TMSCA HIGH SCHOOL

MATHEMATICS

TEST #7 ©

JANUARY 18, 2014

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate $(0.0666...)^{-1} + (0.0090909...)^{-1} - (0.222...)^{-2}$.

A) $\dfrac{631}{891}$ 　　　B) $-\dfrac{29}{198}$ 　　　C) $\dfrac{419}{4}$ 　　　D) $-\dfrac{2663}{132}$ 　　　E) $\dfrac{241}{2}$

2. Susan plans on buying 4 shirts for \$27.95 each, 2 skirts for \$32.99 each and a pair of school shoes for \$54.95. The local tax rate is 8.5%. How much money will she save if she makes her purchases during tax-free weekend?

A) \$19.78 　　　B) \$16.98 　　　C) \$12.65 　　　D) \$19.20 　　　E) \$16.48

3. Events A and B are independent such that $P(A) = 5P(B)$ and $P(A \cup B) = 0.7875$. Find $P(B)$.

A) 0.75 　　　B) 0.20 　　　C) 0.125 　　　D) 0.375 　　　E) 0.15

4. On the diagram, FC is the geometric mean of AB and ED, AF = 4.5, FE = 6.3 and EC = 15.4. Find FB. (nearest tenth)
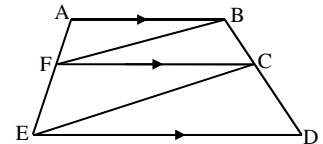
A) 9.2 　　　B) 11.9 　　　C) 11.0 　　　D) 10.7 　　　E) 10.5

5. $f(x) = ax^3 - bx^2$ where $a, b \in \mathbb{Z}^+$. The graph of $f(x)$ has a point of inflection when $x = $_____.

A) $-\dfrac{b}{3a}$ 　　　B) $-\dfrac{2b}{3a}$ 　　　C) $\dfrac{b}{3a}$ 　　　D) $-\dfrac{b}{a}$ 　　　E) $\dfrac{2b}{3a}$

6. Which of the following is an equation of the perpendicular bisector of the segment with endpoints $(12, -5)$ and $(7, 2)$?

A) $5x - 7y = 58$ 　　B) $7x - 5y = -21$ 　　C) $7x + 5y = 64$ 　　D) $5x - 7y = -95$ 　　E) $5x + 7y = -95$

7. The time it takes to empty a cistern is inversely proportional to the number of pumps used. It takes 90 minutes with 4 pumps, how long will it take with 9 pumps?

A) 20 min 　　　B) 25 min 　　　C) 30 min 　　　D) 40 min 　　　E) 45 min

8. Given $x - y = -5$ and $xy = 37$ find the value of $x^3 - y^3$.

A) -310 　　　B) 60 　　　C) 245 　　　D) -680 　　　E) -865

9. The base radius of a cone is 7 cm. and the vertex angle is 48°. Find the total surface area of the cone to the nearest square centimeter.

A) $361\ \text{cm}^2$ 　　B) $532\ \text{cm}^2$ 　　C) $207\ \text{cm}^2$ 　　D) $378\ \text{cm}^2$ 　　E) $422\ \text{cm}^2$

10. Find the area of the circle defined by the equation $x^2 - 4x + y^2 + 12y = 41$.

A) 18π 　　　B) 4π 　　　C) 36π 　　　D) 49π 　　　E) 81π

11. $f(x) = 2x + 5$ and $g(x) = x^2 + 8$. Find $g(f(-x))$.

A) $2x^2 + 21$ 　　B) $4x^2 - 20x + 33$ 　　C) $-2x^2 - 13$ 　　D) $2x^2 + 20x + 17$ 　　E) $4x^2 + 33$

12. The function $f(x)$ shown below is periodic and has a domain $0 \le x \le 40$. Evaluate $f'(37)$.
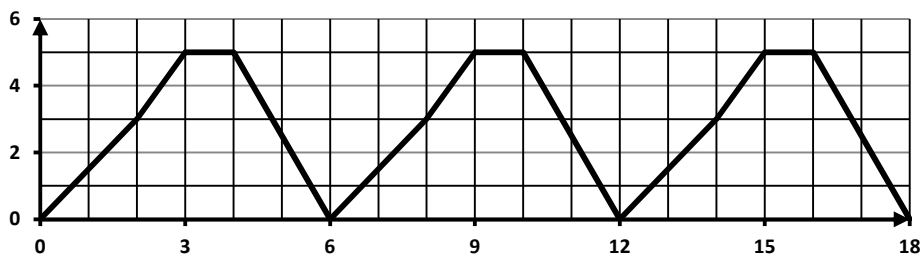
A) 0 　　　B) 1.5 　　　C) 0.5 　　　D) -2 　　　E) undefined

13. $\angle A$ and $\angle B$ are complementary and $m\angle B$ is 22° less than three times $m\angle A$. Find the measure of the supplement of $\angle A$.

