

**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 7 ©
JANUARY 14, 2017**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016-2017 TMSCA Mathematics Test Seven

- Evaluate: $\frac{13}{24} \div 0.555... + 0.85$.
 (A) $\frac{663}{800}$ (B) $\frac{1243}{1080}$ (C) $\frac{73}{40}$ (D) $\frac{49}{24}$ (E) $\frac{1447}{1320}$
- Find the number of positive integral divisors of 296
 (A) 8 (B) 6 (C) 10 (D) 9 (E) 2
- Carrie drives to work every weekday on the highway. Her average daily speeds for the week are 66 mph, 68 mph, 57 mph, 65 mph and 65 mph. What is her average speed for the week? (nearest tenth)
 (A) 64.2 mph (B) 63.9 mph (C) 63.7 mph (D) 64.8 mph (E) 64.0 mph
- If $\frac{x^4 - 5x^2 + 4}{(x^2 + 4x + 4)(x^2 + 2x + 1)} = \frac{x^2 + ax + 2}{x^2 + bx + 2}$, find $\frac{a}{b}$.
 (A) 1 (B) -1 (C) -3 (D) $-\frac{3}{4}$ (E) $-\frac{1}{3}$
- If $m\angle A + m\angle B + m\angle C = 180^\circ$ and $m\angle C + m\angle D = 180^\circ$, then $m\angle A + m\angle B + m\angle C = m\angle C + m\angle D$ is an example of _____property.
 (A) Distributive (B) Transitive (C) Associative (D) Commutative (E) Closure
- Let $X = \{m, a, s, c, o, t\}$, $Y = \{s, p, o, r, t\}$ and $Z = \{p, o, i, n, t, s\}$. Find $(X \cup Z) \cap (Z \cap Y) \cup (X \cap Y)$?
 (A) $\{o, p, s, t\}$ (B) $\{m, o, p, s, t\}$ (C) $\{m, p, s, t\}$ (D) $\{o, s, t\}$ (E) $\{a, m, o, p, s, t\}$
- The equation of a line \overline{AB} is $y = \frac{2}{3}x + \frac{11}{3}$. The line \overline{CD} is parallel to \overline{AB} and includes the point $(-3, 7)$. What is the x -intercept of \overline{CD} ?
 (A) $(9, 0)$ (B) $(-9, 0)$ (C) $(0, 9)$ (D) $(-\frac{27}{2}, 0)$ (E) $(\frac{27}{2}, 0)$
- A triangle is inscribed in a circle. The center of the circle is the intersection of the ____ of the triangle.
 (A) Angle Bisectors (B) Medians (C) Altitudes
 (D) Sides (E) Perpendicular Bisectors
- A set of positive integers has a mean of 18, a median of 15, a mode of 28 and a range of 19. If A, B, C, D and E are the integers arranged from least to greatest, the value of B is?
 (A) 9 (B) 11 (C) 10 (D) 15 (E) 12
- Find the total surface area of a right cone given the radius of the base is 12 ft. and the vertex angle is 35° . (nearest square foot)
 (A) 3462 ft^2 (B) 1889 ft^2 (C) 5740 ft^2 (D) 1957 ft^2 (E) 3325 ft^2

11. $4^3 + 5^3 + \dots + 18^3 =$

- (A) 29205 (B) 29241 (C) 23409 (D) 23373 (E) 29141

12. Two standard dice are rolled. What is the probability that the positive difference in the numbers on the dice is 3?

- (A) $\frac{5}{36}$ (B) $\frac{7}{36}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$ (E) $\frac{1}{3}$

13. All the edges of a cube are expanding at a rate of 4.75 cm/s. How fast is the volume of the cube changing when the length of one edge is 10 cm?

- (A) 142.5 cm³/s (B) 475 cm³/s (C) 775 cm³/s (D) 1425 cm³/s (E) 1275 cm³/s

14. Naming the single 1 at the top as row 0, what is the sum of all the numbers greater than one in the 12th row of Pascal's triangle?

