T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 5 ©

N O V E M B E R 2 3 , 2 0 1 3

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2013-2014 TMSCA Mathematics Test Five

1. Evaluate $\frac{10!}{5!}\bullet\frac{8!}{11!}\div\frac{7!}{6!}$.

(A) $\frac{96}{77}$ (B) 2352 (C) $\frac{48}{11}$ (D) $\frac{56}{33}$ (E) $\frac{6}{11}$

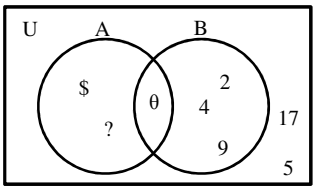
2. $\angle A$ and $\angle B$ are supplementary. If $m\angle A=(7x-5)^\circ$ and $m\angle B=(x^2-13)^\circ$ find the measure of the complement of $\angle A$.

(A) 23° (B) 18° (C) 15° (D) 11° (E) 13°

3. Jay's current age is ¼ of his mother's age. In 10 years, Jay's age will be 2 years less than ½ his mother's age. What is the sum of their current ages?

(A) 28 (B) 38 (C) 45 (D) 55 (E) 35

4. Use the Venn diagram to determine the set $A'\cap B$.



(A) $\{\theta\}$ (C) $\{2,4,5,9,17\}$ (E) $\{\$,?,\theta,2,4,9,\}$

(B) $\{2,4,9\}$ (D) $\{\theta,2,4,5,9,17\}$

5. Karolyn invested $1000 for 4 years in a variable interest account. Her annual interest rates are shown in the table below. What was the average interest for the 4 years?

Year	1	2	3	4
Interest	5% gain	2.4% loss	3% loss	2.7% gain

(A) 3.27% (B) 0.58% (C) 0.52% (D) 0.33% (E) 0.45%

6. $(3x+5)$, $(7x-1)$ and $(2-3x)$ are all factors of

(A) $63x^3-54x^2+79x-10$ (C) $63x^3-54x^2-79x-10$ (E) $-63x^3-54x^2+79x-10$

(B) $-63x^3+54x^2+79x-10$ (D) $-63x^3-54x^2-79x+10$

7. Points *P* and *Q* have coordinates $(-5,8)$ and $(7,-6)$ respectively. Which of the following is an equation of the perpendicular bisector of $\overline{PQ}$?

(A) $6x-7y=-1$ (B) $7x+6y=13$ (C) $6x-7y=-86$ (D) $7x+6y=-6$ (E) $6x-7y=0$

8. What is the sum of the arithmetic sequence 16, 18.4, 20.8, …119.2?

(A) 5678.2 (B) 2906.8 (C) 2622.4 (D) 5125.6 (E) 2974.4

9. A box contains 3 black marbles, 7 red marbles and 11 gold marbles. If Carl draws out 2 marbles 1 at a time without replacement, what are the odds that he will draw gold then black?

(A) 0.085 (B) 0.079 (C) 0.075 (D) 0.081 (E) 0.083

10. $(2x+7)(5x-1)=2x(5x-1)+7(5x-1)$ is an example of _____ property.

(A) Transitive (B) Associative (C) Commutative (D) Distributive (E) Substitution

11. If $\frac{x^4-5x^2+4}{(x^2+4x+4)(x^2+2x+1)}=\frac{x^2+ax+2}{x^2+bx+2}$, find $\frac{a}{b}$.

(A) 0.5 (B) -2 (C) 1 (D) -0.5 (E) -1

12. A theater company sold 400 tickets worth $8000 to their fall performance. If adult tickets cost $22.50 each and children's tickets cost $14.50 each, how many children's tickets were sold?