A)   50.5°   B)   56°   C)   101°   D)   152°   E)   146°

14. If $\dfrac{3x+1}{4x-3}+\dfrac{2x-1}{3x+5}=\dfrac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\dfrac{A+B+C}{P+Q+R}=$

A) $\dfrac{29}{7}$ B) $\dfrac{13}{31}$ C) $\dfrac{33}{8}$ D) $\dfrac{49}{31}$ E) $\dfrac{29}{31}$

15. Let $f(x)=\dfrac{7}{x-6}$. On which of these intervals is f not continuous?

A) $(6,\infty)$ B) $[-6,0]$ C) $[0,\infty)$ D) $[0,6)$ E)   all of these

16. $A=\begin{bmatrix}1 & -1\\ 4 & 7\end{bmatrix}$ and $B=\begin{bmatrix}3 & -5\\ 1 & 0\end{bmatrix}$. Find the product of the elements in $2A-B$.

A)   -294   B)   -168   C)   -630   D)   -63   E)   13

17. The dots are 6 units apart vertically and horizontally. Find the area of the shaded region.

A) 378 units^2 B) 324 units^2 C) 306 units^2 D) 360 units^2 E) 352 units^2
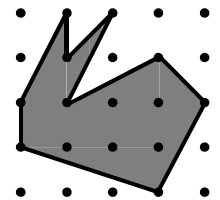
18. If $\int_6^{12} f(x)\,dx=19.5$, find $\int_6^{12}\left[3f(x)+5\right]dx$.

A)   63.5   B)   118.5   C)   42.5   D)   88.5   E)   67.5

19. Blackbeard sailed from his hideout at bearing 275° for 40 miles to island A, then on to island B at a bearing of 122° for 72 miles. How far will Blackbeard sail to go directly back to his hideout? (nearest mile)

A)   58 mi   B)   44 mi   C)   43 mi   D)   41 mi   E)   52 mi

20. The repeating decimal 0.545454… in base 7 can be written as which of the following fractions in base 7?

A) $\dfrac{6}{11_7}$ B) $\dfrac{16}{22_7}$ C) $\dfrac{9}{22_7}$ D) $\dfrac{9}{11_7}$ E) $\dfrac{36}{101_7}$

21. How many distinct arrangements can be made from the letters "DERIVATIVE"?

A)   40320   B)   453600   C)   30240   D)   3628800   E)   604800

22. Solve $2\sin x=\tan x$, for $0<x\le\dfrac{\pi}{2}$.

A) $\dfrac{\pi}{6}$ B) $\dfrac{\pi}{2}$ C) $\dfrac{\pi}{4}$ D) $\dfrac{\pi}{3}$ E)   no solution

23. The polynomial $x^2-2x-15$ is a factor of $x^3+(a-5)x^2+(1-4a)x-30$. Find the value of the constant a.
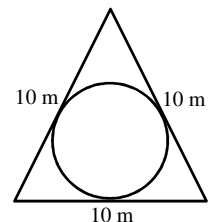
A)   -5   B)   -6   C)   2   D)   3   E)   5

24. The sixth term of a geometric sequence is 25.6 and the twelfth term is 1638.4. Find the second term.

A)   0.8   B)   3.2   C)   1.6   D)   2.4   E)   0.4

25. Find the area of the inscribed circle. (nearest tenth)

A) 26.2 m^2 B) 43.3 m^2 C) 31.4 m^2 D) 56.2 m^2 E) 78.5 m^2

10 m   10 m

10 m

26. Mr. Carl's calculus class has 14 girls and 9 boys. They want to put together a volleyball team for the school charity tournament. If the team must consist of 3 boys and 5 girls, how many different teams could be formed from the class?
A) 45864   B) 168168   C) 121080960   D) 33022080   E) 20020

