



**TMSCA HIGH SCHOOL
MATHEMATICS
TEST #6 [C] ©
DECEMBER 7, 2013**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Evaluate: $36 - 30 + (24 - 18 \times 12) \div 6$

- (A) 42 (B) 24 (C) -6 (D) -26 (E) -31

2. A waiter served three couples at one of his tables. He received a \$5.40 tip from one couple, an \$8.10 tip from another couple and a \$9.00 tip from the third couple. All three of the couples figured an 18% tip on their individual bills. What was the table's total bill before the tip?

- (A) \$126.11 (B) \$375.00 (C) \$147.50 (D) \$135.00 (E) \$125.00

3. If x is 60% less than y and z is 140% more than y , then x is what part of z ?

- (A) $\frac{4}{5}$ (B) $\frac{14}{25}$ (C) $\frac{1}{4}$ (D) $\frac{3}{7}$ (E) $\frac{2}{7}$

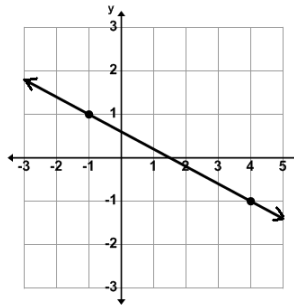
4. On a map legend, $1\frac{1}{4}$ inches represents 50 miles. How far is it in miles, from Maikyurbet, Nevada to Las Myshirt, Nevada, if the distance on the map is $4\frac{1}{2}$ inches?

- (A) 162.5 mi (B) 175 mi (C) 180 mi (D) 281.25 mi (E) 287.5 mi

5. How many distinguishable arrangements can be made from the letters "COEFFICIENT"?

- (A) 990 (B) 226,800 (C) 831,600 (D) 1,247,400 (E) 2,494,800

6. Find the slope of the line through $(0, -1)$ and perpendicular to the line shown ?



- (A) $\frac{5}{2}$ (B) $\frac{2}{1}$ (C) $-\frac{1}{3}$ (D) $-\frac{1}{2}$ (E) $-\frac{2}{5}$

7. The cost per pound of Pecans is \$7.00 and the cost per pound of cashews is \$9.00. What would the cost per pound be for a special mixture of 4 pounds of pecans and 3 pounds of cashews? (nearest cent)

- (A) \$7.75 (B) \$7.86 (C) \$8.00 (D) \$8.14 (E) \$8.50

8. Find k when $15 - 20 + (35 - 25 \times k) \div 5 = -198$

- (A) 5.88 (B) 10 (C) 30 (D) 40 (E) 45.6

9. Roland Bones rolled two dice. What are the odds that the sum of the top faces is less than five?

- (A) $\frac{1}{6}$ (B) $\frac{5}{36}$ (C) $\frac{1}{5}$ (D) $\frac{7}{30}$ (E) $\frac{2}{15}$

10. Millie Watt works for an electric company. She knows that the length of a wire varies directly with the weight of the wire. She weighs 20 feet of wire and finds that it weighs 3 pounds. How long would a roll of wire be if it weighs 25 pounds?

- (A) 42 ft (B) 126 ft 6 in (C) 166 ft 8 in (D) 240 ft (E) 266 ft 4 in

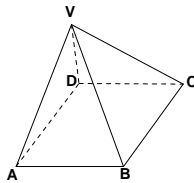
11. The sum of the interior angles of a simple convex octagon is:

- (A) 720° (B) 900° (C) 1080° (D) 1440° (E) 1800°

12. M. T. Tank has a rectangular horse trough that is 5' long, 2.5' wide, and 2' deep. He fills it three-fourths full. How many gallons of water are in the trough? (nearest gallon)

- (A) 101 gal (B) 140 gal (C) 142 gal (D) 164 gal (E) 187 gal

13. Let $AB = 10''$. If the height of the square based pyramid shown is $12''$, then the total surface area of the pyramid is _____. (nearest sq. inch)



- (A) 325 in^2 (B) 400 in^2 (C) 378 in^2 (D) 340 in^2 (E) 360 in^2

14. Jewel received an ankle bracelet and five different charms for 16th birthday. How many different ways can she arrange the charms on her bracelet?

