T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 6 [U I L C] ©

D E C E M B E R 3 , 2 0 1 6

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Evaluate: $1 + 2^0 \times 3 - 20 \div (1 \times 6)$

(A) $-2\frac{2}{3}$ **(B)** $-2\frac{1}{3}$ **(C)** $-1\frac{1}{3}$ **(D)** $\frac{2}{3}$ **(E)** $1\frac{1}{3}$

2. Find the sum of the multiples of 4 that are greater than 0 and less 123.

(A) 1,984 **(B) 1,800** **(C) 1,860** **(D) 1,845** **(E) 1,736**

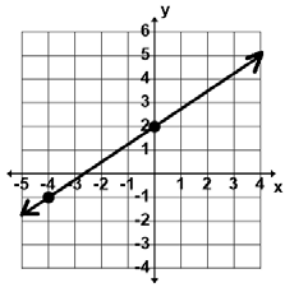
3. Millie What's electric company charges a $10.00 per month base fee plus 8¢ per kilowatt for the first 1,000 kilowatts and 5¢ per kilowatt for any kilowatts over 1,000. What would her electric bill be if she used 1,250 kilowatts during the month?

(A) $72.50 **(B) $102.50** **(C) $110.00** **(D) $126.62** **(E) $172.50**

4. Simplify: $\frac{(n+1)!}{(n-1)!} \times \frac{1}{n} \div \frac{(n)!}{(n+2)!}$

(A) $n^2 + n + 2$ **(B) 1** **(C)** $n^3 - n^2 - 2n$ **(D)** $n^2 + 2n$ **(E)** $n^3 + 4n^2 + 5n + 2$

5. A line perpendicular to the line shown at point (0, 2) contains which of the following points?



(A) (— 3, 6) **(B) (— 4, 3)** **(C) (— 2, 4)** **(D) (— 4, 5)** **(E) (3, — 4)**

6. Al Mund bought some cashews that cost $7.00 per pound and some walnuts that cost $2.25 per pound. How many pounds of walnuts did he buy if he bought 10 total pounds that cost $51.00?

(A) 6 lbs **(B) 5.5 lbs** **(C) 5 lbs** **(D) 4.5 lbs** **(E) 4 lbs**

7. If $10x^2 + 11x - 6 = (2x + 3)(ax + b)$ then $a + b =$ _____.

(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

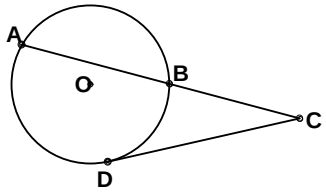
8. An operation " △ " is defined by: $a \triangle b = a^b - ab$. What is the value of $(-1 \triangle 2) \triangle (1 \triangle -6)$?

(A) 322 **(B) 2,166** **(C) 228** **(D) 2,208** **(E) 4**

9. A square has side lengths of 5 cm. If the square's width is doubled and its length is tripled, then the perimeter of the square is what percent of the perimeter of the new rectangle?

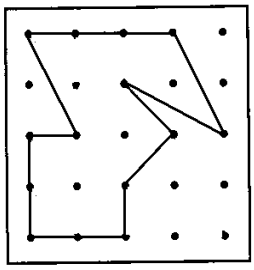
(A) $66\frac{2}{3}\%$ **(B) 40%** **(C)** $33\frac{1}{3}\%$ **(D) 16%** **(E) 10%**

10. Given the circle with center O shown. Find CD if AB = 4" and BC = 3". (nearest tenth).



(A) 2.6 (B) 3.5 (C) 4.6 (D) 5.3 (E) 7.0

11. A rubber band was stretched on the geoboard to form this 10-sided figure. What is its area?



(A) 8 sq. units (B) 8.5 sq. units (C) 9 sq. units (D) 9.5 sq. units (E) 10 sq. units

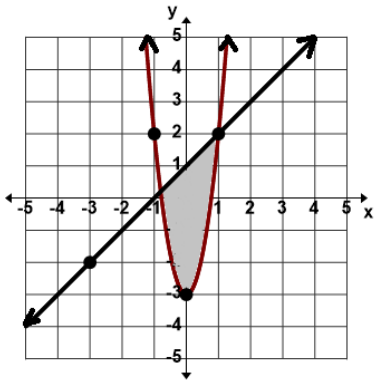
12. Miss Dumas is having a bake sale to raise money for her math team to attend a math camp. She sold 6 cakes and 4 pies for $48.00 to Pam and 5 cakes and 2 pies for $35.00 to Jack. How much would Holly have to pay for 3 cakes and 6 pies?