- (A) 4096 (B) 8190 (C) 2046 (D) 2048 (E) 4094

15. $(\sin x + \cos x)^2 = ?$

- (A) $1 - \sin(2x)$ (B) $\sin(2x) - 1$ (C) $1 + \sin(2x)$ (D) $1 + 2\sin x$ (E) $2\sin x - 1$

16. The graph of the polar equation $r = 1 - 2\cos\theta$ is a _____.

- (A) Rose Curve (B) Lemniscate (C) Circle (D) Cardioid (E) Limacon

17. Working alone, Jim and Joe can plaster a wall in 22 min and 32 min respectively. How fast can they plaster a wall twice as high and four times as long together? (nearest minute)

- (A) 52 min (B) 104 min (C) 26 min (D) 54 min (E) 108 min

18. How many 3-digit numbers exist such that the sum of their digits equals 4?

- (A) 10 (B) 11 (C) 9 (D) 8 (E) 5

19. Solve $2\sin x = \tan x$ for x where $0 < x \leq \frac{\pi}{2}$

- (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) π (E) $\frac{2\pi}{3}$

20. One-half of James' age two years from now plus one-third of his age three years ago is twenty years. Find his age one decade from now.

- (A) 20 (B) 24 (C) 28 (D) 30 (E) 34

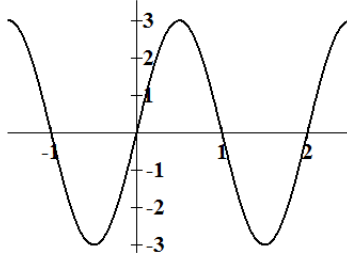
21. Determine the range of $f(\theta) = 7 - 5\sin\left(\frac{2\pi}{3}\theta - 2\right)$.

- (A) [6,8] (B) [6,12] (C) [2,12] (D) [1,3] (E) [-1,-3]

22. What is $\sum_{k=-2}^0 (kx+2)^2$?

- (A) $5x^2+6x+12$ (B) $5x^2+12x+12$ (C) $5x^2-12x+12$
 (D) $5x^2-6x+12$ (E) $5x^2+12x+8$

23. If the equation of the function graphed below is $y = a \sin(bx) + c$, find the value of ab .



- (A) 3π (B) $\frac{3\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) -3π (E) $-\frac{3\pi}{2}$

24. A and B are the roots of $f(x) = 2x^2 - 9x - 45$. Calculate the value of $A^4 - 4A^3B + 6A^2B^2 - 4AB^3 + B^4$.

- (A) $\frac{6561}{16}$ (B) $\frac{194481}{16}$ (C) 20736 (D) $\frac{441}{4}$ (E) $\frac{81}{16}$

25. If $5 + 3i$ is one of the zeros the polynomial $x^3 - 6x^2 - 6x + 136$, then another of its zeros is:

- (A) -4 (B) -2 (C) -1 (D) 2 (E) 4

26. If $\begin{pmatrix} 2 & 3 & a \\ 0 & -1 & -8 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -11 \\ 6a-1 \end{pmatrix}$, find the value of a .

- (A) -2 (B) 1 (C) 0 (D) -1 (E) 3

27. The ratio of width to the length of a rectangle is 5:7. The perimeter is 256.8. What is the area of the rectangle?

- (A) 16028.6 (B) 114.49 (C) 4121.64 (D) 4007.15 (E) 801.43

28. $11_2 + 111_3 + 1111_4 = \text{_____}_5$

- (A) 14413 (B) 1233 (C) 3321 (D) 410 (E) 401

29. Find the equation of the directrix of the parabola with the equation $2x^2 - 4x + y + 4 = 0$

- (A) $y = \frac{8}{15}$ (B) $x = -\frac{15}{8}$ (C) $x = -\frac{8}{15}$ (D) $x = \frac{8}{15}$ (E) $y = -\frac{15}{8}$

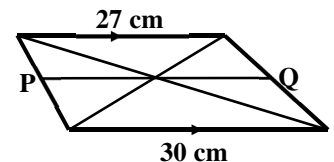
30. Find $\lim_{x \rightarrow -\infty} \frac{8+5x^2-7x^3}{2x^3-11}$

