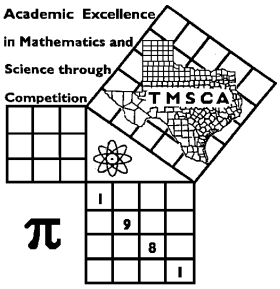

TMSCA HIGH SCHOOL MATHEMATICS TEST# 8 © JANUARY 23, 2016

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test 8

1. Evaluate: $1\div(1+4)\times5-\frac{3}{8}+17\times(18)^0$

A. $\frac{2453}{8}$ B. $\frac{333}{8}$ C. $\frac{141}{8}$ D. $\frac{237}{8}$ E. $\frac{673}{40}$

2. There are 27 students on a bus on their way to a math meet. Eighteen plan on competing in number sense, 14 plan on competing in math and 6 are not competing in either. How many students plan on competing in both number sense and math?

A. 4 B. 13 C. 5 D. 11 E. 14

3. Find the greatest common divisor of 306, 680 and 1190.

A. 17 B. 2 C. 19 D. 34 E. 65

4. Simplify: $\left(a^{-4}\div b^2\right)^{-2}\times a^3\div b^{-2}$

A. $a^{11}b^2$ B. $a^{11}b^{-2}$ C. $a^{-5}b^4$ D. $a^{11}b^6$ E. $a^{-5}b^6$

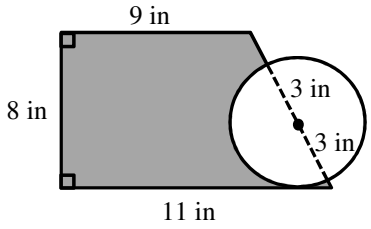
5. It takes 5 typists working 4 hours per day to type a manuscript in 18 days. How many days would it take 6 typists working 5 hours per day?

A. 15 B. 8 C. 9 D. 10 E. 12

6. Brandy walked from home to school at 4 mph and ran back at 10 mph. The entire trip took 32 minutes. How far is Brandy's house from the school? (nearest tenth)

A. 1.2 mi B. 2.1 mi C. 1.8 mi D. 1.5 mi E. 1.6 mi

7. Find the area of the shaded region. (nearest tenth)



A. 70.6 in^2 B. 61.2 in^2 C. 65.9 in^2 D. 46.8 in^2 E. 51.7 in^2

8. The intersection of the three altitudes of a triangle is called a(n)_____.

A. Center B. Orthocenter C. Centroid D. Incenter E. Circumcenter

9. If $\frac{2x-5}{x+2}-\frac{3x+4}{x-3}=\frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

A. 3.5 B. -3 C. 2.5 D. 2 E. -3.5

10. A transversal intersects two parallel lines such that the measure of one interior angle is $\left(x^2+3x\right)°$ and the measure of the opposite interior angle is $(11x+9)°$. What is the measure of the first angle?

A. 88° B. 108° C. 72° D. 90° E. 92°

11. The fundamental period of the graph of $y=2-5\sin^2(3x)$ is:

A. $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$ C. π D. $\frac{\pi}{2}$ E. $\frac{3\pi}{2}$

12. Given the arithmetic sequence $22,a,b,44.5,c,...$, find the value of $a+b+c$.

A. 144.75 B. 118.5 C. 131.25 D. 66.5 E. 89

13. A farmer has a cylindrical water tank sitting on the circular base. The radius of the tank is 3 feet and the height is 8 feet. If the tank is already one-third full, how much water must he add to fill the tank? (nearest gallon)

A. 1692 gal B. 141 gal C. 94 gal D. 1128 gal E. 539 gal

14. Given $\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 1 \\ 32 \end{bmatrix}$, find $m+n$.

A. -2 B. 12 C. -6 D. -9 E. -7

15. Find the mean value over the interval $[-2,8]$ of $f(x) = 2x^2 - 3x$.

A. $\frac{770}{3}$ B. 90 C. 9 D. $\frac{77}{3}$ E. $\frac{51}{5}$

16. $\left[(1+2+3+4+...+28+29)\times 30\right] \div \left[(31+32+33+34+...+58+59)\times 60\right] = ?$

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. 9 D. $\frac{1}{9}$ E. 6

17. Which of the following prime numbers are considered to be Germain Primes?

I. 5 II. 7 III. 23 IV. 29

A. I & II B. III & IV C. I, III & IV D. II, III & IV E. All of these

18. The repeating decimal $0.50505050...$ in base 6 can be written as which of the following fractions in base 6 in simplified form?