(A) 200 (B) 275 (C) 150 (D) 125 (E) 225

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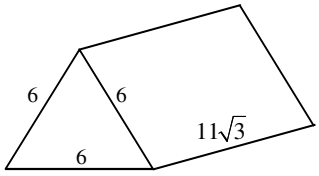
13. $\frac{dy}{dx}\sqrt{3x^2+5} =$

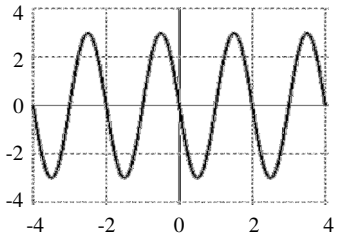
(A) $\frac{-6x}{\sqrt{3x^2+5}}$ (B) $3x\sqrt{3x^2+5}$ (C) $6x\sqrt{3x^2+5}$ (D) $\frac{1}{2\sqrt{3x^2+5}}$ (E) $\frac{3x}{\sqrt{3x^2+5}}$

14. Find the surface area of the triangular prism.



(A) $216\sqrt{3}$ (B) $246\sqrt{3}$ (C) $198\sqrt{3}$ (D) $234\sqrt{3}$ (E) 648

15. The function $f(x) =$_____ will produce this graph.



(A) $3\sin(\pi x+1)$ (C) $3\sin(x+\pi)$ (E) $\sin(\pi(x-1))+3$

(B) $3\sin(\pi x-1)$ (D) $3\sin(\pi(x-1))$

16. Considering the "1" at the top of Pascal's triangle as row 0, the third number in the 18th row of Pascal's triangle is the _____th triangular number.

(A) 18 (B) 15 (C) 17 (D) 19 (E) 16

17. The faces of a regular icosahedron are_____.

(A) Triangles (B) Squares (C) Pentagons (D) Hexagons (E) Octagons

18. If *f* is continuous on the closed interval $[a,b]$ and *k* is any number between $f(a)$ and $f(b)$, then there is at least one number *c* in $[a,b]$ such that $f(c)=k$. This is the _____ ,

(A) Sandwich Theorem (C) Rolle's Theorem (E) Fundamental Theorem of Calculus

(B) Intermediate Value Theorem (D) Fundamental Theorem of Algebra

19. $\frac{\cot^2 t}{\csc t} =$

(A) $\sec t+\cos t$ (B) $\sec t-\cos t$ (C) $\csc t+\sin t$ (D) $\sec t-\sin t$ (E) $\csc t-\sin t$

20. If $5^x \cdot 25^{2y} = 1$ and $3^{5x} \cdot 9^y = \frac{1}{9}$, then $x+y =$

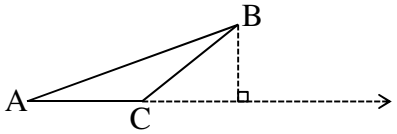
(A) $\frac{1}{3}$ (B) $-\frac{4}{9}$ (C) $-\frac{5}{9}$ (D) $-\frac{1}{3}$ (E) $\frac{4}{9}$

21. If $A+B=11$ and $AB=24$, then $A^2+B^2 =$

(A) 598 (B) 73 (C) 554 (D) 169 (E) 97

22. On triangle ABC shown, $m\angle BAC = \frac{\pi}{6}$ radians, $AB=12$ and $AC=9$.

Find the area of triangle ABC.



(A) 27 (B) $27\sqrt{3}$ (C) 36 (D) 54 (E) $36\sqrt{3}$

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23. $A=\begin{bmatrix}3 & 5\\ 7 & -2\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 0 & 4\end{bmatrix}$. Calculate $\det(2A+B)$.

(A) -119 (B) -20 (C) 9 (D) 133 (E) -126

24. $4\log a^3b-2\log\frac{a}{b^2}+\frac{1}{2}\log ab=$

(A) $\log a^9b$ (B) $\frac{1}{2}\log a^{11}b^9$ (C) $\log\left(a^{10}\sqrt{ab}\right)$ (D) $\frac{1}{2}\log a^5b$ (E) $\frac{1}{2}\log a^{21}b^{17}$

25. $f''(x)=2$, $f'(2)=5$ and $f(2)=10$. Evaluate $f(8)$.

