

**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 8 ©
JANUARY 21, 2017**

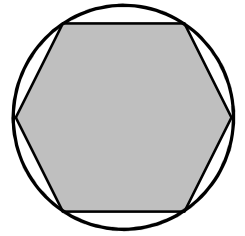
GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016-2017 TMSCA Mathematics Test Eight

- What is $0.58333... \div 0.125 + 1.875 - 0.41666... \times 1.6$
 - $\frac{47}{8}$
 - $-\frac{3}{8}$
 - $\frac{49}{8}$
 - 7
 - $\frac{109}{24}$
- Gemma started her weekend with \$150. She spent \$27.72 eating out on Friday night and \$11.75 on a movie Saturday. On Sunday, she bought 2 books for \$7 each, 1 DVD for \$22.50 and a coffee drink for \$4.75. If the tax on the books, DVD and coffee was 8.25%, how much money did Carla have left?
 - \$84.12
 - \$69.28
 - \$76.28
 - \$65.88
 - \$73.72
- $(x+3)(x-7) = (x-7)(x+3)$ and $9(2x+7) = (2x+7)9$ are examples of the _____ property of equality.
 - associative
 - commutative
 - addition
 - distributive
 - multiplication
- Travelling via I-45, the distance between Houston and Dallas is 238 miles. Kathalee drove from Houston to Dallas on I-45 at an average speed of 72 mph. Meanwhile, Bill left Houston and travelled through Waco then on to Dallas for a total trip distance of 273 miles at an average speed of 68 mph. What was the positive difference in their travel times? (nearest minute)
 - 71 min
 - 43 min
 - 18 min
 - 66 min
 - 38 min
- Six workers can paint a wall in 20 minutes. How long will it take four workers at the same individual rate to paint a wall twice as long and twice as high?
 - 80 min
 - 180 min
 - 120 min
 - 90 min
 - 110 min
- Which of the following is not a one-to-one function?
 - $y = 3x^5$
 - $y = 3x^4$
 - $\log(x-8)$
 - $y = e^{-2x}$
 - all are one to one
- If $L = \{l, i, z, a, r, d\}$, $M = \{m, u, s, c, r, a, t\}$ and $H = \{h, e, r, o, n\}$ then $(L \cup M) \cap (M \cup H)$ contains how many elements?
 - 6
 - 7
 - 5
 - 8
 - 9
- Given that $\angle P$ is supplementary to $\angle Q$; $m\angle R = 52^\circ$; and $\angle Q$ is complementary to $\angle R$, find $m\angle P$.
 - 142°
 - 138°
 - 42°
 - 38°
 - 134°
- If p and q are the zeros of the function $f(x) = 15x^2 - 49x - 204$ then $pq^2 + p^2q =$
 - $\frac{3332}{75}$
 - $\frac{3332}{225}$
 - $\frac{3060}{49}$
 - $-\frac{3332}{225}$
 - $-\frac{3332}{75}$
- Find the total volume of a right cone given the radius of the base is 9 ft. and the vertex angle is 35° . (nearest gallon)
 - 1629 gal
 - 6792 gal
 - 18112 gal
 - 4887 gal
 - 13034 gal

11. The regular hexagon in the illustration is inscribed in the circle. If a dart thrown at random strikes inside the circle, what are the odds that it will land in the shaded region? (nearest hundredth)