A. $\frac{6}{43_6}$ B. $\frac{10}{11_6}$ C. $\frac{10}{111_6}$ D. $\frac{5}{43_6}$ E. $\frac{5}{11_6}$

19. Which of the following is an equation of the perpendicular bisector of the line segment with endpoints $(-7,8)$ and $(3,-6)$.

A. $5x-7y=-17$ B. $5x-7y=-9$ C. $7x+5y=-9$ D. $7x+5y=9$ E. $5x-7y=3$

20. $(2-2i)^5 =$

A. $128-128i$ B. $-128-128i$ C. $-128i$ D. $128+128i$ E. $-128+128i$

21. The point (x, y) is a point of inflection on $f(x) = \frac{\sin x}{1+\cos x}$, where $0 < x \le 3\pi$. Find the value of x.

A. 0 B. π C. $\frac{\pi}{2}$ D. $\frac{\pi}{6}$ E. 2π

22. Mr. Data gives a ten question quiz to his class. When he is done grading, he gives the following frequency table to his class and offers extra credit to the first student to find the mean. What is the mean number of questions the students got right on the quiz? (nearest hundredth)

Questions Right	2	3	4	5	6	7	8	9	10
Number of Students	1	4	2	5	4	8	4	2	1

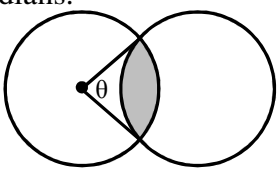
A. 6.00 B. 6.03 C. 5.97 D. 6.28 E. 6.09

23. The binomial $x-2$ is a factor of $6x^3 + Ax^2 + Ax + 30$. Find the value of A.

A. 1 B. -6 C. 11 D. -13 E. 9

24. The illustration shows two congruent circles each with a radius of 23 cm and $\theta = 1.3$ radians. Find the area of the shaded region. (nearest square centimeter)

A. 344 cm^2 B. 178 cm^2 C. 89 cm^2 D. 688 cm^2 E. 676 cm^2



25. If $f(x)=x^4$ and $g(x)=x-2$, then $g(f(2x-1))=$

A. $16x^4-32x^3+24x^2-8x-1$ C. $16x^4-4x^3+6x^2-4x-1$ E. $16x^4+4x^3+6x^2+4x-1$

B. $16x^4+32x^3+24x^2+8x+1$ D. $16x^4-32x^3+24x^2-8x+1$

26. What is the units digit of 2016^{2016}?

A. 2 B. 4 C. 0 D. 6 E. 8

27. James has a drawer full of unmatched socks containing 6 black, 15 white and 11 gray socks. If James reaches into the drawer to pull out 2 socks one at a time without replacement, what is the probability that he will pick two socks of the same color?

A. $\frac{105}{496}$ B. $\frac{175}{496}$ C. $\frac{175}{321}$ D. $\frac{105}{391}$ E. $\frac{10}{31}$

28. Which of the following is NOT a member of the solution set of $3|1-2x|+9<14$?

A. $\frac{7}{6}$ B. $-\frac{1}{5}$ C. $-\frac{1}{3}$ D. $\frac{4}{9}$ E. $\frac{12}{11}$

29. Given $\frac{A}{2x-5}+\frac{B}{x-3}=\frac{x-7}{2x^2-11x+15}$, calculate $A\times B$.

A. $\frac{7}{6}$ B. $-\frac{1}{5}$ C. $-\frac{1}{3}$ D. $\frac{4}{9}$ E. -36

30. Given that *A* and *B* are independent events, $p(A)=0.6$ and $p(A\cup B)=0.8$, calculate $p(A\cap B)$.

A. 0.5 B. 0.2 C. 0.4 D. 0.12 E. 0.3

31. Find $f(-2)+f(1)+f(3)$ if $f(x)=\begin{cases} x-8, & x\le 0 \\ -x^2, & 0<x\le 1.5 \\ \frac{3}{2}x & x>1.5 \end{cases}$.