(A) 64 (B) 76 (C) 72 (D) 74 (E) 80

26. $\sum_{k=0}^{15}k\left(k^2+1\right)=$

(A) 11130 (B) 74400 (C) 14520 (D) 1300 (E) 3390

27. How many distinct arrangements can be made using all of the letters in the word ASPARAGUS?

(A) 90720 (B) 60480 (C) 362880 (D) 30240 (E) 120960

28. If $\int_0^a\cos x\,dx=C$ then $\int_{-a}^{a}3\cos x\,dx=$

(A) 0 (B) $2C$ (C) $3C$ (D) $6C$ (E) $12C$

29. If $\frac{7x+13}{x^2+2x-3}=\frac{A}{x+3}+\frac{B}{x-1}$, then $AB=$

(A) 7 (B) -6 (C) 10 (D) -3 (E) 6

30. Given $a_0=-1$, $a_1=2$ and $a_n=2a_{n-1}+3a_{n-2}$ find a_5.

(A) 62 (B) 181 (C) 59 (D) 122 (E) 184

31. A belt joins two pulleys with diameters of 15 cm and 25 cm. The larger rotates at a rate of 96 rpm. The smaller rotates at a rate of _____ rpm.

(A) 267 (B) 58 (C) 160 (D) 242 (E) 145

32. A triangle is inscribed in a circle. The center of the circle is the intersection of the _____ of the triangle.

(A) Perpendicular Bisectors (B) Altitudes (C) Medians (D) Angle Bisectors (E) Sides

33. Classify the graph of $16x^2-8xy+y^2-10x+5y=0$.

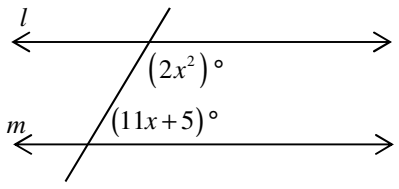
(A) Circle (B) Parabola (C) Centroid (D) Hyperbola (E) Ellipse

34. Solve $\frac{3}{x}+\frac{5}{x+y}=4$ for y.

(A) $\frac{4x^2-8x}{1-x}$ (B) $\frac{x^2-2x}{x-5}$ (C) $\frac{x^2-8x}{x-5}$ (D) $\frac{4x^2-8x}{3-4x}$ (E) $\frac{4x^2}{x-1}$

35. In the diagram, $l\,//\,m$. Find the value of x.

(A) 12.5 (B) 10 (C) 4 (D) 1 (E) 7



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36. Given $f(x)=\sin x$ and $g(x)=x+\pi$ calculate $f\left(g\left(\frac{\pi}{6}\right)\right)$.

(A) $\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $-\frac{1}{2}$ (E) $\frac{\sqrt{3}}{2}$

37. P and Q are the roots of $f(x)=6x^2-4x-15$. Calculate $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$.

(A) $\frac{130321}{1296}$ (B) $\frac{6859}{216}$ (C) $\frac{8836}{81}$ (D) $-\frac{1}{216}$ (E) $\frac{130321}{50625}$

38. $\frac{\cot\theta}{\sec\theta}=$

(A) $\sec\theta-\cos\theta$ (B) $\csc\theta-\sin\theta$ (C) $\csc\theta+\sin\theta$ (D) $\sec\theta+\cos\theta$ (E) $\csc\theta+\cos\theta$

39. The length of one edge of a regular tetrahedron is $6\sqrt{3}$ cm. The surface area of the tetrahedron is _____ cm^2.

(A) 108 (B) $36\sqrt{3}$ (C) $108\sqrt{3}$ (D) 324 (E) $324\sqrt{3}$

40. If y varies directly with x and $y=28$ when $x=200$, calculate y when $x=117$.

(A) 18.72 (B) 16.38 (C) 24.57 (D) 26.91 (E) 14.04

41. Calculate the slope of the tangent line to $2x^3+3x^2y-y^2=-2$ at the point $(-1,3)$.

(A) -4 (B) 6 (C) -6 (D) 4 (E) 3

42. If $v_1=\langle -2,8\rangle$ and $v_2=\langle 7,-6\rangle$ then the angle between the vectors is ____°. (nearest degree)

(A) 32 (B) 122 (C) 58 (D) 46 (E) 145

43. Given $(3+2i)^4+(3-2i)^3=a+bi$ calculate $a+b$.

(A) 202 (B) 56 (C) 276 (D) -202 (E) -54

44. The ratio of width to length of a rectangle is 5:11 and the area is 343.75 in^2. What is the perimeter of the rectangle?