- (A) 4 (B) $-\frac{7}{2}$ (C) -4 (D) $\frac{7}{2}$ (E) does not exist

31. Let $f(x) = x^2 - x - 6$ and $g(x) = 3x - 1$. Find $(g \circ f)(-1)$.
- (A) 14 (B) 5 (C) -6 (D) -13 (E) 0
32. Use the Fibonacci-type sequence $a, b, -1, c, -5$ to find the value of $a + b + c$.
- (A) -4 (B) -1 (C) -5 (D) -7 (E) 6
33. If $a - b = 7$ and $ab = -8$, then $a^3 - b^3 = ?$
- (A) 231 (B) 225 (C) 273 (D) 175 (E) 119
34. A group agrees to share equally the cost of a \$48,000 piece of machinery. If they can find two more group members, each member's share will decrease by \$4000. How many are presently in the group?
- (A) 4 (B) 5 (C) 3 (D) 6 (E) 2
35. Quadrilateral ABCD has vertices $(-9, 3)$, $(-5, 6)$, $(2, 1)$ and $(8, -2)$ respectively. What is the area of ABCD?
- (A) 27 (B) 43 (C) 16 (D) 32 (E) 31
36. If $a_0 = -1$, $a_1 = 3$, $a_2 = 5$ and $a_n = (a_{n-3})(a_{n-1}) + a_{n-2}$ for $n \geq 3$, then $a_6 = ?$
- (A) -15 (B) -7 (C) -20 (D) 13 (E) -6
37. Find the slope of the tangent line to the function $f(x) = 5x^3 - 2x^2 + 11x - 1$ when $x = -\frac{5}{2}$.
- (A) 115 (B) $\frac{379}{4}$ (C) $\frac{459}{4}$ (D) $\frac{737}{8}$ (E) $\frac{953}{8}$
38. What is the area of a sector with a central angle of $\frac{3\pi}{8}$ in a circle with diameter 32 cm?
- (A) 36π (B) 48π (C) 96π (D) 24π (E) 12π
39. The letters in the world TUESDAY are arranged in a line. How many distinct arrangements are possible that begin with D and end with S?
- (A) 5040 (B) 240 (C) 720 (D) 120 (E) 360

40. $PQ =$ _____ cm. (nearest cm)

- (A) 28.4 (B) 28.3 (C) 28.1 (D) 28.6 (E) 28.7



41. The function $f(x) = \frac{10x^2 - 39x + 35}{4x^2 - 25}$ has a vertical asymptote at $x = V$ and a horizontal asymptote at $y = H$. Find $V + H$.

- (A) $\frac{5}{4}$ (B) $-\frac{5}{4}$ (C) 0 (D) $-\frac{5}{2}$ (E) $-\frac{5}{4}$

42. Given $f(x) = 2x + 5$ and $g(x) = x^2 - 3$ find $g(f(x))$.

- (A) $4x^2 + 22$ (B) $2x^2 - 1$ (C) $4x^2 + 20x + 22$ (D) $2x^2 + 10x + 11$ (E) $4x^2 - 1$

43. If $\int_2^k \frac{1}{x+2} dx = \ln 2$

- (A) 6 (B) 8 (C) 0 (D) 2 (E) 4

44. Find the sum of the infinite series: $-1.2 - 0.9 - 0.675 - 0.50625 \dots$

- (A) -3.3 (B) -3.7 (C) -4.8 (D) -4.5 (E) -3.6

45. Find the area of the ellipse defined by the equation $49x^2 - 294x + 16y^2 + 160y = -57$

- (A) 784π (B) 35π (C) 32π (D) 27π (E) 28π

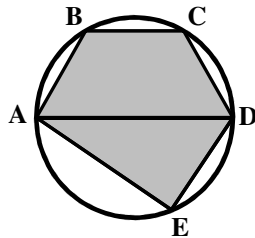
46. Find the population standard deviation of the set of numbers $\{18, 21, 22, 28, 28, 32\}$. (nearest tenth)

- (A) 4.8 (B) 5.7 (C) 5.1 (D) 5.3 (E) 4.6

47. What are the coordinates of the relative maximum of the function $f(x) = -3x + x^3$?

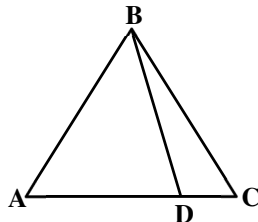
- (A) (1, -2) (B) (-1, 2) (C) (-1, 0) (D) (-2, 1) (E) (2, 2)

48. On the picture shown, \overline{AD} is a diameter, $AD = 30$ cm, $AE = 16$ cm and $\widehat{AB} \cong \widehat{BC} \cong \widehat{CD}$. The area of the shaded region is _____ cm^2 . (nearest whole number)



- (A) 318 (B) 517 (C) 483 (D) 495 (E) 503

49. The triangle ABC is equilateral and AD is triple CD. Calculate $m\angle BDC$.



- (A) 74° (B) 120° (C) 81° (D) 106° (E) 99°

50. Evan is going to take a quiz with 5 questions. The probability that Evan will get any single question on a multiple-choice quiz right is 0.25. Evan knows the answer to one of the questions. If Evan needs to get 4 or 5 questions right to pass, what is the probability that he will pass by answering the one he knows and randomly guessing on the rest? (nearest thousandth)

- (A) 0.016 (B) 0.051 (C) 0.367 (D) 0.034 (E) 0.023

51. $f(x) = x^{e^x}$. Find $f'(x)$.