- (A) 120 (B) 40 (C) 30 (D) 24 (E) 20

15. If you slice a right circular cone with a plane parallel to the base of the cone, the intersection is a(n) _____.

- (A) line (B) circle (C) elongated ellipse (D) parabola (E) hyperbola

16. Let $f(x) = \frac{6x+5}{5x-6}$. Find $f^{-1}(4)$.

- (A) $2\frac{1}{8}$ (B) $2\frac{1}{14}$ (C) $1\frac{7}{19}$ (D) $\frac{19}{26}$ (E) $1\frac{6}{7}$

17. The ratio of the square of the semimajor axis to the foci is called the _____.

- (A) asymptote (B) directrix (C) eccentricity (D) focal distance (E) locus

18. Phil Upp has a 1600 gallon water tank. His well pumps water into the tank at a rate of 7 gallons a minute. He can fill the tank in 2.5 hours if he uses his well and a garden hose. If his well goes dry, how long would it take him to fill the tank using just the garden hose? (nearest minute)

- (A) 7 hrs 16 min (B) 9 hrs 38 min (C) 10 hrs 40 min (D) 8 hrs 10 min (E) 3 hrs 49 min

19. How many 3-digit numbers exist such that the sum of their digits equals 5?

- (A) 10 (B) 12 (C) 15 (D) 18 (E) 30

20. Roland A. Long is building a ramp, so his grandfather can use his scooter to get to the deck porch. The deck is 6 feet from the ground. How long will the ramp be if the angle of elevation of the ramp with the ground is 20° ? (nearest inch)

- (A) 6' 5" (B) 7' 0" (C) 16' 6" (D) 17' 7" (E) 20' 5"

21. If $\sin A = \frac{1}{2}$, where $\frac{\pi}{2} < A < \pi$, and $\cos B = \frac{1}{2}$, where $\frac{3\pi}{2} < B < 2\pi$, then $\sin(A + B) - \cos(A + B) = ?$

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

22. The graph of the polar equation $r = 6 + 3\cos(\theta)$ is a:

- (A) dimpled cardioid (B) looped cardioid (C) lemniscate
(D) looped limaçon (E) convex limaçon

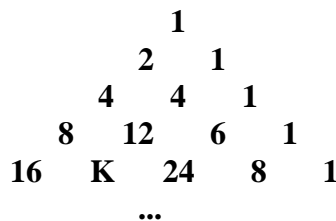
23. The repeating decimal 0.232323... in base 6 can be written as which of the following fractions in base 6?

- (A) $\frac{15}{35}_6$ (B) $\frac{23}{55}_6$ (C) $\frac{11}{23}_6$ (D) $\frac{3}{11}_6$ (E) $\frac{5}{12}_6$

24. Simplify to the form $a + bi$: $(5 + \sqrt{-80})(3 - \sqrt{-45})$

- (A) $-7 - 3\sqrt{5}i$ (B) $45 + 3\sqrt{5}i$ (C) $27 + 27\sqrt{5}i$ (D) $75 - 3\sqrt{5}i$ (E) $17 + 7\sqrt{5}i$

25. Find K if the triangular pattern shown below continues.



- (A) 32 (B) 28 (C) 24 (D) 20 (E) 16

26. $\sum_{k=-1}^1 (kx + 1)^2 = ?$

- (A) 3 (B) $x^2 + 3$ (C) $2x^2 + 3$ (D) $2x^2 + 1$ (E) $x^2 + 2$

27. Solve for X: $\frac{1}{\log_{18}x} + \frac{1}{\log_{12}x} - \frac{1}{\log_6x} = \frac{1}{2}$

- (A) 6 (B) 3 (C) 576 (D) 1,296 (E) 2,592

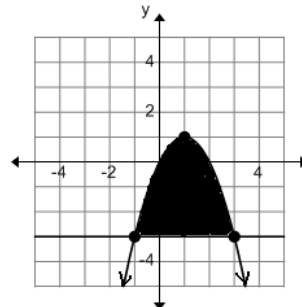
28. Which of the following numbers is in row 13 of Pascal's triangle?

- (A) 1,715 (B) 1,287 (C) 1,234 (D) 924 (E) 791

29. Evaluate: $\int_{-n}^n (2x^3 + 4) dx$

- (A) n^4 (B) $8n$ (C) $n^2 - 8n$ (D) $n - 4$ (E) $n^4 - 4n$

30. Find the area of the shaded region in square units.



- (A) 11.333... (B) 10.666... (C) 10 (D) 9.75 (E) 9.5

31. Find the slope of the line tangent to the curve $y = x^2 - 6x + 9$ at the point (1, 4).

- (A) -4 (B) -3 (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) 2

32. How many positive integers less than or equal to 2013 are divisible by neither 2 nor 3?

- (A) 671 (B) 1,678 (C) 2,013 (D) 1,006 (E) 335

33. $101_2 + 111_3 + 121_5 = \underline{\hspace{2cm}}_4$

- (A) 131 (B) 312 (C) 1431 (D) 303 (E) 1235

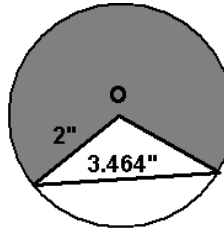
34. A line crosses the y-axis at $y = -3$ and goes through the point (2, 2). Another line crosses the x-axis at $x = 3$ and goes through the point $(-2, -2)$. The lines intersect at (x, y) . Find $x + y$.