(A) 40 in (B) 200 in (C) 80 in (D) 50 in (E) 100 in

45. In how many ways can a class of 18 students be split into three groups of 2, 6 and 10 students?

(A) 2840292 (B) 812323512 (C) 1225224 (D) 6694974 (E) 15148224

46. When $f(x)=\frac{x^2+1}{2-x}$ evaluate $f'(3)$.

(A) 2 (B) 5 (C) 7 (D) 4 (E) 3

47. The Real value solution set of $|3x+1|+3<11$ is

(A) $\left\{x\left|-\frac{7}{3}<x<3\right.\right\}$ (B) $\left\{x\left|-3<x<-\frac{7}{3}\right.\right\}$ (C) $\left\{x\left|\{x<-3\}\cup\left\{x>\frac{7}{3}\right\}\right.\right\}$ (D) $\left\{x\left|-3<x<\frac{7}{3}\right.\right\}$ (E) $\left\{x\left|\left\{x<-\frac{7}{3}\right\}\cup\{x>3\}\right.\right\}$

48. $\int_{-3}^{7} f(x)\,dx=9$. Calculate $\int_{-3}^{7}\left(3+2f(x)\,dx\right)$

(A) 21 (B) 39 (C) 45 (D) 48 (E) 36

49. How many positive perfect cubes are factors of $(3!)(4!)(7!)$?

(A) 5 (B) 7 (C) 4 (D) 6 (E) 8

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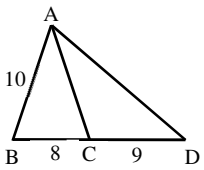
50. Given that $\overline{AB} \cong \overline{AC}$, find the area of triangle *ABD*.



(A) $17\sqrt{21}$ (B) 102 (C) $34\sqrt{21}$ (D) $34\sqrt{42}$ (E) $17\sqrt{42}$

51. The roots of $f(x) = x^3 + ax^2 + bx + c$ are -2, 4 and 11. $a + b + c =$

(A) 82 (B) 115 (C) -88 (D) 101 (E) 89

52. The chord $\overline{AB}$ has a length of 14 cm and the circle has a diameter of 50 cm. How far is $\overline{AB}$ from the center of the circle?

(A) 48 cm (B) 21 cm (C) 24 cm (D) 45 cm (E) 37 cm

53. Calculate $4 - \frac{3^2}{2!} - \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} - \frac{3^6}{6!} - \frac{3^7}{7!} + \ldots$ to the nearest ten-thousandth.

(A) -0.8295 (B) -0.8489 (C) -1.0464 (D) 1.0510 (E) -0.1424

54. If $\csc\theta = -3$ and $\cos\theta > 0$, then $\tan\theta =$

(A) $\frac{\sqrt{2}}{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{3}$ (D) $-\frac{\sqrt{3}}{4}$ (E) $-\frac{\sqrt{2}}{4}$

55. If $f(x) = ax^5 + bx^3 + cx + 7$ and $f(9) = 48$, then $f(-9) =$

(A) 41 (B) -34 (C) 48 (D) -48 (E) 55

56. Quadrilateral ABCD has vertices $(-7,3)$, $(-4,6)$, $(5,5)$ and $(9,-2)$ respectively. What is the area of ABCD?

(A) 77 (B) 61 (C) 45 (D) 67 (E) 71

57. Solve $e^{2x} - 10e^x + 21 = 0$.

(A) 0, $\log 21$ (B) $\log 3, \log 7$ (C) $\ln 3, \ln 7$ (D) 0, $\ln 21$ (E) 3, 7

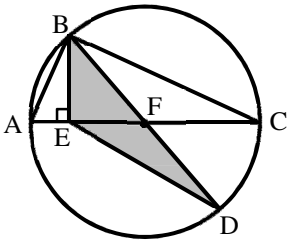
58. Given the set of integers in ascending order $\{a,b,c,d,e\}$ has a median of 14 a mean 16.4, mode 11 and range of 16. Find the value of *d*.

(A) 27 (B) 21 (C) 17 (D) 19 (E) 14

59. $3456_7 - 2345_6 - 1234_5 =$ _____$_{10}$.

(A) -123 (B) 891 (C) 1641 (D) 2029 (E) 503

60. On the diagram of circle F below, $2AE = EF$. If the area of triangle ABC is 72 cm^2, find the area of the shaded region.



