



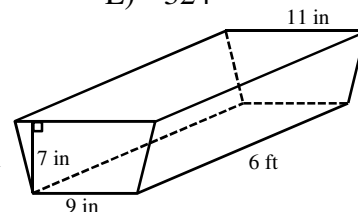
**TMSCA HIGH SCHOOL
MATHEMATICS
TEST #7 ©
JANUARY 17, 2015**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

- $6^2(0.\overline{15}) + 6^3(0.\overline{21}) + 6 \div 8.25 =$
 A) $\frac{14159}{275}$ B) 101 C) $\frac{2217}{22}$ D) $\frac{5013}{50}$ E) 52
- The GDP of the United States in 1980 was approximately 2.8625×10^{12} dollars. In 2010 it was approximately 1.49583×10^{13} dollars. What was the percent increase in the GDP during the 30 year period between 1980 and 2010? (nearest tenth of a percent)
 A) 4.2% B) 80.9% C) 62.3% D) 422.6% E) 622.6%
- In a survey of 150 juniors, researchers found that 75 students were taking a math course, 49 were taking a science course and 12 were taking both math and science. How many students were not taking either?
 A) 26 B) 50 C) 38 D) 62 E) 56
- The line a passes through the points $(-12, 5)$ and $(4, -2)$. The line b is perpendicular to line a and passes through the point $(2, -7)$. Which of the following is an equation of line b ?
 A) $16x + 7y = 17$ B) $7x + 16y = -98$ C) $16x - 7y = 81$ D) $7x - 16y = 126$ E) $16x - 7y = 17$
- The center of a circle circumscribed around a triangle has a center at the intersection of the _____ of the triangle.
 A) Medians B) Angle Bisectors C) Radii D) Altitudes E) Perpendicular Bisectors
- Joe has been pricing Speed-Pass train fares for a group trip to New York. Three adults and four children must pay \$132. Two adults and three children must pay \$94. Find the price of an adult's ticket.
 A) \$18.86 B) \$20 C) \$18.80 D) \$18 E) \$21.50
- Given that A and B are independent events. If the probability of A is triple the probability of B and $p(A \cap B) = 0.3072$ calculate $p(A)$.
 A) $\frac{8}{25}$ B) $\frac{1}{3}$ C) $\frac{32}{625}$ D) $\frac{24}{25}$ E) $\frac{96}{625}$
- On a map of Texas, one inch represents 75 miles. On this same map, El Paso is approximately $10\frac{3}{4}$ inches from Texarkana. Approximate the actual distance from El Paso to Texarkana.
 A) 563 miles B) 806 miles C) 1000 miles D) 751 miles E) 657 miles
- Determine the number of non-negative integer solutions to $p + q + r = 25$.
 A) 351 B) 325 C) 312 D) 338 E) 324

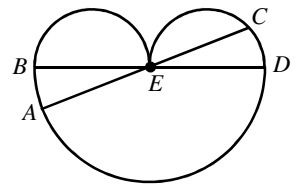
- Calculate the capacity of the length of guttering shown in the diagram. (nearest tenth of a gallon)



- A) 19.6 gal B) 21.8 gal C) 14.1 gal D) 15.7 gal E) 16.9 gal

- Ninety-two miles per hour equals _____ feet per second. (nearest whole number)
 A) 8096 B) 135 C) 45 D) 485760 E) 4116
- The measure of one interior angle of a regular dodecagon is _____°.
 A) 120 B) 135 C) 144 D) 148 E) 150

13. The figure shown is composed of three semicircles where E is the midpoint of the diameter \overline{BD} . If $m\angle BEA = 30^\circ$ and $AE = 10$ cm then $EC =$ _____ cm.



- A) $\frac{15}{2}$ B) 10 C) $5\sqrt{3}$ D) $\frac{15\sqrt{3}}{2}$ E) $\frac{5\sqrt{3}}{2}$

14. $\frac{x^3 - 7x^2 - 4x + 28}{x^3 + 2x^2 - 49x - 98} \div \frac{x + 2}{x^2 - 49} =$

- A) $\frac{x^2 - 5x - 14}{x + 7}$ B) $x^2 - 5x - 14$ C) $\frac{x^2 + 9x + 14}{x - 7}$ D) $\frac{x^2 + 5x - 14}{x - 2}$ E) $\frac{x^2 - 9x + 14}{x + 2}$

15. Phil, Frank and Forest can each peel a pound of potatoes in 8 minutes, 6 minutes and 4 minutes respectively. How much longer will it take Phil to peel 20 pounds of potatoes alone than it would take Frank and Forest working together?