27. In a survey of 75 students, 32 listened to country music, 41 listened to pop music and 18 listened to both. How many students listened to neither?
A) 2   B) 27   C) 20   D) 9   E) 18

28. If the 1 at the top of Pascal's triangle is row 0, what is the sum of the numbers in row 9?
A) 256   B) 254   C) 1024   D) 512   E) 510

29. The geometric mean of 2 numbers is 672 and the arithmetic mean is 1250. Find the Heronian mean.
A) 1586   B) $\dfrac{3172}{3}$ C) $\dfrac{3554}{3}$ D) $\dfrac{2696}{3}$ E) 1348

30. For a triangle ABC with sides AB = 12, BC = 10.44 and $m\angle A = 60°$, there are two possible areas. Find the smaller area.
A) $30\sqrt{3}$ B) $21\sqrt{3}$ C) $42\sqrt{3}$ D) $15\sqrt{3}$ E) $35\sqrt{3}$
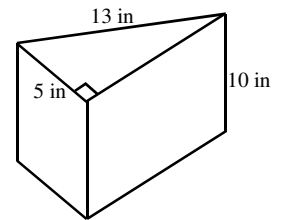
31. $f(x) = \dfrac{3x-2}{2x+4}$. $f^{-1}(3) =$
A) -2   B) $\dfrac{7}{10}$ C) $-\dfrac{10}{9}$ D) $-\dfrac{14}{3}$ E) -3

32. Given the Fibonacci –type sequence -12, A, B, 2, C, -1… , find $A+B+C$.
A) -7   B) -1   C) -3   D) 5   E) 1

33. Find the volume of the right triangular prism.

A) 320 in^3 B) 300 in^3 C) 360 in^3 D) 350 in^3 E) 325 in^3

34. Circle P has a radius of 7 cm. $\overline{AB}$ is a chord of circle P and $AB = 5.6$ cm. How far is $\overline{AB}$ from the center of circle P? (nearest tenth)
A) 2.0 cm   B) 6.4cm   C) 3.2 cm   D) 0.6 cm   E) 4.2 cm
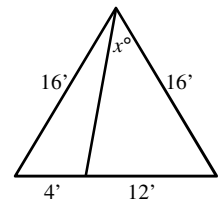
35. $f(x) = 7x^4 + 6x^3 - 9x^2 + 11x - 12$. $f''(-4) =$
A) 1182   B) -1432   C) 1208   D) 1470   E) 1326

36. $(2-7i)^3 - (3+4i) = a+bi$. $a+b =$
A) 544   B) -80   C) -12   D) -34   E) -20

37. Find the value of x in the triangle diagram. (nearest degree)

A) 44   B) 45   C) 48   D) 47   E) 46

38. Find the constant term in the binomial expansion of $\left(2x + \dfrac{4}{x^2}\right)^6$

A) 3840   B) 10240   C) 4096   D) 256   E) 960

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39. A plane flew 1750 km in 6 hours with a tail wind of a constant velocity. It then flew back 540 km in 3 hours with a head wind of the same velocity. Find the speed of the wind.

A) $254\frac{4}{9}$ kmh^{-1} B) $55\frac{5}{6}$ kmh^{-1} C) $198\frac{1}{9}$ kmh^{-1} D) $235\frac{5}{6}$ kmh^{-1} E) $63\frac{2}{9}$ kmh^{-1}

40. An airplane travelling at an altitude of 47,000 ft is 90 ground miles from its destination. If it flies in a straight line to land, what is the angle of depression? (nearest tenth)

A) 1.6°   B) 8.3°   C) 5.6°   D) 4.4°   E) 3.7°

41. Find the Real number solution set of $|6-2x|\geq 4$.

A) $(-\infty,\infty)$ B) $(-\infty,-5]\cup[-1,\infty)$ C) $[1,5]$ D) $(-\infty,1]\cup[5,\infty)$ E) $[-5,-1]$

42. Find PQ.

24 cm

P   Q

30 cm
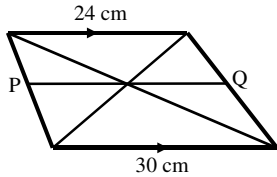
A) $\frac{80}{3}$ cm   B) 27 cm   C) $\frac{82}{3}$ cm   D) $12\sqrt{5}$ cm   E) $18+4\sqrt{5}$ cm

