T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 8 ©

J A N U A R Y 2 4 , 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. $\sqrt[3]{\frac{1}{5}+\left(-\frac{1}{5}\right)^2-\frac{3}{125}}=$

A) $-\frac{1}{5}$ B) $\frac{3}{5}$ C) $\frac{\sqrt[3]{17}}{5}$ D) $\frac{2\sqrt[3]{4}}{5}$ E) $\frac{8\sqrt[3]{4}}{5}$

2. In a class of 102 students, there are 45 students studying French, 50 studying Spanish and 22 studying neither language. If a French student is chosen at random, what is the probability he/she will not be taking Spanish?

A) $\frac{1}{3}$ B) 1 C) $\frac{2}{3}$ D) $\frac{5}{17}$ E) $\frac{7}{10}$

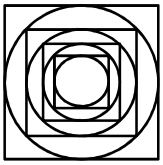
3. Clarence has 30% and 80% salt solutions. How much of the 30% solution should he use if he wants to obtain 2 liters of 50% solution? (nearest mL)

A) 1000 mL B) 933 mL C) 1067 mL D) 800 mL E) 1200 mL

4. $\overleftrightarrow{CM}$ is the perpendicular bisector of $\overline{AB}$, and M is the midpoint of $\overline{AB}$. The points have coordinates $A(12,a)$, $B(18,11)$, $M(15,14)$ and $C(c,24)$. What is the value of $a+c$?

A) 42 B) 25 C) 23 D) 40 E) 17

5. A circle is inscribed in a square, then a square is inscribed in the circle, then a circle is inscribed in the smaller square followed by a circle inscribed in the smaller square and so on. If this pattern is continued infinitely and one side of the largest square is 4 inches, what is the sum of the areas of all of the squares?



A) $\frac{64}{3}$ in^2 B) 32 in^2 C) 16 in^2 D) 24 in^2 E) $\frac{32}{3}$ in^2

6. A red die and a green die are both rolled and the top numbers on each are recorded. Given that the number on the red die is even, what is the probability that the sum of the dice will be prime?

A) $\frac{7}{18}$ B) $\frac{1}{2}$ C) $\frac{2}{9}$ D) $\frac{5}{18}$ E) 0

7. What is the amplitude of the graph of the function $f(x)=\frac{2}{3}\cos\left(3\left(\theta-\frac{\pi}{2}\right)\right)$?

A) $\frac{4}{3}$ B) 3 C) $\frac{\pi}{2}$ D) π E) $\frac{2}{3}$

8. Find the range of the mean, median and mode of 3.75, 3.5, 4.25, 2.75, 3.5, 5.25, 4, 5.

A) 2.25 B) 4 C) 2.75 D) 0.5 E) 3.5

9. Charlie can dig a cellar in 15 hours and Barry can dig a cellar the same size in 18 hours. How long would it take them to dig a cellar that is twice as long and twice as deep working together? (nearest minute)

A) 16 hr. 36 min. B) 16 hr. 22 min. C) 32 hr. 22 min. D) 24 hr. 36 min. E) 32 hr. 44 min.

10. A collection of 9 books including 4 books by Dickens are arranged on a shelf. How many different arrangements are possible if the books by Dickens are shelved together?

A) 2880 B) 1451520 C) 17280 D) 362880 E) 90720

11. What is the length of the longest straight rod that will fit completely in an open box that is a cube with side lengths of 3 ft?

A) $3\sqrt{2}$ ft. B) 9 ft. C) 6 ft. D) $3\sqrt{3}$ ft. E) $3\sqrt{6}$ ft.

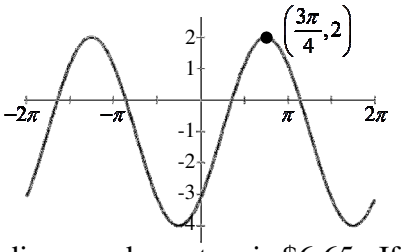
12. Find the base a such that $386_a = 272_b$ and $146_a = 102_b$.

A) 7 B) 9 C) 8 D) 10 E) 11

13. Find the value of c for which the roots of $6x^2 - 25x + c = 0$ are in a ratio of 2:3.

A) 25 B) 36 C) 30 D) 45 E) 18

14. The graph shows $f(x) = a\sin(x+b)+c$. $f(x) =$



A) $2\sin\left(x+\frac{\pi}{4}\right)-1$ B) $3\sin\left(x+\frac{3\pi}{4}\right)-1$ C) $-2\sin\left(x+\frac{3\pi}{4}\right)-1$ D) $-3\sin\left(x+\frac{3\pi}{4}\right)-1$ E) $-3\sin\left(x+\frac{\pi}{4}\right)-1$

15. The total value of the money in a jar containing 42 coins made up of nickels, dimes and quarters is \$6.65. If there are three times as many quarters as there are dimes, how many dimes are in the jar?

A) 6 B) 18 C) 7 D) 21 E) 8

16. If $a_0 = 4$, $a_1 = 6$ and $a_n = (a_{n-2})^2 - 3a_{n-1}$, then $a_5 =$

A) -122 B) 1398 C) 8494 D) 642 E) 2130

17. Find C if the remainder of $7x^5 - 9x^4 - 11x^3 + 17x + C$ divided by $x-2$ is 17.

A) -297 B) -185 C) -9 D) 331 E) 59

18. What is the sum of all the numbers in the 18th row of Pascal's triangle?

A) 131072 B) 524288 C) 65536 D) 1048576 E) 262144

19. $\csc\theta - \sin\theta =$

A) $\tan\theta\cot\theta$ B) $\tan\theta\cos\theta$ C) $\cot\theta\csc\theta$ D) $\cot\theta\cos\theta$ E) $\cot\theta\sin\theta$

20. There are two values of k for which $\det\begin{pmatrix} k & 1 \\ 3 & k+7 \end{pmatrix} = 95$. What is smaller value of k?