(A) $32.00 (B) $37.75 (C) $39.00 (D) $41.50 (E) $43.00

13. If $\sqrt{x^4\left(\sqrt{x^3(\sqrt{x})}\right)} = \sqrt[n]{x^k}$ **, where k and n are relatively prime and x > 0, then k = ?**

(A) 6 (B) 8 (C) 9 (D) 23 (E) 24

14. Which of the following system of inequalities would be best represented by the shaded region shown?



(A) $y \le x + 1$, $y \ge 5x^2 - 3$

(B) $y \le x - 1$, $y \ge 3x^2 + 3$

(C) $y \ge x + 1$, $y \le x^2 + 3$

(D) $y \le x + 1$, $y \ge x^2 + 3$

(E) $y \ge x + 1$, $y \le 5x^2 - 3$

15. Using Blaise Pascal's triangle, determine what the third number in the 11^{th} row will be.

1	(row 0)
1 1	(row 1)
1 2 1	(row 2)
1 3 3 1	(row 3)
1 4 6 4 1	(row 4)
1 5 10 10 5 1	(row 5)
...	(row ...)

(A) 66 (B) 60 (C) 55 (D) 50 (E) 45

16. A plane has traveled 26 miles on a course heading 48º east of north. How far north has the plane traveled at this point, to the nearest tenth of a mile?

(A) 14.4 miles (B) 17.4 miles (C) 18.0 miles (D) 19.3 miles (E) 24.4 miles

17. Determine the period of $f(x) = 4\cos[3\pi(x + 2)] - 1$.

(A) (B) 3 (C) $1\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{2}{3}$

18. How many 3-digit numbers exist such that the sum of their digits equals 12?

(A) 56 (B) 66 (C) 73 (D) 78 (E) 90

19. If $\log(ab) = 6$ and $\log(a/b) = \frac{1}{3}$, what is the value of the $\log(a)$?

(A) 2 (B) 3 (C) $3\frac{1}{6}$ (D) $3\frac{2}{3}$ (E) $5\frac{2}{3}$

20. Find $a + b + c + d$ given the Fibonacci characteristic sequence: 1, a, b, 11, 17, c, d, ...

(A) 67 (B) 78 (C) 84 (D) 95 (E) 113

21. Which of the following functions is neither even nor odd? f(x) = _____.

(A) $|x| - 2$ (B) x^3 (C) $x^2 + 4$ (D) $\frac{1}{x^2+1}$ (E) $(x - 2)^2$

22. The fraction $\frac{15}{23}$ in base 6 can be written as which of the following decimals in base 6?

(A) $0.1121212..._6$ (B) $0.4222..._6$ (C) $0.3444..._6$ (D) $0.2333..._6$ (E) $0.242424..._6$

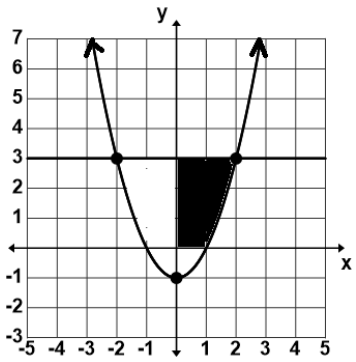
23. Let $f'(x) = 6x^2 + 2x - 1$ and $f(-1) = 3$. Find $f(1)$.

(A) -3 (B) -1 (C) 0 (D) 2 (E) 5

24. Let $f(x) = \frac{x-2}{2x+1}$, where $x \neq -\frac{1}{2}$. Find $f^{-1}(x)$.

(A) $\frac{-x-2}{2x-1}$ **(B)** $\frac{1-2x}{2+x}$ **(C)** $\frac{2-x}{1-2x}$ **(D)** $\frac{x+2}{2x-1}$ **(E)** $\frac{-x+2}{2x-1}$

25. The area (in square units) of the dark shaded region shown is:



(A) 4 **(B)** $4\frac{1}{3}$ **(C)** $4\frac{2}{3}$ **(D)** 5 **(E)** $5\frac{1}{3}$

26. Simplify: $\frac{(n+1)!(n-2)!}{n!(n-1)!}$

(A) $n^2 + n$ **(B)** $\frac{n+1}{n-1}$ **(C)** $\frac{n-2}{n+2}$ **(D)** $\frac{n+1}{n-2}$ **(E)** $\frac{n^2+n}{n-1}$

27. The 1952 senior class at Millersview High School consisted of 5 girls (Alice, Betty, Carol, Dawn, Ellie) and 6 boys (Frank, George, Hal, Igor, Justin, Keith). A girl was chosen for prom queen and a boy for prom king. What is the probability that Carol and Keith were chosen?

(A) $3\frac{1}{3}\%$ **(B)** $6\frac{2}{3}\%$ **(C)** $16\frac{2}{3}\%$ **(D)** 20% **(E)** $33\frac{1}{3}\%$

28. The *I Screem* Shop specializes in triple scoop sundaes. How many different triple scoop sundaes could be made using these flavors: vanilla, chocolate, strawberry, cherry, lime, and lemon?