43. Evaluate: $\log_3 18 + \log_3 54 - \log_3 144 + \log_3 4$

A) 4   B) 3   C) 27   D) 2   E) 6

44. Find the slope of the graph of $2x^2+3y^2=11$ at the point $(-2,1)$.

A) $\frac{4}{3}$ B) $\frac{3}{4}$ C) $\frac{19}{3}$ D) $-\frac{4}{3}$ E) $-\frac{19}{3}$

45. One edge of a regular tetrahedron is 7.2 m. What is the surface area? (nearest tenth)

A) 119.2 m^2 B) 89.8 m^2 C) 75.1 m^2 D) 119.7 m^2 E) 104.8 m^2

46. Find the remainder of $\left(x^7-3x^6-4x^3+11x^2-9\right)\div(x+4)$.

A) -28749   B) 4247   C) -13223   D) 3655   E) 4007

47. Which of the following is an equation of the line through $(4,3)$ with an x-intercept of 2.6?

A) $10x-y=37$ B) $7x-15y=-7$ C) $15x-7y=39$ D) $x-10y=-26$ E) $x+10y=17$

48. Solve $2^{2x+1}-2^{x+1}+1=2^x$ for x.

A) -1, 0   B) $\frac{1}{2}$, 1   C) -1, $\frac{1}{2}$ D) 0, 1   E) 0, $\frac{1}{2}$

49. Find the equation of the directrix of the parabola with the equation $2x^2-4x+y+4=0$.

A) $y=\frac{8}{15}$ B) $x=-\frac{15}{8}$ C) $x=-\frac{8}{15}$ D) $y=-\frac{15}{8}$ E) $x=\frac{8}{15}$

50. If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$. This is the _____ ,

(A)   Sandwich Theorem   (C)   Rolle's Theorem   (E) Fundamental Theorem of Calculus

(B)   Intermediate Value Theorem   (D)   Fundamental Theorem of Algebra

51. $f(x)=ax^5+bx^3+cx+8$. If $f(-3)=72$ then $f(3)=$

A) -56   B) 64   C) -72   D) -80   E) 58

52. The polar coordinates of point P are $\left(-6, \frac{3\pi}{4}\right)$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian coordinate plane?

A) QI   B) QII   C) QIII   D) QIV   E) x - axis

53. What is the angle between the vectors $\langle 2, -5 \rangle$ and $\langle -1, -8 \rangle$. (nearest degree)

A) 151°   B) 84°   C) 29°   D) 61°   E) 96°

54. The areas of the bases of a frustum are A_1 and A_2, and the height is 17 inches. The Heronian mean of A_1 and A_2 is 248π in^2. Find the volume of the frustum.

A) $\frac{4216\pi}{3}$ in^3 B) 2108π in^3 C) $\frac{1054\pi}{3}$ in^3 D) 4216π in^3 E) $\frac{2108\pi}{3}$ in^3

55. Find $8 - \frac{8^3}{3!} + \frac{8^5}{5!}...$ correct to 4 decimal places.

A) 195.7333   B) -0.1455   C) 0.9894   D) -6.7997   E) 0.1392

56. A particular model of car has an advertised gas mileage of 32 mpg for in-town driving. Upon investigation, a consumer discovers that the gas mileage is normally distributed with a standard deviation of 3 mpg. What is the probability that the driver will get over 33 mpg for in-town driving?

A) 16%   B) 63%   C) 84%   D) 37%   E) 27%

57. The diameters of the concentric circles on the right are 10 cm, 8 cm and 4 cm. If a dart hits the figure at random, what is the probability that it will **not** land in the shaded area?

A) 0.64   B) 0.48   C) 0.36   D) 0.52   E) 0.56
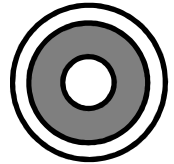
58. In triangle *ABC*, the 3 medians intersect at point *M*. If $AM = 18$ in, and point *P* is the midpoint of $\overline{BC}$ then the length of the median from point *P* is
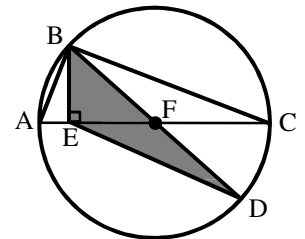
A) 12 in   B) 36 in   C) 15 in   D) 24 in   E) 27 in

59. $f(x) = x^{e^x}$. Find $f'(x)$.