- (A) 0.21 (B) 4.78 (C) 0.83 (D) 1.21 (E) 2.14

12. If $\frac{5x-4}{x-2} + \frac{x-2}{3x-2} = \frac{ax^2+bx+c}{3x^2-8x+4}$ then $a+b+c$

- (A) -7 (B) -14 (C) 2 (D) -22 (E) 7

13. Which of the following prime numbers are considered to be both Mersenne and Germain primes?

- I. 3 II. 7 III. 13 IV. 17

- (A) I & III (B) II & III (C) II only (D) I only (E) none of these

14. How many distinct 4-letter arrangements can be made with the letters in “MCQUEENEY”?

- (A) 15,120 (B) 1044 (C) 990 (D) 864 (E) 840

15. The points $P(-2,11)$, $Q(6,k)$ and $R(-6,55)$ are collinear. Find the value of k .

- (A) 99 (B) -99 (C) -33 (D) -55 (E) -77

16. Let $f(x) = x^3 - 5$ and $g(x) = \sqrt[3]{-27x} - 1$. Calculate $f(g(-1))$.

- (A) 3 (B) 59 (C) -2 (D) -3 (E) 69

17. What is the constant term in the expansion of $\left(x^2 + \frac{3}{x}\right)^6$?

- (A) 405 (B) 540 (C) 1215 (D) 729 (E) 567

18. Determine the range of $f(\theta) = -3 + 5\cos\left(\frac{4\pi}{3}\theta - 2\pi\right)$.

- (A) $[7, -13]$ (B) $[-8, 2]$ (C) $[-2, 8]$ (D) $[-3, 7]$ (E) $[3, -7]$

19. The intersection of the three medians in a triangle is called the _____.

- (A) Incenter (B) Orthocenter (C) Center (D) Centroid (E) Circumcenter

20. Given that the binomial $x+2$ is a factor of $3x^4 + 35x^3 + 3ax^2 + 60x - a$, calculate the value of a .

- (A) 32 (B) -16 (C) -96 (D) -32 (E) 64

21. Simplify: $\tan\left(\frac{\pi}{2} - \theta\right)\sin\left(\frac{\pi}{2} - \theta\right)\cos\left(\frac{\pi}{2} - \theta\right)$

- (A) $\sin^2 \theta$ (B) $-\sin^2 \theta$ (C) $\cos^2 \theta$ (D) $-\csc^2 \theta$ (E) $\sin(2\theta)$

22. Using the following array, determine the sum of all the numbers in the 19th row.

1					(row 1)		
3	5				(row 2)		
7	9	11			(row 3)		
13	15	17	19			(row 4)	
21	23	25	27	29			(row 5)
...					(...)		

- (A) 6859 (B) 6878 (C) 6840 (D) 6175 (E) 6156

23. The ADA recommends that wheelchair ramps have no more than a 5° angle of elevation. Perry needs to build a ramp up to a porch that stands 0.72 meters off the ground. How long should Perry make the ramp itself? (nearest centimeter)

- (A) 213 cm (B) 823 cm (C) 75 cm (D) 826 cm (E) 518 cm

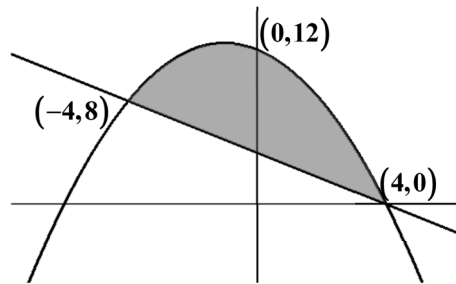
24. Given the arithmetic sequence 17, a, b, 27, c, ..., find $a + b + c$.

- (A) $\frac{217}{3}$ (B) 86 (C) $\frac{223}{3}$ (D) $\frac{209}{3}$ (E) $\frac{230}{3}$

25. Find the sum of all the x-values of the critical points of $f(x) = (x + 2)^5(x^2 - 1)^4$.

- (A) -2 (B) $-\frac{28}{13}$ (C) -3 (D) $-\frac{42}{13}$ (E) -4

26. Find volume of the solid generated when the shaded region bounded by the parabola and the line in the illustration is rotated 360° around the line $y = -2$. (nearest cubic unit)



- (A) 785 (B) 3271 (C) 1041 (D) 5147 (E) 2466

27. $(212121_3 + 121212_3) \times 2_3 = \underline{\hspace{2cm}}$,

- (A) 1440 (B) 1443 (C) 2222220 (D) 2886 (E) 1880

28. $[(2 + 4 + 6 + 8 + \dots + 36 + 38) \times 40] \div [(42 + 44 + 46 + 48 + \dots + 76 + 78) \times 80] = ?$

- (A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{4}$

29. If an integral factor of 132, not including 1 or 132 is chosen at random, what are the odds that it is a multiple of 3?

- (A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{4}$ (D) 2 (E) $\frac{4}{3}$

30. Find $\lim_{x \rightarrow -\infty} \frac{9 + 6x^2 - 11x^3}{2x^4 - 7}$

- (A) 0 (B) $-\frac{11}{7}$ (C) $\frac{11}{2}$ (D) $-\frac{11}{2}$ (E) does not exist

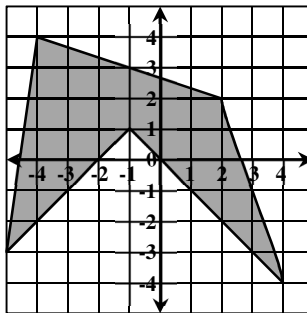
31. If $f''(x) = 24x - 6$ and $f(1) = -48$ and $f(2) = -80$, then $f(-1) =$ _____.

