})$

- A) $-123 + 36i\sqrt{2}$ B) $117 + 36i\sqrt{2}$ C) $123 - 24i\sqrt{2}$ D) $-117 + 36i\sqrt{2}$ E) $123 + 24i\sqrt{2}$

41. Let $a_1 = 12$, $a_2 = 5$ and $a_n = 2a_{n-1} - 3a_{n-2}$. Find a_5 .

- A) 147 B) -56 C) -339 D) -159 E) -33

42. Simplify $a^4 \times b^3 \div a^{-1} \times b^{-4} \times a^2 \div b^2 \div b^{-3}$.

- A) $\frac{a^5}{b^2}$ B) a^7 C) a^7b D) a^5b^2 E) a^5

43. Let A and B be the roots of $f(x) = 8x^2 + 18x - 35$. Find the value of $A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$.

- A) $\frac{6561}{256}$ B) $\frac{16}{6561}$ C) $\frac{6561}{16}$ D) $\frac{256}{81}$ E) $\frac{256}{6561}$

44. If $f(a) = g(a) = 0$, $f'(a)$ and $g'(a)$ exist and $g'(a) \neq 0$ then $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(a)}{g'(a)}$. This is _____.

- A) Definition of Derivative C) L'Hopital's Rule E) Fundamental Theorem of Calculus
 B) Intermediate Value Theorem D) Rolle's Theorem

45. Given $GCF(36, k) = 12$ and $LCM(36, k) = 252$, find the value of k .

- A) 84 B) 48 C) 42 D) 63 E) 126

46. Solve: $\log_2 x + \log_2(x - 6) = 4$

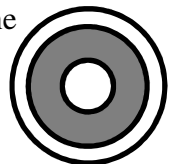
- A) 9 B) 6 C) -3 D) 4 E) 8

47. Set $S = \{1, 2, 3, 4, 5, 6, 7\}$. How many 5-element subsets of set S are there?

- A) 32 B) 128 C) 49 D) 21 E) 35

48. The diameters of the concentric circles on the right are 14 cm, 12 cm and 6 cm. If a dart hits the figure at random, what is the probability that it will land outside the shaded area? (nearest %)

- A) 55% B) 27% C) 59% D) 45% E) 73%



49. The letters in the word PARALLEL are arranged in a line. How many of distinct arrangements are possible?

- A) 3360 B) 10080 C) 6720 D) 40320 E) 720

50. The points $A(-8, 0)$, $B(1, 12)$, $C(12, 3)$ and $D(3, -5)$ are plotted and connected to form the convex quadrilateral $ABCD$. What is the area of this quadrilateral?

- A) 153 units² B) 87 units² C) 213 units² D) 170 units² E) 173 units²

51. Let $f(x) = ax^5 + bx^3 + cx - 8$ and $f(-6) = 14$. Calculate $f(6)$.

- A) -28 B) 22 C) -30 D) -14 E) -6

52. $11_3 + 11_4 + 11_5 + 11_6 + 11_7 + 11_8 + 11_9 = \underline{\hspace{2cm}}_{10}$.
- A) 88 B) 44 C) 52 D) 49 E) 72
53. What is the area of a regular hexagon in terms of the length, s , of one side?
- A) $\frac{3s^2\sqrt{3}}{2}$ B) $\frac{s^2\sqrt{3}}{6}$ C) $\frac{s^2\sqrt{3}}{4}$ D) $\frac{3s^2\sqrt{3}}{4}$ E) $\frac{2s^2\sqrt{3}}{3}$
54. How many solutions are there to the equation $11x + 9y = 216$, where x and y are whole numbers?
- A) 0 B) 2 C) 3 D) 4 E) 1
55. How many petals does the graph of the curve $r = 2 \cos 5\theta$ have?
- A) 4 B) 10 C) 7 D) 2 E) 5
56. Simplify to the nearest ten-thousandth place: $1 + (1.5) + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!} + \dots$
- A) 0.0707 B) 0.4055 C) 4.4817 D) 0.9975 E) 0.0262
57. $\det \begin{bmatrix} \cos A & \sin A \\ \sin B & \cos B \end{bmatrix} =$
- A) $\sin(A+B)$ B) $\cos(A+B)$ C) $\cos(A-B)$ D) $\sin(2B)$ E) $\cos(2A)$
58. Find the range, or ranges, of values of K for which $f(x) = Kx^2 - 4x + 5 - K$ has two distinct real roots.
- A) $1 < K < 4$ B) $K < 1$ and $K > 4$ C) $K < -6$ and D) $K < \frac{4}{5}$ E) $-6 < K < 2$
59. How many numbers in the form a^4 , where $a \in \mathbb{Z}^+$ are factors of $(3!)(7!)(9!)$?
- A) 5 B) 7 C) 9 D) 6 E) 8
60. What is the centroidal mean of 30 and 50? (nearest tenth)
- A) 40.0 B) 38.7 C) 40.8 D) 38.9 E) 41.1