(A) 120 **(B)** 60 **(C)** 56 **(D)** 35 **(E)** 20

29. Coach Hobb's UIL team consists of 5 boys and 7 girls. How many different 5-member teams consisting of 3 girls and 2 boys could he make up?

(A) 792 **(B)** 700 **(C)** 420 **(D)** 350 **(E)** 210

30. Who was the first Greek mathematician who recognized fractions as numbers; thus he allowed positive rational numbers for the coefficients and solutions.

(A) Agnesi **(B)** Byron **(C)** Cantor **(D)** Diophantus **(E)** Euler

31. If A= {a, c, u, t, e}, O = {o, b, t, u, s e}, R = {r, i, g, h, t} and T = {t, r, i, a, n, g, l, e} then $(A \cup R) \cap (O \cup T)$ contains how many distinct elements?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

32. Cal Q. Later's four math team members scored 164, 94, 120, and 142 on the Fall SAC test. What was the difference between the mean and the median of the team's scores?

(A) 0 (B) 1 (C) 2 (D) 6 (E) 8

33. Find the greatest common divisor of 88, 120, and 152.

(A) 2 (B) 4 (C) 8 (D) 12 (E) 16

34. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{20}?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

35. If the roots of $x^3 + 5x^2 + cx + d = 0$ are 2, 4, and k, then $c + d$ equals:

(A) — 58 (B) — 11 (C) 11 (D) 30 (E) 35

36. If $2x - y = 3$, $x - y = 2$ and $ax - y = 4$, then a equals:

(A) 4 (B) 3 (C) 1 (D) — 1 (E) — 5

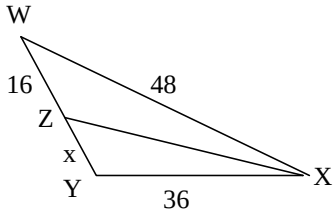
37. Joy Ryder drove to town to hang out with friends. She averaged only 15 mph because of construction. On her way home she averaged 35 mph. The total driving time to town and back was 2 hours. How long did it take Joy to drive to town?

(A) 1 hr 40 min (B) 1 hr 10 min (C) 1 hr 24 min (D) 35 min (E) 50 min

38. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + ... + \frac{1}{36}$ = _____.

(A) $\frac{1}{5}$ (B) $\frac{4}{5}$ (C) $\frac{9}{11}$ (D) $\frac{5}{7}$ (E) $\frac{7}{9}$

39. Find WY if $\angle WXZ \cong \angle YXZ$.



(A) 28 (B) $21\frac{1}{3}$ (C) 14 (D) 12 (E) not enough information given

40. Les Square drew a rectangle with the length being twice the width. He drew a second rectangle with the length and width being 4 inches shorter than the original rectangle. The area of the second rectangle was 44 sq. inches less than the area of the original one. What was the length of the original rectangle?

(A) 2.5 " (B) 5 " (C) 6 " (D) 10 " (E) 12.5 "

41. The point of concurrency of the perpendicular bisectors of the sides of a triangle is called the:

(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) line of Euler

42. The number 28 is a member of which of the following sets of numbers?
(E)vil (D)eficient (H)appy (P)olite

(A) H & P only (B) H only (C) D, H, & P, but not E
(D) E, D, & H, but not P (E) all of them

43. If $a_1 = -2$, $a_2 = 3$, and $a_n = (a_{n-1})(a_{n-2}) - (a_{n-2})$ where $n > 2$ then a_5 = ?

(A) -15 (B) -10 (C) -4 (D) 6 (E) 64

44. Which of the following is an identity for $\frac{1}{\cos x} - \frac{\cos x}{1 + \sin x}$, where $\cos x \neq 0$?

(A) $\tan x$ (B) $\frac{1}{1 - \sin x}$ (C) $\sin x$ (D) $-\frac{1}{1 + \sin x}$ (E) $\cot x$

45. Point B lies on $\overline{AC}$ and points D and E exist such that $\overline{DB} \perp \overline{AC}$, $\overline{EC} \perp \overline{BC}$, AB = 1", BC = 2", CE = 3", BD = 4", AE intersects DB at point F, and all 6 points are coplanar. Find m∠CAE. (nearest degree).

(A) 27 ° (B) 18 ° (C) 34 ° (D) 45 ° (E) 53 °

46. Let vector u = (1, 4) and vector v = (−5, −3). Find the measure of the angle with initial side u and terminal side v. (nearest second)

(A) 163°4' 21" (B) 135° 0' 0" (C) 156° 38' 7" (D) 141° 20' 51" (E) 45° 0' 0"

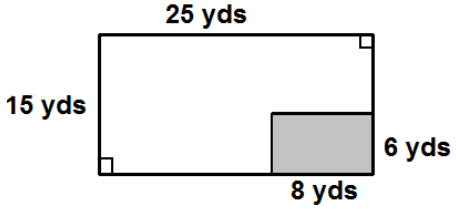
47. Find C if the remainder when $3x^3 + 2x^2 + x + C$ is divided by $x + 2$ is 1.