- (A) 78 (B) 55 (C) -51 (D) -48 (E) 46

32. Ellipse $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$ has foci (x_1, y_1) and (x_2, y_2) . Find the value of $y_1 + y_2$.

- (A) -2 (B) 2 (C) -1 (D) 1 (E) 0

33. The coordinates of the vertices of the pentagon shown are all integers. What is the area of the pentagon?



- (A) 46.5 units² (B) 27 units² (C) 34.5 units² (D) 28 units² (E) 33.5 units²

34. Simplify: $(a^2 \div b^4)^{-3} \div a^8 \times b^5$.

- (A) $\frac{b^{14}}{a^{13}}$ (B) $\frac{b^7}{a^{14}}$ (C) $\frac{1}{a^7 b^{14}}$ (D) $\frac{b^{14}}{a}$ (E) $\frac{b^{17}}{a^{14}}$

35. Calculate the sample standard deviation of the set of numbers {12, 16, 28, 32, 38}. (nearest hundredth)

- (A) 9.77 (B) 10.35 (C) 9.26 (D) 9.52 (E) 10.92

36. Two years from now, Zack's age will be triple Xerxes age? A year ago, the sum of their ages was 22. How old is Xerxes now?

- (A) 8 (B) 6 (C) 16 (D) 5 (E) 19

37. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$ be the terms of the Fibonacci sequence. Find f_{36} .

- (A) 9,227,465 (B) 14,930,352 (C) 5,702,887 (D) 24,157,817 (E) 3,524,578

38. In how many distinct ways can a group of nine diners be seated at a round table?

- (A) 40,320 (B) 362,880 (C) 181,440 (D) 20,160 (E) 17,280

39. Which of the following functions expresses the area, A , of an equilateral triangle in terms of the length of the apothem, a ?

- (A) $A = \frac{3a^2\sqrt{3}}{2}$ (B) $A = 3a^2\sqrt{3}$ (C) $A = \frac{a^2\sqrt{3}}{4}$ (D) $A = \frac{3a^2\sqrt{3}}{4}$ (E) $A = \frac{a^2\sqrt{3}}{2}$

40. A fair tetrahedral die with sides numbered 1, 2, 3 and 4 is rolled and the number on the down side is recorded? What is the expected value of a single roll?

- (A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) $\frac{5}{4}$ (D) $\frac{7}{2}$ (E) $\frac{7}{4}$

41. If $\begin{bmatrix} 3 & -3 \\ a & 6 \end{bmatrix} \times \begin{bmatrix} 7 & b \\ 2 & -7 \end{bmatrix} = \begin{bmatrix} 15 & 30 \\ -2 & -48 \end{bmatrix}$ then $a + b = ?$

- (A) 2 (B) -3 (C) -1 (D) 3 (E) 1

42. The function $f(x) = \frac{2x^2 - 9x - 35}{4x^2 - 25}$ has a removable discontinuity at the point (a, b) . What is the value of b ?

- (A) -2.5 (B) 0 (C) -9.5 (D) 0.95 (E) -0.25

43. The number 567 in base 8 is equivalent to the number k in base 4. Find the sum of the digits in the number k .

- (A) 9 (B) 8 (C) 7 (D) 11 (E) 12

44. $(-2 - 3\sqrt{-20})(7\sqrt{-8})$

- (A) $84\sqrt{10} + 28\sqrt{2}i$ (B) $168\sqrt{5} - 28\sqrt{2}i$ (C) $-84\sqrt{10} + 28\sqrt{2}i$
 (D) $168\sqrt{5} - 28i\sqrt{2}$ (E) $84\sqrt{10} - 28\sqrt{2}i$

45. The lengths of the sides of triangle PQR are the roots of $f(x) = x^3 - 19x^2 + 108x - 162$. Find the area of triangle PQR. (nearest tenth)

- (A) 62.9 (B) 22.3 (C) 4.7 (D) 9.5 (E) 7.9

46. How many integral values of n exist such that $n \geq 1$ and $\frac{(n+4)!}{(n+1)!} \leq 250$

- (A) 1 (B) 4 (C) 3 (D) 5 (E) 8

47. Simplify: $(3\log_7 X - 3\log_7 Y) + (\log_7 Y^2 - 4\log_7 X^3)$.

- (A) $\log_7\left(\frac{1}{X^9}\right)$ (B) $\log_7\left(\frac{1}{X^9Y}\right)$ (C) $\log_7\left(\frac{Y}{X^9}\right)$ (D) $\log_7(X^9Y)$ (E) $\log_7(X^{12})$