- (A) $e^x x^{e^x - 1}$ (B) $e^x x^{e^x} \left(\frac{1}{x} + \ln x \right)$ (C) $e^x \left(\frac{1}{x} + \ln x \right)$
 (D) $e^x x^{e^x + 1}$ (E) $e^x x^{e^x} (x + \ln x)$

52. If $f(x) = \frac{x^2 + x}{x - 3}$, then $f''(-3) = ?$

- (A) $\frac{2}{3}$ (B) -1 (C) $\frac{1}{9}$ (D) $-\frac{1}{3}$ (E) $-\frac{1}{9}$

53. A ship initially at a position A travels 6 km on a bearing of 55° followed by 16 km on a bearing of 150° to reach a final position B. Find the distance from A to B. (nearest meter).

- (A) 8346 m (B) 20789 m (C) 18690 m (D) 16591 m (E) 13518 m

54. Which of the following series converges?

- (A) $\sum_{n=1}^{\infty} \left(\frac{3}{2} \right)^n$ (B) $\sum_{n=0}^{\theta} \frac{n+1}{2n+1}$ (C) $\sum_{n=1}^{\infty} \frac{n}{100n+1000}$ (D) $\sum_{n=0}^{\infty} \frac{3}{2^n}$ (E) $\sum_{n=1}^{\infty} \log n$

55. What is the 10^{-8} digit in the expansion of $1 + (x - 2) + \frac{(x - 2)^2}{2!} + \frac{(x - 2)^3}{3!} + \frac{(x - 2)^4}{4!} + \dots$ when $x = 2.5$.

- (A) 2 (B) 1 (C) 7 (D) 6 (E) 5

56. Let $f(x) = ax^5 + bx^3 + cx - 8$ and $f(-6) = 27$. Calculate $f(6)$.

- (A) -35 (B) 19 (C) -27 (D) -43 (E) 51

57. How many solutions are there to $7x + 9y = 2017$ such that $x, y \in \mathbb{Z}^+$.

- (A) 32 (B) 37 (C) 35 (D) 30 (E) 33

58. Two numbers are in a ratio of 7 to 9. If the lesser number is reduced by 36 and the larger number is reduced by 27, the resulting ratio is 1 to 2. What is the product of the numbers?

- (A) 1458 (B) 5103 (C) 252 (D) 900 (E) 4032

59. A light bulb is placed on a pole 32 ft above a straight, horizontal path. A man is walking away from the pole at a rate of 4 ft/s. If the man is 6 feet tall, the tip of his shadow moves along the ground at a rate of _____ ft/s. (nearest hundredth foot per second)

- (A) 4.92 (B) 5.08 (C) 0.92 (D) 1.08 (E) 4.98

60. How many numbers in the form a^3 , where $a \in \mathbb{Z}^+$ are factors of $(3!)(7!)(9!)$.

- (A) 7 (B) 11 (C) 9 (D) 8 (E) 10

Test Seven Answer Key

1. C	21. C	41. C
2. A	22. C	42. C
3. E	23. A	43. A
4. B	24. B	44. C
5. B	25. A	45. E
6. A	26. A	46. A
7. D	27. D	47. B
8. E	28. E	48. D
9. C	29. B	49. D
10. D	30. B	50. B
11. A	31. D	51. B
12. D	32. C	52. E
13. D	33. D	53. D
14. E	34. A	54. D
15. C	35. E	55. C
16. E	36. D	56. D
17. B	37. C	57. A
18. A	38. B	58. B
19. A	39. D	59. A
20. E	40. A	60. E

Test Seven Select Solutions

3. Treat each trip as if it has a distance of 1. Then the average daily speed over all trips will be

$$\frac{5}{\frac{1}{66} + \frac{1}{68} + \frac{1}{57} + \frac{1}{65} + \frac{1}{65}} \approx 64.0 \text{ mph.}$$

11. Use the sum of cubes then subtract the first three:

$$\sum_{k=1}^{18} k^3 = \left(\frac{18(19)}{2}\right)^2 = 29241,$$

$$4^3 + 5^3 + \dots + 18^3 = 29241 - 1 - 8 - 27 = 29205$$

13. Let x be the length of one edge and V be the volume of the cube, then: $\frac{dx}{dt} = 4.75 \text{ cm/s}$, $V = x^3$, $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$ and at the given edge-length, $\frac{dV}{dt} = 3(10)^2(4.75) = 1425 \text{ cm}^3/\text{s}$.