2014-2015 TMSCA Mathematics Test Five Answers

1. C	21. A	41. B
2. C	22. D	42. B
3. E	23. C	43. A
4. B	24. C	44. C
5. A	25. A	45. A
6. C	26. B	46. E
7. E	27. B	47. D
8. B	28. C	48. D
9. C	29. E	49. A
10. A	30. C	50. E
11. B	31. B	51. C
12. D	32. E	52. D
13. B	33. C	53. A
14. C	34. B	54. C
15. B	35. E	55. E
16. D	36. B	56. C
17. B	37. E	57. B
18. D	38. D	58. B
19. C	39. A	59. E
20. A	40. E	60. C

2013-2014 TMSCA Mathematics Test Five Select Solutions

<p>13. $0.32 = p(A)p(B) = 2(p(B))^2$ and $p(B) = 0.4$ $p(A \cup B) = p(A) + p(B) - p(A)p(B)$ So $p(A \cup B) = 0.8 + 0.4 - (0.8)(0.4) = 0.88$</p> <p>15. $x^2 + y^2 = (x - y)^2 + 2xy =$ $37^2 + 2(620) = 2609$</p> <p>21. $\frac{11^2 + 10^2 + 1}{2} = 111$</p> <p>26. Evaluate $f(2) = 24 + 8 - 14 + k = 18 + k = m$</p> <p>28. $\left(\frac{dy}{dx}\right)^2 = 1$, so $(\sec^2 \theta)^2 = 1$, $\cos \theta = \pm 1$ at any multiple of π.</p> <p>31. $(-7 - 2)^2 = 81$</p> <p>33. $61 + 94 = 155$</p> <p>34. The rate of an individual worker is $\frac{1}{6}$, so to paint a fence that is 4 times as large with 6 workers $6\left(\frac{1}{6}\right)t = 4$ and $t = 4$ hours.</p> <p>36. The measure of each arc between two consecutive letters is 36°. $\angle BHF$ is an inscribed angle with an intercepted arc of 144°, so $m\angle BHF = 72^\circ$.</p> <p>38. Use law of cosines. $(AC)^2 = 14^2 + 10^2 - 2(14)(10)\cos 60 \approx 12.5$</p> <p>43. The expression is the binomial expansion of $(A + B)^4$. $\left(-\frac{18}{8}\right)^4 = \frac{6561}{256}$.</p> <p>45. $36k = 12(252)$, so $k = 84$</p> <p>46. $\log_2(x^2 - 6x) = 4$, so $16 = x^2 - 6x$ and $x = 8$ or $x = -2$. 8 is the only solution that works in the original equation.</p>	<p>51. $X - 8 = 14$, so $X = 22$. $-22 - 8 = -30$</p> <p>53. A hexagon is 6 equilateral triangles, so $A = 6\left(\frac{s^2\sqrt{3}}{4}\right) = \frac{3s^2\sqrt{3}}{2}$</p> <p>56. McLaurin series for $e^{1.5} \approx 4.48168907\dots$</p> <p>57. $\det = \cos A \cos B - \sin A \sin B =$ $\cos(A + B)$</p> <p>58. $16 - 4(K)(K + 5) > 0$, so $K < 1$ and $K > 4$.</p> <p>60. $\frac{2}{3}\left(\frac{(30 + 50)^2 - (30)(50)}{30 + 50}\right) = 40.8\bar{3}$</p>
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