48. If $f(x) = 3x^2 - 4x$, then $\lim_{h \rightarrow 0} \frac{f(6+h) - f(6-h)}{2h}$ is

- (A) 30 (B) 32 (C) 26 (D) 6 (E) undefined

49. If $\frac{x+9}{x-9} + \frac{x-9}{x+9} = 2 + \frac{B}{(x-9)(x+9)}$ where $B \in \mathbb{Z}^+$ then $B = ?$

- (A) 81 (B) 162 (C) 81 (D) 324 (E) 648

50. If $x - \frac{1}{x} = 21$, then $x^3 - \frac{1}{x^3} = ?$

- (A) 9303 (B) 9282 (C) 9324 (D) 9261 (E) 8860

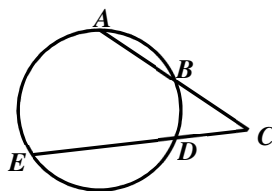
51. Dairy Joy Ice Cream Parlor has 3 types of cones and 8 flavors of ice cream. How many distinct 3-scoop cones could a customer order?

- (A) 360 (B) 1001 (C) 210 (D) 495 (E) 120

52. The repeating decimal 0.4333... in base 5 can be written as which of the following fractions in base 5 simplified form?

- (A) $\frac{32}{40_5}$ (B) $\frac{31}{20_5}$ (C) $\frac{4}{10_5}$ (D) $\frac{2}{10_5}$ (E) $\frac{34}{40_5}$

53. \overline{AC} and \overline{EC} are both secants of the circle shown. Find $m\widehat{BD}$ if $m\widehat{AE} = 131^\circ$ and $m\angle C = 35^\circ$.



- (A) 48° (B) 61° (C) 39° (D) 59° (E) 27°

54. Find the range, or ranges of values k can take for $kx^2 - 8x + 10 - k = 0$ to have two distinct rational solutions.

- (A) $(2, 8)$ (B) $(-\infty, -12) \cup (4, \infty)$ (C) $(-\infty, 2) \cup (8, \infty)$
 (D) $(-8, -2)$ (E) $(-\infty, -8) \cup (-2, \infty)$

55. Find $8 - \frac{8^3}{3!} + \frac{8^5}{5!} - \frac{8^7}{7!} \dots$ correct to 4 decimal places.

- (A) 195.7333 (B) -6.7997 (C) -0.1455 (D) 0.9894 (E) 0.1392

56. How many 3-digit numbers can be made with the digits 0, 0, 2, 4, 6 and 8?

- (A) 52 (B) 48 (C) 26 (D) 96 (E) 54

57. Find the shortest distance between the x -intercept of the line $3x + 7y = 21$ to the line $5x - 6y = -48$. (nearest tenth)

- (A) 4.6 (B) 10.6 (C) 7.7 (D) 4.5 (E) 8.1

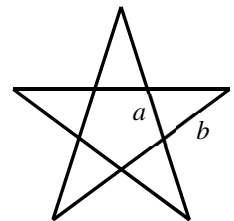
58. If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c , except possibly at c itself, and if

$\lim_{x \rightarrow c} h(x) = L = \lim_{x \rightarrow c} g(x)$ then $\lim_{x \rightarrow c} f(x)$ exists and is equal to L . This theorem is known as:

- (A) Sandwich Theorem (B) Rolle's Theorem (C) Fundamental Theorem of Calculus
 (D) Intermediate Value Theorem (E) Fundamental Theorem of Algebra

59. Given the pentagram shown, find a if $b = 8^\circ$. (nearest tenth)

- (A) 7.6 in (B) 4.2 in (C) 2.4 in (D) 6.2 in (E) 4.9 in



60. Let $f(x) = ax^4 + bx^2 + x - 8$ and $f(-6) = 27$. Calculate $f(6)$.

- (A) 33 (B) 41 (C) 39 (D) -43 (E) 47

Test Eight Answer Key

1. A	21. C	41. E
2. D	22. A	42. D
3. B	23. D	43. A
4. B	24. C	44. E
5. C	25. D	45. E
6. B	26. E	46. C
7. B	27. D	47. B
8. A	28. A	48. B
9. E	29. B	49. D
10. C	30. A	50. C
11. B	31. E	51. A
12. C	32. A	52. E
13. D	33. D	53. B
14. B	34. E	54. C
15. E	35. E	55. D
16. A	36. D	56. A
17. C	37. B	57. B
18. B	38. A	58. A
19. D	39. B	59. E
20. A	40. B	60. C

Test Eight Select Solutions

5. Each worker works at a rate of $1/120$ of the wall per minute. To paint a wall with four times the area, it will take for workers $\frac{4}{\frac{1}{120} + \frac{1}{120} + \frac{1}{120} + \frac{1}{120}} = 120$ minutes.