A. -4.5 B. -7.5 C. -6.5 D. -5.5 E. -8.5

32. The center of the circle $x^2+y^2+6x-2y-6=0$ is _____units from the origin?

A. $\sqrt{10}$ B. 4 C. $2\sqrt{2}$ D. $\sqrt{6}$ E. 3

33. What is the coefficient of the x^2 term in the expansion of $(7x-5)(x-3)^5$?

A. 702 B. 1485 C. 4185 D. 1350 E. 675

34. A 96-ft rope is cut into *n* pieces of increasing lengths that form an arithmetic sequence. The shortest and longest pieces are 2.5 feet and 9.5 feet long respectively. What is the value of the common difference of the sequence?

A. $\frac{4}{5}$ ft B. $\frac{15}{32}$ ft C. $\frac{1}{4}$ ft D. $\frac{7}{15}$ ft E. $\frac{1}{2}$ ft

35. Which of the following is an equation of the normal line of the function $f(x)=\frac{1}{x^2+1}$ when $x=1$

A. $4x-2y=5$ B. $4x-2y=3$ C. $x+2y=2$ D. $x+2y=0$ E. $x-2y=2$

36. A regular pentagon of side length of 4 feet is inscribed in a circle. Find the area of the circle. (nearest tenth)

A. 13.9 ft^2 B. 19.2 ft^2 C. 131.6 ft^2 D. 23.8 ft^2 E. 36.4 ft^2

37. Evaluate: $\lim_{x\to 0}\dfrac{e^x-(1-x)}{x}$.

A. -1 B. 0 C. $\dfrac{1}{2}$ D. 2 E. Does not exist

38. How many distinguishable arrangements can be made using all of the letters in "CELEBRATE"?

A. 60480 B. 120960 C. 362880 D. 10080 E. 30240

39. The Key Club at a particular high school consists of 8 girls and 5 boys. The club has enough money to send a delegation of 2 girls and 2 boys to the state conference. In how many ways can the delegation be chosen?

A. 1120 B. 280 C. 715 D. 918 E. 160

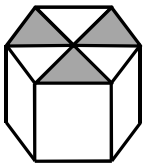
40. $\dfrac{1}{\csc\theta+\cot\theta}=$

A. $\cot\theta-\cos\theta$ B. $\csc\theta-\cot\theta$ C. $\sec\theta-\cot\theta$ D. $\cot\theta-\sin\theta$ E. $\cos\theta-\tan\theta$

41. A prankster removed the labels from all of the cans in Barry's pantry. He has four cans he knows are soup, 2 stews and 2 chowders. If Barry decides to open cans until he opens a chowder then stop, find $E(x)$, the number of cans Barry should expect to open to successfully open his first can of chowder.

A. $\dfrac{5}{3}$ B. $\dfrac{4}{3}$ C. 2 D. $\dfrac{3}{2}$ E. $\dfrac{7}{6}$

42. A candy company designed a box in the shape of a prism with regular hexagonal bases and square lateral faces as shown. If each edge is 3 cm. long, what is the volume of the box?