A) $e^x x^{e^x - 1}$

C) $e^x\left(\frac{1}{x} + \ln x\right)$

E) $e^x x^{e^x + 1}$

B) $e^x x^{e^x}\left(\frac{1}{x} + \ln x\right)$

D) $e^x x^{e^x}(x + \ln x)$

60. On the circle with center F, $EC = 7AE$. The area of triangle ABC is 108 cm^2. Find the area of the shaded region.

A) 94.5 cm^2 B) 40.5 cm^2 C) 81 cm^2 D) 84 cm^2 E) 96 cm^2

2013-2014 TMSCA Mathematics Test Seven Answers

1. C	21. B	41. D
2. A	22. D	42. A
3. E	23. E	43. B
4. C	24. C	44. A
5. C	25. A	45. B
6. A	26. B	46. A
7. D	27. C	47. C
8. D	28. D	48. A
9. B	29. B	49. D
10. E	30. D	50. B
11. B	31. D	51. A
12. B	32. B	52. D
13. D	33. B	53. C
14. C	34. B	54. D
15. C	35. A	55. C
16. A	36. D	56. D
17. C	37. E	57. D
18. D	38. A	58. E
19. D	39. B	59. B
20. B	40. C	60. C

2013-2014 TMSCA Mathematics Test Seven Select Solutions

3. $p(A \cup B) = p(A) + p(B) - p(A) \cdot p(B)$
for independent events. So
$0.7875 = 5p(B) + p(B) - 5(p(B))^2$
Solve quadratic for $p(B) = 0.15$

4. The segment parallel to the bases of the trapezoid that has the length of the geometric mean of the bases divides the trapezoid into two similar trapezoids, so
$\frac{4.5}{6.3} = \frac{FB}{15.4}$ and $FB = 11$.

8. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$ and
$(x-y)^2 = x^2 - 2xy + y^2$, so
$x^3 - y^3 = (-5)(25 + 3 \cdot 37) = -680$

17. $A = \frac{2I + P}{2} - 1 = \frac{10 + 9}{2} - 1 = 8.5$ square
units on the diagram, but if the linear scaling is 1:6, then the area scaling is 1:36 so the area is $8.5(36) = 306$ square units.

18.
$\int_6^{12} [3f(x) + 5] dx = 3\int_6^{12} f(x) dx + \int_6^{12} 5 dx$
$= 3(19.5) + 30 = 88.5$

21. There are 10 total letters with 3 letters that each repeat twice, so the number of
arrangements = $\frac{10!}{(2!)(2!)(2!)} = 453600$.

22. $x^2 - 2x - 15 = (x-5)(x+3)$, so
$f(-3) = f(5) = 0$ and
$5^3 + 5^2(a-5) + 5(1-4a) - 30 = 0$ solve
linear equation to get $a = 5$.

29. The Heronian mean 1/3 of the way from the arithmetic mean to the geometric mean. Here the difference is 578. So the
Heronian mean is $1250 - \frac{578}{3} = \frac{3172}{3}$.

30. Use law of cosines to set up quadratic
$10.44^2 = AC^2 + 12^2 - 2(AC)12\cos 60$.
The smaller solution is about 5 so the
smaller area is $0.5(12)(5)\sin 60 = 15\sqrt{3}$.

38. $_6C_4 (2x)^4 \left(\frac{4}{x^2}\right)^6 = 3840$

42. The harmonic mean is the parallel that passes through the intersection of the diagonals.
$PQ = \frac{2(24)(30)}{(24 + 30)} = \frac{80}{3}$

53. The angle between the vectors can be found using the two definitions of dot product.
$(2)(-1) + (-5)(-8) =$
$\sqrt{2^2 + (-5)^2} \cdot \sqrt{(-1)^2 + (-8)^2} \cos\theta$
So $\theta \approx 29°$.

54. The volume of a frustum is equal to the product of the Heronian mean of the bases and the height of the frustum.
$248\pi(17) = 4216\pi$

55. MacClaurin series for sine.
$\sin 8 \approx 0.9894$

56. The heights of triangles ABC, EBF and EFD are all equal. The bases of EBF and EFD are each $\frac{3}{8}$ of the base of ABC, so the area of the shaded region is
$2\left(\frac{3}{8}\right)108 = 81$