9. $pq^2 + p^2q = pq(q + p)$ which is the sum of the roots times the product of the roots or for a quadratic $\frac{c}{a}\left(-\frac{b}{a}\right) = \frac{-204}{15}\left(-\frac{-49}{15}\right) = -\frac{3332}{75}$.

14. There are three distinct groups of arrangements to count, no repeats, 2-E arrangements and 3-E arrangements: ${}_7P_4 + ({}_6C_2)\binom{4!}{2!} + ({}_6C_1)\binom{4!}{3!} = 1044$.

17. The constant term using binomial theorem is $({}_6C_2)(x^2)^2\left(\frac{3}{x}\right)^4 = 1215$.

20. Evaluate $f(-2) = 48 + (-280) + 12a - 120 - a = 0$ then $a = 32$.

21. $\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$, $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ and $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$, so $\tan\left(\frac{\pi}{2} - \theta\right)\sin\left(\frac{\pi}{2} - \theta\right)\cos\left(\frac{\pi}{2} - \theta\right) = \frac{\cos \theta}{\sin \theta}(\cos \theta)(\sin \theta) = \cos^2 \theta$.

22. The sum of the number in each row is the perfect cube of the row number for $19^3 = 6859$.

26. First, find the equations of both the parabola and line: $y_1 = -\frac{1}{2}x^2 - x + 12$ and $y_2 = -x + 4$, then use the washer method to set up the volume $\pi \int_{-4}^4 [(y_1 + 2)^2 - (y_2 + 2)^2] dx \approx 2466$

28. $\frac{\frac{19}{2}(2+38)(40)}{\frac{19}{2}(42+78)(80)} = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$

37. The nth term is $\frac{\phi^n}{\sqrt{5}}$ where $\phi = \frac{1+\sqrt{5}}{2}$ (golden ratio) or $f_{36} = \frac{\phi^{36}}{\sqrt{5}} = 14,930,352$.

38. Nine people in a row can be seated in $9!$ ways, but at a round table, rotations of the same order do not count as distinct arrangements so the total number of arrangements is $(n-1)! = 8! = 40,320$.

40. On a fair die, each outcome is equally likely, so the expected value of an individual roll is $1(0.25) + 2(0.25) + 3(0.25) + 4(0.25) = \frac{5}{2}$.

41. Use the definition of matrix multiplication for $3b + 21 = 30$ and $7a + 12 = -2$ then $b = 3$, $a = -2$ and $a + b = 1$.

45. Use Heron's formula where the sum of the roots is 19 and $s = 9.5$. The area is $\sqrt{9.5f(9.5)} \approx 7.9$.

48. This is the definition of the derivative of $f(x) = 3x^2 - 4x$ when $x = 6$ for $36 - 4 = 32$.

49. Use the number sense relationship $\frac{p}{q} + \frac{q}{p} = 2 + \frac{(p-q)^2}{pq}$ for $B = 18^2$ or $(-18)^2 = 324$.

50. $\left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2} = 441$ so $x^2 + \frac{1}{x^2} = 443$. Finally, $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = 21(444) = 9324$.

51. $3 \times ({}_{8+3-1}C_3) = 360$.

$10_5 n = 4.333..._5$
52. $\frac{n}{4n} = \frac{0.433..._5}{3.4_5}$ and $n = \frac{3.4}{4}_5 = \frac{34}{40}_5$. This cannot be simplified because in base 10 the equivalent fraction is $\frac{19}{20}$.

53. $\frac{131^\circ - m\widehat{BD}}{2} = 35^\circ$ for $m\widehat{BD} = 61^\circ$.

54. Use the discriminant $(-8)^2 - 4(k)(10-k) > 0$.

55. This is the Maclaurin Series expansion of $\sin 8 \approx 0.9894$.

57. Use the distance between a point and a line for $(7, 0)$ and $5x - 6y + 48 = 0$ for $d = \frac{|5(7) + 0(-6) + 48|}{\sqrt{25 + 36}} \approx 10.6$