- (A) $-\frac{6}{7}$ (B) 0 (C) 1 (D) $1\frac{1}{6}$ (E) $1\frac{5}{7}$

35. Saul D. Wudkutter has 5 pieces of wood whose lengths are 4 inches, 5 inches, 7 inches, 10 inches, and 12 inches. How many different triangles can he make?

- (A) 10 (B) 8 (C) 6 (D) 5 (E) 4

36. Given the circle O with radius and chord shown, find the shaded area. (nearest tenth)



- (A) 6.9 sq. in (B) 7.5 sq. in (C) 7.8 sq. in (D) 8.4 sq. in (E) 8.8 sq. in
37. How many integral values of n exist such that $n \geq 3$ and $\frac{(n-1)!}{(n-3)!} \leq 72$
- (A) 24 (B) 18 (C) 12 (D) 8 (E) 6
38. The *Saul T. See* yacht starts at buoy #1 and sails 5 km on a bearing of 30° to buoy #2. Then he turns and sails on a bearing of 345° to buoy #3 which is due north of buoy #1. How far is buoy #3 from buoy #1? (nearest tenth)
- (A) 7.1 km (B) 9.7 km (C) 13.7 km (D) 9.2 km (E) 6.4 km
39. The sum of the coefficients of the 1st term in the expansion of $(x + 1)^2$, the 2nd term of $(x + 1)^3$, the 3rd term of $(x + 1)^4$, and 4th term of $(x + 1)^5$ is:
- (A) 24 (B) 20 (C) 16 (D) 14 (E) 12
40. Find, to the nearest degree, the measure of the acute angle between $y = 3x - 1$ and $y = x + 3$.
- (A) 63° (B) 18° (C) 27° (D) 72° (E) 34°
41. Leonardo Pisa numbered 10 blank cards from 2 to 11 and mixed them up in a bag. He drew out two cards at the same time and placed them on the table. What is the probability that both of them were Fibonacci numbers?
- (A) $26\frac{2}{3}\%$ (B) $10\frac{10}{11}\%$ (C) $18\frac{2}{11}\%$ (D) $13\frac{1}{3}\%$ (E) $21\frac{9}{11}\%$
42. The person considered to be the first mathematician to use algebraic notation and symbolism was _____.
- (A) Agnesi (B) Archimedes (C) Boole (D) Diophantus (E) Zeno
43. Which "trapezoidal mean" can be used to find the volume of a frustrum of a cone?
- (A) heronian (B) harmonic (C) geometric (D) centroidal (E) algebraic
44. The *Claud Hopper* shoe store reduced the original selling price of a pair of shoes by 20%, resulting in a profit of 4% over the cost of the shoes. What percentage of profit would the store have made if the shoes had been sold for the original price?
- (A) 8% (B) 16% (C) 24% (D) 30% (E) $33\frac{1}{3}\%$

45. If $\frac{5}{2y} - \frac{3x}{4} = \frac{1}{6}$, then x equals _____.

- (A) $\frac{12+9y}{10}$ (B) $\frac{30-2y}{9y}$ (C) $\frac{60}{18y+1}$ (D) $\frac{30-9y}{2y}$ (E) $\frac{-10}{9y-12}$

46. Find the remainder when 6^{2014} is divided by 7.

- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0

47. A circle with center O is inscribed inside $\triangle PQR$ with side lengths 11 cm, 14 cm, and 19 cm. Find the radius of the circle. (nearest tenth)

- (A) 3.5 cm (B) 4.2 cm (C) 4.8 cm (D) 6.6 cm (E) 9.6 cm

48. $\triangle ABC$ and $\triangle PQR$ exist such that $\angle BAC \cong \angle PRQ$, $\angle ACB \cong \angle RQP$, $AB = 15$, $AC = 18$, $PR = 45$, and $PQ = 42$. Find $QR - BC$.