- A) 112 minutes B) 40 minutes C) 12 minutes D) 48 minutes E) 160 minutes

16. $\frac{d}{dx}(x \sqrt[3]{2x}) =$

- A) $\frac{4 \sqrt[3]{2x}}{3}$ B) $\frac{2 \sqrt[3]{2x}}{3}$ C) $\frac{5 \sqrt[3]{2x}}{3}$ D) $\frac{4 \sqrt[3]{4x}}{3}$ E) $\frac{2 \sqrt[3]{4x}}{3}$

17. How many distinct arrangements are possible of the letters in the word "SASSAFRASS"?

- A) 3628800 B) 5040 C) 25200 D) 30240 E) 820

18. Larry consistently makes 60% of the free throws he attempts. If he tries to shoot six times, what is the probability that he makes at least four?

- A) $\frac{3969}{15625}$ B) $\frac{972}{3125}$ C) $\frac{1424}{3125}$ D) $\frac{14086}{15625}$ E) $\frac{1701}{3125}$

19. Which of the following is not a triangular number?

- A) 1225 B) 3916 C) 1711 D) 5040 E) 820

20. If A and B are the roots of $f(x) = 6x^2 - 13x - 63$, calculate $A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$.

- A) $\frac{2825761}{1296}$ B) $\frac{28561}{20736}$ C) $\frac{2825761}{20736}$ D) $\frac{1427161}{1296}$ E) $\frac{28561}{1296}$

21. At the beginning of a day trip, Daryn had no money, but his three brothers did. Adam gave Daryn one-sixth of his money. Brad gave Daryn one-third of his money and Carl gave Daryn one-fifth of his money. Each gave Daryn the same amount of money. What fraction of the brothers' money did Daryn have after the exchange?

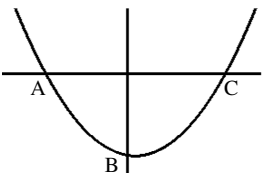
- A) $\frac{3}{14}$ B) $\frac{3}{8}$ C) $\frac{7}{10}$ D) $\frac{3}{10}$ E) $\frac{3}{11}$

22. Given $\ln 7 = P$ and $\ln 4 = Q$ which of the following is equal to $\ln(6.125)$?

- A) $2P \div \frac{3}{2}Q$ B) $P^2 - 2Q$ C) $P^2 \div 2Q$ D) $P^2 - \frac{3}{2}Q$ E) $2P - \frac{3}{2}Q$

23. Solve $1 - \sin^2 \theta = 1 - \cos^2 \theta$ where $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.

- A) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ B) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ C) π D) $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$ E) $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