(A) 19 (B) −27 (C) −2 (D) 45 (E) −35

48. Find the distance between the foci of the hyperbola, $25x^2 - 4y^2 = 100$. (nearest tenth)

(A) 7.3 (B) 9.2 (C) 10.0 (D) 10.8 (E) 14.0

49. Luce Phit lost a ring in her rectangular garden. She roped off the shaded rectangular region shown below to start looking for the ring. What are the odds she will find it in that region? (nearest whole percent)



(A) 5% (B) 13% (C) 15% (D) 20% (E) 25%

50. Find an equation of the line tangent to the curve $y = 2 - 3x^2$ at $x = -1$.

(A) $y = -6x - 7$ (B) $y = 7x - 5$ (C) $y = 2 - 6x$ (D) $y = 6x + 5$ (E) $y = 6x + 7$

51. Which of the functions are concave up on an open interval containing x = 0?

I. ln(x) II. x^2 III. cos(x) IV. $\frac{1}{x^2 - 1}$

(A) I only (B) II only (C) II & III (D) II & IV (E) II, III, & IV

52. A lock's combination consists of three digits. The first digit is a power of 2, the second digit is a prime number, and the last digit is a factor of 10. How many unique combinations fit this criteria?

(A) 64 (B) 48 (C) 36 (D) 12 (E) 11

53. The graph of $g(x) = (x^3 + 2x^2 - 3x) \div (x^2 + 2x - 3)$ has vertical asymptotes at:

(A) $x = 1, -3$ (B) $x = 0$ (C) $x = 1$ (D) $x = -3$ (E) g(x) has no vertical asymptotes

54. A garage sale has 6 items left for sale. Three of the items cost $3.00 each. Two items cost $5.00 each. The last item cost $2.00. A customer comes in and buys one of the items. What is the mathematical expectation value of the purchase? (nearest cent)

(A) $3.50 (B) $2.50 (C) $3.83 (D) $2.67 (E) $3.33

55. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_n = 1{,}597$ then n is:

(A) 17 (B) 14 (C) 11 (D) 9 (E) 8

56. Point P(— 1 , 3) undergoes several transformations to point Q. First, it is reflected across the y–axis. Then, it is reflected across the x-axis. Then it is translated 5 units vertically in the positive direction and horizontally 2 units in the negative direction. What are the coordinates of point Q?

(A) (4, 2) (B) (— 1, 3) (C) (— 3, 1) (D) (1, — 2) (E) (— 1, 2)

57. If $\frac{4x - 1}{x + 4} - \frac{Ax + B}{3x + 2} = \frac{10x^2 + 10}{3x^2 + 14x + 8}$, where A and B are constants, then A — B equals:

(A) 9 (B) 5 (C) 1 (D) — 1 (E) — 3

58. Let f(x) = 2x + 1, g(x) = 3x — 2, h(x) = x + 1, and $f\big(g(h(x))\big) = -9$. Find x.

(A) $1\frac{1}{7}$ (B) $\frac{5}{6}$ (C) $-\frac{1}{2}$ (D) — 1 (E) — 2

59. $134_8 \times 25_8 =$ _____ $_8$

(A) 132 **(B)** 3864 **(C)** 3350 **(D)** 3414 **(E)** 3614

60. A function, $g(x) = x^2 + bx + c$, **exists such that** $g(1) = 2$ **and** $g(2) + g(3) = 5$. **Find** $g(-1)$.

(A) -5 **(B)** $-3\frac{1}{3}$ **(C)** -2 **(D)** 2 **(E)** $8\frac{2}{3}$

2016-17 TMSCA HS Math Test #6
Answer Key

1. D	**21. E**	**41. D**
2. C	**22. B**	**42. A**
3. B	**23. E**	**43. E**
4. E	**24. A**	**44. A**
5. A	**25. C**	**45. D**
6. E	**26. B**	**46. B**
7. A	**27. A**	**47. A**
8. B	**28. C**	**48. D**
9. B	**29. D**	**49. C**
10. C	**30. D**	**50. D**
11. E	**31. D**	**51. B**
12. C	**32. B**	**52. B**
13. D	**33. C**	**53. E**
14. A	**34. B**	**54. A**
15. C	**35. D**	**55. A**
16. B	**36. B**	**56. E**
17. E	**37. C**	**57. B**
18. B	**38. E**	**58. E**
19. C	**39. A**	**59. E**
20. C	**40. D**	**60. E**