14. If the top of Pascal's triangle is the 0th row, then the n th row of Pascal's triangle has a sum of 2^n . The sum of the non-one numbers in the 12th row is $2^{12} - 2 = 4094$.

17. The new wall has an area that is 8 times the original, so the time for them to plaster the new wall together is

$$\frac{8}{\frac{1}{22} + \frac{1}{32}} \approx 104 \text{ min.}$$

$$24. A^4 - 4A^3B + 6A^2B^2 - 4AB^3 + B^4 = (A - B)^4 = (7.5 - (-3))^4 = \frac{194481}{16}.$$

26. Use the definition of matrix multiplication:
 $2(1) + 3(-3) + a(2) = -11$ for $a = -2$.

32. $a + b = -1$ and $-1 + c = -5$ for $c = -4$ and $a + b + c = -1 - 4 = -5$.

$$33. (a - b)^2 = a^2 - 2ab + b^2 \text{ for } a^2 + b^2 = (a - b)^2 + 2ab = 49 - 16 = 33$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) = 7(33 - 8) = 175.$$

38. For central angle measures using radians:

$$A_{\text{sector}} = \frac{r^2\theta}{2} = \frac{16^2}{2} \times \frac{3\pi}{8} = 48\pi.$$

39. If the beginning and end are set, then there are ${}_5P_5 = 120$ distinct arrangements.

40. The segment in a trapezoid that is parallel to both bases and passes through the intersection of the diagonals has a length equal to the harmonic mean of the two bases:

$$\frac{2(27)(30)}{27 + 30} \approx 28.4.$$

43. $\ln(k + 2) - \ln 4 = \ln 2$, so $\frac{k + 2}{4} = 2$, $8 = k + 2$ and $k = 6$.

44. The series is geometric with $a = -1.2$ and $r = 0.75$, so the sum is $\frac{-1.2}{1 - 0.75} = -4.8$.

45. Complete each square for $49(x^2 - 6x + 9) + 16(y^2 + 10y + 25) = -57 + 441 + 400$, then change to standard form for an ellipse:

$$\frac{(x - 3)^2}{16} + \frac{(y + 5)^2}{49} = 1 \text{ and an area of } (4)(7)\pi = 28\pi.$$

48. Because of the three congruent arcs, the trapezoid is a composite of three congruent equilateral triangles with sides of 15 cm. The triangle ADE is a right triangle because it is inscribed in a semicircle and $AD = \sqrt{644}$ cm. The area of the shaded region is:

$$3 \times \frac{15^2\sqrt{3}}{4} + \frac{16\sqrt{644}}{2} \approx 495 \text{ cm}^2.$$

50. Evan can miss no more than one of the remaining four questions. Use binomial theorem

$$p(\text{all right}) + p(\text{exactly 1 miss}) = (0.25)^4 + 4(0.25)^3(0.75) \approx 0.051$$

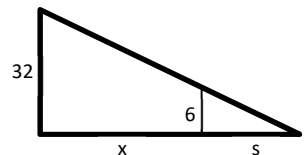
51. Use logarithmic differentiation: $\ln y = e^x \ln x$, so

$$\frac{1}{y} \frac{dy}{dx} = e^x \left(\frac{1}{x}\right) + \ln x (e^x) \text{ and } \frac{dy}{dx} = ye^x \left(\frac{1}{x} + \ln x\right) = e^x x^{e^x} \left(\frac{1}{x} + \ln x\right).$$

55. This is a series expansion of $e^{(2.5-2)} \approx 1.648721271$ for a 10⁻⁸ digit of 7.

$$59. \frac{32}{6} = \frac{x + s}{s} \text{ for } 13s = 3x,$$

$$13 \frac{ds}{dt} = 3 \frac{dx}{dt} \text{ and } \frac{ds}{dt} = \frac{3}{13} \times 4 = \frac{12}{13}.$$



The shadow is moving and expanding, so the tip of the shadow is moving at a rate of $4 + \frac{12}{13} \approx 4.92 \text{ ft/sec}$.