24. Let $f(x) = x^4 + x^2 + x + 3$. According to Descartes' Rule of Signs, how many possible negative real roots are there?
 A) 0 or 1 B) 0 or 2 C) 1 or 3 D) 0 E) 1 only
25. $\sum_{k=2}^{\infty} \frac{2}{3} \left(-\frac{3}{4}\right)^k =$
 A) $\frac{8}{21}$ B) $-\frac{2}{7}$ C) $\frac{8}{3}$ D) $\frac{3}{14}$ E) $\frac{3}{2}$
26. When the polynomial $f(x) = 3x^5 + Ax^4 + 7x^2 + Bx - 13$ is divided by $x - 1$ the remainder is -13 . When $f(x)$ is divided by $x + 1$ the remainder is -23 . Calculate $A + B$
 A) 18 B) -12 C) -10 D) -14 E) 40
27. A prom committee charged \$22 for a single ticket and \$36 for a couple ticket. If the ticket sales totaled \$12972 and they expect 690 people to attend based on ticket sales, how many singles bought tickets?
 A) 157 B) 138 C) 532 D) 276 E) 266
28. In triangle ABC , $AB = 12$ cm, $BC = 22$ cm and $\angle B$ is obtuse. The area of the triangle is 66 cm². Find the measure of $\angle B$.
 A) 120° B) 135° C) 150° D) 105° E) 100°
29. A class contains 12 girls and 14 boys. The teacher selects four students at random. What are the odds that all of the students selected are girls?
 A) $\frac{77}{1150}$ B) $\frac{99}{2891}$ C) $\frac{77}{1073}$ D) $\frac{99}{2990}$ E) $\frac{45}{91}$
30. The graph of a parabola $f(x) = 3(x - p)(x - q)$ is shown. The coordinates of the points shown are $A(-5, 0)$, $B(0, b)$ and $C(6, 0)$. Find the value of b .
 A) -90 B) -30 C) -45 D) -150 E) -120
- 
31. $12_4 + 23_5 + 34_6 + 45_7 + 56_8 + 67_9 = \text{_____}_{10}$
 A) 181 B) 175 C) 237 D) 154 E) 214
32. Find the slope of the line tangent to the curve $2x^2 - 5y^3 = 13$ at $(-3, 1)$.
 A) $-\frac{19}{15}$ B) $-\frac{2}{15}$ C) $\frac{19}{5}$ D) $\frac{4}{5}$ E) $-\frac{2}{5}$
33. How many solutions are there to $7x + 8y = 221$ where $x, y \in \mathbb{Z}^+$.
 A) 4 B) 24 C) 7 D) 18 E) 8
34. Given the sequence 4, 7, 14, 25, 40, 59, a, b, c , calculate the value of $a + b + c$.
 A) 424 B) 331 C) 140 D) 175 E) 249
35. A right circular cone has a radius of 13 in. and is 18 in. tall. What is the greatest number of fluid ounces or water that could be poured into the cone before flowing over? (nearest fl. oz.)
 A) 5296 B) 1270 C) 3810 D) 1765 E) 2444

36. Given that $\sin \theta = \frac{5}{13}$ and $\frac{\pi}{2} \leq \theta \leq \pi$, find $\sin 2\theta$.

- A) $\frac{120}{169}$ B) $-\frac{70}{169}$ C) $\frac{119}{169}$ D) $-\frac{71}{169}$ E) $-\frac{120}{169}$

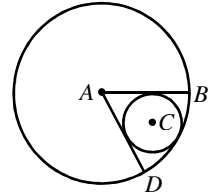
37. The table below shows the performance of an investment each year for 5 years. What is the average percentage growth for the investment over the 5 years. (nearest tenth of a percent)

Year	1	2	3	4	5
Percent Growth	+10%	-6%	+9%	-4%	+8%

- A) 3.4% B) 7.4% C) 7.0% D) 18 E) 3.2%

38. Circle C is tangent to circle A and to both radii of circle A shown. If $m\angle DAB = 60^\circ$ calculate the ratio of the area of circle C to the area of circle A .

- A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{1}{9}$ D) $\frac{1}{4}$ E) $\frac{1}{6}$



39. Find the number of positive integral divisors of 960.

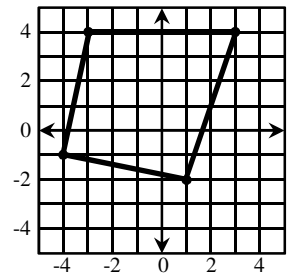
- A) 28 B) 24 C) 27 D) 26 E) 30

40. Two elements of the set $\{1, 2, 3, 5, 8, 13\}$ are selected at random. What is the probability that the sum of the two elements is also an element of the set?

- A) $\frac{2}{9}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{4}{15}$ E) $\frac{2}{5}$

41. The coordinates of the quadrilateral shown are integers. Calculate the area of the quadrilateral.

- A) 29 B) 28 C) 31 D) 32 E) 30



42. Solve the equation $\frac{dy}{dx} = \frac{dx}{dy}$ when $y = \cos^2(2x)$ and $0 \leq x \leq \frac{\pi}{2}$.

- A) $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \frac{7\pi}{16}$ B) $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{24}, \frac{11\pi}{24}$ C) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ D) $\frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ E) $\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$

43. Classify the graph of with the equation: $4x^2 + 11xy - 9y^2 + 2x - 3y + 5 = 0$

- A) Circle B) Ellipse C) Centroid D) Hyperbola E) Parabola

44. Given that x varies directly with $y - 3$, and $x = 9$ when $y = 6$. Find the value of y when $x = 37$.

- A) $\frac{138}{37}$ B) $\frac{46}{3}$ C) $\frac{69}{37}$ D) $\frac{40}{3}$ E) $\frac{64}{37}$