A. $\dfrac{27\sqrt{3}}{2}$ cm^3 B. $\dfrac{81\sqrt{3}}{2}$ cm^3 C. $27\sqrt{3}$ cm^3 D. $54\sqrt{3}$ cm^3 E. $\dfrac{27\sqrt{3}}{4}$ cm^3

43. Given $a_1=-1$, $a_2=\dfrac{3}{2}$ and $a_n=\dfrac{a_{n-2}}{a_{n-1}}$ find the value a_6.

A. $\dfrac{81}{16}$ B. $\dfrac{243}{32}$ C. $\dfrac{27}{8}$ D. $-\dfrac{243}{32}$ E. $\dfrac{8}{27}$

44. Let $f(x)=\dfrac{6x^3+5x-8}{2x^2-7}$ and $s(x)$ be the slant asymptote of $f(x)$. Calculate $s(-2)$.

A. -6 B. -4 C. 4.5 D. 6 E. -4.5

45. On the triangle ABC, $m\angle A=35^\circ$, $BC=4$ cm and $AC=6.5$ cm. Find the sum of the two possible values of AB. (nearest hundredth)

A. 14.42 B. 10.25 C. 7.5 D. 10.65 E. 13.55

46. What is the equation of the directrix of a parabola with the equation $x^2+24y-8x=-16$?

A. $x=4$ B. $y=-6$ C. $y=-4$ D. $x=-6$ E. $y=6$

47. Given $f''(x)=18x-10$, $f'(2)=24$ and $f(2)=19$, calculate $f(1)$.

A. -2 B. 6 C. 7 D. 5 E. -1

48. What is the smallest angle formed by the hour and minute hand of a clock at 2:20 pm?

A. 50° B. 45° C. 60° D. 55° E. 65°

49. Let $a=\log x$, $b=\log y$ and $c=\log z$. What is $\log\left(\frac{x^2\sqrt{y^3}}{z^3}\right)$ in terms of *a, b* and *c*?

A. $\frac{3ab}{2}-3c$ B. $2a+\frac{3}{2}b-3c$ C. $\frac{a^2\sqrt{b^3}}{c^3}$ D. $-\frac{3ab}{2c}$ E. $a^2+\sqrt{b^3}-c^3$

50. The line $y=20x-7$ is tangent to the curve $y=5x^3+ax^2+bx+11$ at the point (1, 13). Find the value $a+b$.

A. -7 B. -11 C. -3 D. 7 E. 3

51. The operation € is defined as A€B = A^3+B^3. Compute 3€(1€2).

A. 27 B. 1008 C. 756 D. 42876 E. 36

52. The function f is such that $\int_{-1}^{3} f(x)\,dx=7$. What is the value of $\int_{3}^{-1}(2f(x)+3)\,dx$?

A. 17 B. -26 C. 10 D. -17 E. 26

53. The equation $3x^2-5x+k=0$ always has two positive roots when which of the following is true?

A. $-\frac{25}{12}<x<\frac{25}{12}$ B. $0<k<\frac{25}{12}$ C. $-\frac{25}{12}<x<0$ D. $-\frac{12}{25}<x<0$ E. $-\frac{12}{25}<x<\frac{12}{25}$

54. Find the area in square units of a scalene triangle whose vertices are $(-2,-1)$, $(2,12)$ and $(3,3)$.

A. 27.5 B. 30.5 C. 22.5 D. 25.5 E. 24.5

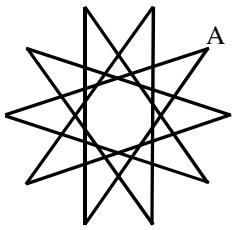
55. Let *a*, *b* and *c* be real numbers such that $c=a+b+16$, $c^2=a^2+b^2$ and $ab=10$. Find the numeric value of $16c$.

A. 118 B. 236 C. 123 D. 246 E. 126

56. $654_7+543_6+432_5=$ _____$_{10}$.