(A) 36 cm^2 (B) 28 cm^2 (C) 48 cm^2 (D) 24 cm^2 (E) 12 cm^2

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2013-2014 TMSCA Mathematics Test Five Answers

1. C	21. B	41. A
2. B	22. A	42. E
3. E	23. E	43. E
4. B	24. E	44. C
5. C	25. B	45. C
6. E	26. C	46. D
7. A	27. D	47. D
8. E	28. D	48. D
9. A	29. C	49. D
10. D	30. A	50. A
11. E	31. C	51. E
12. D	32. A	52. C
13. E	33. B	53. B
14. A	34. D	54. E
15. D	35. E	55. B
16. C	36. D	56. B
17. A	37. C	57. C
18. B	38. B	58. D
19. E	39. C	59. E
20. D	40. A	60. C

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2013-2014 TMSCA Mathematics Test Five Select Solutions

8. $d = 18.4 - 16 = 2.4$
$n = \frac{119.2 - 16}{2.4} + 1 = 44$
$S_{44} = \frac{44}{2}(16 + 119.2) = 2974.4$

11. There will be two common factors in the numerator and denominator because the reduced fraction has quadratics. The factors will be $(x+1)$ and $(x+2)$. Synthetic or long division yields $\frac{x^2 - 3x + 2}{x^2 + 3x + 2}$ and $\frac{b}{a} = -1$

16. ${}_{18}C_2 = 153$ which is the 17th triangular number

19. $\frac{\cot^2 t}{\csc t} = \frac{\csc^2 t - 1}{\csc t} = \csc t - \sin t$

20. $5^x \cdot 5^{4y} = 1$ and $3^{5x} \cdot 3^{2y} = 3^{-1}$
$x + 4y = 0$ and $5x + 2y = -2$
Adding the two equations yields
$6x + 6y = -2$ divide by 6 and $x + y = -\frac{1}{3}$

21. $A^2 + B^2 = (A+B)^2 - 2AB = 121 - 2(24) = 73$

24. $\log a^{12}b^4 + \log a^{-2}b^4 + \log a^{1/2}b^{1/2}$
$= \log a^{21/2}b^{17/2} = \frac{1}{2}\log a^{21}b^{17}$

27. # of letters = 9, 3-A's and 2-S's
$\frac{9!}{(3!)(2!)} = 30240$

28. $f(x) = \cos x$ is an even function so
$\int_{-a}^{a} \cos x dx = 2\int_{0}^{a} \cos x dx$
$\int_{-a}^{a} 3\cos x dx = 6\int_{0}^{a} \cos x dx = 6C$

29. $7x + 13 = A(x-1) + B(x+3)$
Let $x = 1$ and $B = 5$
Let $x = -3$ and $A = 2$, so $AB = 10$

33. $B^2 - 4AC = 64 - 4(16)(1) = 0$, parabola

37. $P - Q = \frac{\sqrt{b^2 - 4ac}}{a} = \frac{\sqrt{376}}{6}$
$(P-Q)^4 = \frac{376^2}{6^4} = \frac{8836}{81}$

38. $\frac{\cot\theta}{\sec\theta} = \frac{\cos^2\theta}{\sin\theta} = \frac{1 - \sin^2\theta}{\sin\theta} = \csc\theta - \sin\theta$

42. $\cos\theta = \frac{-2(7) + 8(-6)}{\sqrt{(-2)^2 + (8)^2} \cdot \sqrt{(7)^2 + (-6)^2}}$
$\theta \approx 145^\circ$

45. $({}_{18}C_2)({}_{16}C_6) = 1225224$

48. $= \int_{-3}^{7} 3dx + 2\int_{-3}^{7} f(x)dx = 30 + 2(9) = 48$

51. $x^3 + ax^2 + bx + c = (x+2)(x-4)(x-11)$
The sum of the coefficients on the right equals the product of the sum of the coefficients of each factor.
$1 + a + b + c = 3(-3)(-10) = 90$
so $a + b + c = 89$

60. Triangles *ABC, EBF* and *EFD* all have the same heights $3EF = AC$, so
area triangle $EBF = \frac{1}{3}(72) = 24$
and area of *EBF* = area of *EFD*, so the area of the shaded region is 48 cm^2

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