45. Let $f(x) = 2x - 3$ and $g(x) = \frac{x+3}{2}$. Find $f\left(g\left(\frac{a-4}{5}\right)\right)$.

- A) $\frac{a-4}{5}$ B) $\frac{a-6}{10}$ C) $\frac{3a-12}{5}$ D) $\frac{2a-8}{15}$ E) $\frac{2a-8}{5}$

46. $\frac{A}{x-7} + \frac{B}{x+2} = \frac{-4x-53}{x^2-5x-14}$. $A+B =$

- A) -57 B) -4 C) -49 D) 14 E) -5

47. If $y = \frac{9}{x}$ and $x + y = 12$, then $x^3 + y^3 =$
 A) 1701 B) 1512 C) 1404 D) 1620 E) 1728
48. Let a, b, c and d be integers such that $a < b$, $b < c$ and $c = d$. The mode of the four numbers is 22. The range of the four numbers is 16, and the mean of the four numbers is 16. What is the value of b ?
 A) 6 B) 15 C) 14 D) 10 E) 16
49. Simplify: $\left(\frac{a^2}{b^{-2}}\right) \div a^3 \times b^5$
 A) $\frac{b^7}{a}$ B) $\frac{1}{ab^3}$ C) a^5b^3 D) $\frac{b^3}{a}$ E) a^2b
50. Find the area of triangle ABC if $m\angle A = 50^\circ$, $m\angle C = 80^\circ$ and $AC = 12$ cm. (nearest square centimeter)
 A) 71 cm^2 B) 72 cm^2 C) 143 cm^2 D) 13 cm^2 E) 142 cm^2
51. If $s(x)$ is the slant asymptote of $f(x) = \frac{x^2 + 5x + 8}{x + 3}$, then $s(5) =$
 A) 5 B) 3 C) 2 D) 7 E) 8
52. Given $f''(x) = 2$, $f'(2) = 5$ and $f(2) = 10$ calculate $f(7)$.
 A) 56 B) 60 C) 15 D) 49 E) 4
53. Evaluate $\int_{-8}^5 [3f(x) - 3] dx$ if $\int_{-8}^5 f(x) dx = 95$.
 A) 282 B) 276 C) 246 D) 285 E) 324
54. How many perfect cubes are factors of $(3!)(4!)(5!)$?
 A) 4 B) 7 C) 5 D) 4 E) 6
55. If $\frac{x-7}{x+4} + \frac{x+4}{x-7}$ is equal to the mixed number $A + \frac{B}{(x+4)(x-7)}$, then $B =$
 A) 64 B) 121 C) 8 D) 11 E) 9
56. The areas of the bases of a frustum A_1 and A_2 , and the height is 18 inches. The Heronian mean of A_1 and A_2 is $311\pi \text{ in}^2$. Find the volume of the frustum.
 A) $1866\pi \text{ in}^3$ B) $622\pi \text{ in}^3$ C) $5598\pi \text{ in}^3$ D) $16794\pi \text{ in}^3$ E) $11196\pi \text{ in}^3$
57. The ratio of angle measures in a triangle is 2:3:4. If the longest side of the triangle is 18 in., what is the area of the triangle? (nearest tenth)
 A) 91.6 in^2 B) 183.1 in^2 C) 109.1 in^2 D) 218.3 in^2 E) 142.5 in^2
58. Given $z^2 = 21 - 20i$ and $z^3 = 65 - 142i$ find z .
 A) $13 - 5i$ B) $5 - 2i$ C) $44 - 122i$ D) $2 - 5i$ E) $10 - 2i$
59. The discriminant of $f(x) = (3x - 5)(2x + 7)$ is:
 A) 821 B) 961 C) -719 D) 1801 E) 121
60. $(\ln x)^2 + (\ln x^2)^2 + (\ln x^3)^2 + \dots + (\ln x^{15})^2 =$
 A) $1240(\ln x)^2$ B) $\ln x^{240}$ C) $14400 \ln(x^2)$ D) $1240 \ln x^2$ E) $(\ln x)^{240}$