A. 681 B. 1629 C. 657 D. 682 E. 412

57. All ten points on the star shown are congruent. What is the measure of angle A?



A. 45° B. 30° C. 32° D. 36° E. 40°

58. $\ln(x)^2+\ln(x^3)^2+\ln(x^5)^2+.....+\ln(x^{29})^2=$

A. $(\ln x)^{450}$ B. $(\ln x)^{255}$ C. $255(\ln x)$ D. $\ln x^{420}$ E. $\ln x^{450}$

59. If $y=\frac{9}{x}$ and $x+y=12$, then $x^3+y^3=$

A. 1404 B. 1701 C. 1512 D. 1620 E. 1728

60. $\sum_{k=2}^{\infty}\frac{2}{3}\left(-\frac{3}{4}\right)^k=$

A. $\frac{8}{21}$ B. $-\frac{2}{7}$ C. $\frac{8}{3}$ D. $\frac{3}{14}$ E. $\frac{3}{2}$

2015-2016 TMSCA Mathematics Test Eight Answers

1. C	21. E	41. A
2. D	22. B	42. B
3. D	23. D	43. D
4. D	24. B	44. A
5. E	25. A	45. D
6. D	26. D	46. E
7. C	27. B	47. D
8. B	28. C	48. A
9. C	29. E	49. B
10. B	30. E	50. C
11. A	31. C	51. C
12. B	32. A	52. B
13. D	33. C	53. B
14. A	34. D	54. E
15. D	35. B	55. A
16. A	36. E	56. C
17. C	37. D	57. D
18. B	38. A	58. E
19. A	39. B	59. A
20. E	40. B	60. D

2015-2016 TMSCA Mathematics Test Eight Select Solutions

5. (5 typist)(72 hours)(hourly rate of one typist) = 1 job, so (hourly rate) = 1/360. To find the days , (6 typists)(5 hours per day)(1/360)(number of days) = 1 solve for 12 days.

6. Let *t* be the time it took to walk to school. Then $4t = 10\left(\frac{32}{60} - t\right)$ for $t \approx 0.38$ hours and the distance to school will be $4(0.38) \approx 1.5$ miles

16. $\frac{\frac{1}{2}(29)(30)(30)}{\frac{1}{2}(29)(90)(60)} = \frac{1}{6}$

18. $\begin{array}{rcl} 100n_6 & = & 50.\overline{50}_6 \\ \hline 55n_6 & = & 50_6 \end{array}$, so $\frac{50}{55_6} = \frac{30}{35_{10}} = \frac{6}{7_{10}} = \frac{10}{11_6}$

24. Half of the shaded region can be found by using $A = \frac{r^2\theta}{2}$, so the whole shaded region can be found using $r^2\theta = 23^2(1.3) \approx 688$.

29. $A(x-3) + B(2x-5) = x-7$. Let $x = 3$ for $B = -4$ and $x = \frac{5}{2}$ for $A = 9$ and a product of -36.

30. Use $0.8 = 0.6 + p(B) - 0.6p(B)$ for $p(B) = 0.5$ and $p(A \cap B) = (0.6)(0.5) = 0.3$.

32. Complete the squares for $(x+3)^2 + (y-1)^2 = 14$ which is a circle with a center at $(-3,1)$ and a distance from the origin of $\sqrt{(-3)^2 + 1^2} = \sqrt{10}$.

33. There will be two terms of the binomial expansion that will have an exponent of 2 when they are multiplied by terms in $(7x-5)$.

$7x\left(5(x)(-3)^4\right) + (-5)\left(10(x)^2(-3)^3\right) = 4185$

34. Use the sum of the sequence $96 = \frac{n}{2}(12)$ for $n = 16$, then $9.5 = 2.5 + (16-1)d$ and $d = \frac{7}{15}$.

35. Use L'Hopital's rule and evaluate $\lim_{x \to 0} \frac{e^x + 1}{1} = 2$.

41. $E(x) = 1\left(\frac{2}{4}\right) + 2\left(\frac{2}{4} \times \frac{2}{3}\right) + 3\left(\frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}\right) = \frac{5}{3}$

45.Solve the equation $13 = 5 + a + b + 11$ for $a + b = -3$.

55. $(c-16)^2 = (a+b)^2$, so $c^2 - 32c + 256 = a^2 + 2ab + b^2$ and $-32c + 256 = 2ab$ or $-32c + 256 = 20$ and $16c = 118$.

58. This can be rewritten as $2\ln x + 6\ln x + 10\ln x + ... + 58\ln x$, the coefficients form an arithmetic sequence with a sum of 450, so the sum of the sequence is $\ln x^{450}$.

59. $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right) = 12\left(144 - 3(9)\right) = 1404$