- (A) 54 (B) 6 (C) 47 (D) 3 (E) 40

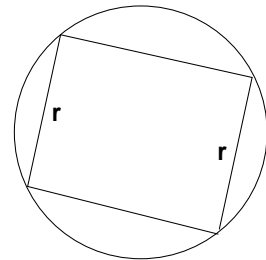
49. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 3 & 6 \\ 2 & 3 & 4 \end{bmatrix}$. Find $|A|$.

- (A) 48 (B) 18 (C) 6 (D) -6 (E) -12

50. If $a_1 = -1$, $a_2 = 1$ and $a_n = (a_{n-2} - a_{n-1})(a_{n-2})$, where $n \geq 3$, then a_7 equals:

- (A) -6 (B) -1 (C) 2 (D) 6 (E) 7

51. A rectangle is inscribed in the circle of radius r . Find the area of the rectangle.



- (A) $\frac{\sqrt{3}r^2}{2}$ (B) $2r^2$ (C) $\frac{\sqrt{2}r^2}{2}$ (D) $\sqrt{2}r^2$ (E) $\sqrt{3}r^2$

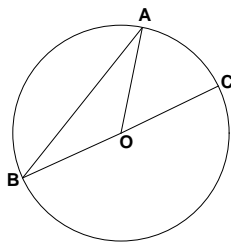
52. Determine the range of $f(x) = 1 - 3\cos(6x + 10)$.


- (A) $[-2, 4]$ (B) $[-2, 2]$ (C) $[-3, 4]$ (D) $[-3, 3]$ (E) $[-1, 5]$

53. Find C if the remainder when $4x^3 - 2x^2 + C$ is divided by $x + 3$ is -63 .

- (A) 9 (B) 12 (C) 30 (D) 42 (E) 63

54. Let $x^5 + x^4 - px^3 - x^2 - qx + 1 = 0$, where $p, q > 0$. According to Descartes' Rule of Signs, how many possible negative roots are there?
- (A) 5, 3, or 1 (B) 3 or 1 (C) 4, 2, or 0 (D) 2 or 0 (E) 0
55. Let $f(x) = \frac{7x-11}{11x+7}$. Find $f'(-1)$.
- (A) -4 (B) $-0.222\dots$ (C) 4.5 (D) 9 (E) 10.625
56. Frank Linn was flying his kite. The kite was at a height of 40 feet. Frank was carrying it horizontally at a rate of 3 ft/sec. At what rate is the string being pulled out when the length of the string released is 50 ft? (nearest tenth)
- (A) 2.0 ft/sec (B) 2.3 ft/sec (C) 1.8 ft/sec (D) 3.8 ft/sec (E) 4.0 ft/sec
57. Penni Les has six one-dollar bills and three envelopes. In how many different ways can she put the bills in the envelopes?
- (A) 18 (B) 28 (C) 60 (D) 120 (E) 216
58. Huey, Dewey, and Louie are trying to solve this problem. The probability that Huey will solve it is $\frac{2}{3}$, that Dewey will solve it is $\frac{3}{4}$, and that Louie will solve it is $\frac{1}{2}$. What is the probability that at least one of three will solve it? (nearest percent)
- (A) 50% (B) 68% (C) 75% (D) 96% (E) 100%
59. Points A, B, and C lie on the circle with the radius, diameter, and center O shown. Find $m\angle AOC$ if the $m\angle ABO = 30^\circ$.



- (A) 15° (B) 30° (C) 45° (D) 60° (E) 75°
60. The Mayan number system consists of three symbols, \bullet — . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. What base 10 number is represented by:



- (A) 10 (B) 13 (C) 17 (D) 43 (E) 70

**2013-14 TMSCA HS Math Test #6
Answer Key**

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|-------|-------|-------|
| 1. D | 21. D | 41. D |
| 2. E | 22. E | 42. D |
| 3. E | 23. D | 43. A |
| 4. C | 24. D | 44. D |
| 5. E | 25. A | 45. B |
| 6. A | 26. C | 46. D |
| 7. B | 27. D | 47. A |
| 8. D | 28. B | 48. E |
| 9. C | 29. B | 49. E |
| 10. C | 30. B | 50. A |
| 11. C | 31. A | 51. E |
| 12. B | 32. A | 52. A |
| 13. E | 33. B | 53. E |
| 14. D | 34. B | 54. B |
| 15. B | 35. C | 55. E |
| 16. B | 36. D | 56. C |
| 17. B | 37. D | 57. B |
| 18. A | 38. C | 58. D |
| 19. C | 39. B | 59. D |
| 20. D | 40. C | 60. B |