2014-2015 TMSCA Mathematics Test Seven Answers

1. E	21. A	41. C
2. D	22. E	42. B
3. C	23. D	43. D
4. C	24. B	44. B
5. E	25. D	45. A
6. B	26. C	46. B
7. D	27. B	47. C
8. B	28. C	48. C
9. A	29. B	49. A
10. B	30. A	50. A
11. B	31. A	51. D
12. E	32. D	52. B
13. C	33. E	53. C
14. E	34. B	54. E
15. A	35. D	55. B
16. A	36. E	56. C
17. B	37. E	57. A
18. E	38. C	58. A
19. D	39. A	59. B
20. E	40. D	60. A

2013-2014 TMSCA Mathematics Test Seven Select Solutions

7. $p(A) = 3p(B)$ and for independent events, $p(A \cap B) = p(A) \cdot p(B)$, so $0.3072 = p(B) \cdot 3p(B)$, so $p(B) = 0.32$ and $p(A) = 0.96 = \frac{24}{25}$

9. If $p + q + r = n$, then the number of non-negative integral solutions is the $(n+1)^{\text{th}}$ triangular number. For this problem: $\frac{26(27)}{2} = 351$

16. Rewrite as

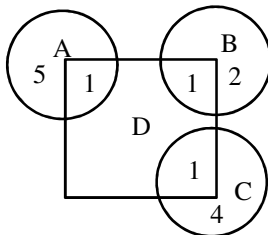
$$\frac{d}{dx} (x^{4/3} \cdot \sqrt[3]{2}) = \frac{4}{3} x^{1/3} \cdot \sqrt[3]{2} = \frac{4\sqrt[3]{2}x}{3}$$

17. There are 10 total letters, A repeats 3 times and S repeats 5 times, so the number of possible arrangements is $\frac{10!}{(3!)(5!)} = 5040$

20. The sum of the roots for $f(x)$ equals $-\frac{b}{a} = \frac{13}{6}$ and the expression shown is the binomial expansion of $(A+B)^4 = \left(\frac{13}{6}\right)^4$ or $\frac{28561}{1296}$

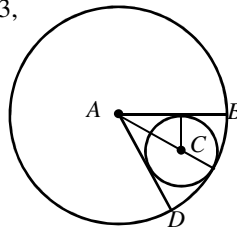
21. Let each brother give Daryn a dollar and make the following Venn diagram. So

Daryn has $\frac{3}{14}$ of the money.



37. $\sqrt[3]{(1.10)(0.94)(1.09)(0.96)(1.08)} \approx 1.0316$ or 3.2% average gain.

38. Draw in a radius of circle C forming a special right triangle. Let the radius of C have a length of 1. Then $AC = 2$ and the radius of circle A is 3, so the ratio of the radii is $\frac{1}{3}$ and the ratio of areas is $\frac{1}{9}$.



39. The prime factorization of 960 is $2^6 \cdot 3^1 \cdot 5^1$. Adding one to each exponent and multiplying: $7 \cdot 2 \cdot 2 = 24$.

42. $y' = 2 \cos(2x) \cdot (-\sin(2x))(2)$ or $-2 \sin(4x)$. This is equal to 1 or -1 when $\sin(4x) = \pm \frac{1}{2}$, so $4x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ and $x = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{24}, \frac{11\pi}{24}$.

45. The two functions are inverse functions, so the composite function is the same as the input or $\frac{a-4}{5}$.

46. Multiply by the common denominator to get $A(x+2) + B(x-7) = -4x - 53$. Let $x = -2$ to get $B = 5$, then let $x = 7$ to get $A = -9$, so $A+B = -4$.

47. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ or $(x+y)[(x+y)^2 - 3xy]$ for which we can use $12(12^2 - 3 \cdot 9) = 1404$.

53. $\int_{-8}^5 [3f(x) - 3] dx = 3 \int_{-8}^5 f(x) dx - \int_{-8}^5 3 dx$ or $3(95) - 3(5+8) = 246$.

56. The volume of a frustum is the of the product of the height and the Heronian mean of the areas of the bases, or $V = (18)(311\pi)$.

60. This can be rewritten as $(\ln x)^2 + 4(\ln x)^2 + 9(\ln x)^2 + \dots + 225(\ln x)^2$ So the sum of the squares from 1² to 15² will be the coefficient hence: $\frac{15(16)(31)}{6} (\ln x)^2 = 1240(\ln x)^2$