A) 14 B) -7 C) -14 D) 21 E) 7

21. What is the coefficient of the quadratic term of the derivative of $f(x) = 9x^4 - 3x^3 + 5x^2 + 6x - 9$?

A) 36 B) 3 C) -9 D) 5 E) 6

22. What is the area of the region enclosed by the graphs of $f(x) = 2x^2 + x - 21$ and $g(x) = 15x - 33$?

A) $\frac{125}{3}$ B) 36 C) $\frac{17}{3}$ D) $\frac{230}{3}$ E) $\frac{505}{3}$

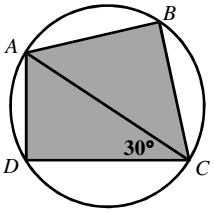
23. A satellite has ten solar power cells. There is a 60% that a single cell will fail in the first five years of operation. If the satellite needs at least two functional cells to continue operating, what is the probability that the satellite will be operational at the end of five years? (nearest thousandth)

A) 0.121 B) 0.984 C) 0.954 D) 0.994 E) 0.833

24. Simplify: $a^3b^3 \div a^{-3}b^2 \times a^5 \div (a^7b^7)$.

A) $\frac{a^4}{b^2}$ B) $\frac{a^4}{b^6}$ C) $\frac{a}{b^2}$ D) $\frac{a}{b^6}$ E) $\frac{1}{a^6b^8}$

25. The illustration shown is a quadrilateral inscribed in a circle. If $\overline{AC}$ is a diameter and $\overline{AB} \simeq \overline{BC}$, what is the probability that a dart landing randomly in the circle would land in the shaded region?



A) $\frac{2+\sqrt{2}}{\pi}$ B) $\frac{1+\sqrt{3}}{2\pi}$ C) $\frac{1+\sqrt{3}}{\pi}$ D) $\frac{1+\sqrt{2}}{2\pi}$ E) $\frac{2+\sqrt{3}}{2\pi}$

26. $\left(2+\frac{1}{x^3}\right)\div\left(\frac{1}{x^2}-2\right)=$

A) $\frac{x^3+2}{x-2x^3}$ B) $\frac{2x^3+1}{1-2x^3}$ C) $\frac{2x^3+1}{x-2x^3}$ D) $\frac{2x^3+1}{2x-x^3}$ E) $\frac{x^3+2}{2x-x^3}$

27. The quantity x varies inversely with y^2. If $y=-3$ when $x=6$, what is the value of x when $y=6$?

A) -3 B) $-\frac{1}{2}$ C) 24 D) $\frac{3}{2}$ E) 18

28. Given $\sin\theta=-\frac{\sqrt{3}}{2}$ and $\frac{\pi}{2}\le\theta\le\frac{3\pi}{2}$, calculate $\cos 2\theta$.

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) 1 E) $-\frac{\sqrt{3}}{2}$

29. The operation ∂ is defined so that $a\partial b=\frac{a^2+b^2}{a+b}$. Evaluate $2\partial(-1\partial 3)$.

A) $\frac{29}{7}$ B) $\frac{10}{3}$ C) 4 D) 1 E) $\frac{13}{5}$

30. Solve for x: $2xy+3x-5=y-x$

A) $\frac{5-y}{6}$ B) $\frac{y-5}{2y+2}$ C) $\frac{y+5}{5y+1}$ D) $\frac{y+5}{2y+4}$ E) $\frac{5}{y+4}$

31. The matrix multiplication $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ results in a 90° counter-clockwise rotation of the point (x,y) around the origin. $\begin{pmatrix}a & b\\ c & d\end{pmatrix}=$

A) $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ B) $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ C) $\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ D) $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ E) $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

32. What is the digit in the tens place in the sum: $(1!)+(2!)+(4!)+(8!).....+(256!)$?

A) 1 B) 2 C) 3 D) 4 E) 0

33. The table below shows the yearly return on a \$10,000 investment each year for 5 years. What is the average apr for the 5-year period? (nearest tenth of a percent)

Year	1	2	3	4	5
Return	+5.5%	+7.3%	-11.2%	+2.8%	+3.7%

A) 7.16% B) 1.6% C) 1.4% D) 8.1% E) 2.1%

34. $6\frac{1}{4}\%$ of $\left(\frac{1}{6}\div 0.41666...\right)=$

A) 0.025 B) 2.5 C) 0.43403 D) 0.0234375 E) 0.046296

35. $223_6+425_6+205_6=$ _____$_6$

A) 1253 B) 1301 C) 3101 D) 1303 E) 1151

36. If $\frac{A}{x+5}+\frac{B}{x+2}=\frac{2x-28}{x^2+7x+10}$, then $A+B=$

A) 14 B) 8 C) 2 D) 6 E) 10

37. The total surface area of a regular tetrahedron is $108\sqrt{3}$ cm^2. What is the perimeter of one face of the tetrahedron?

A) 18 cm B) $6\sqrt{3}$ cm C) 9 cm D) $9\sqrt{3}$ cm E) $18\sqrt{3}$ cm

38. Which of the following is **not** a solution to $f(x)\geq\left|25-\sqrt{x^2}\right|$?

A) $(-3,22)$ B) $(-8,18)$ C) $(9,14)$ D) $(0,30)$ E) $(4,21)$

39. Given $f(x)=2x+5$ and $g(x)=x^3-1$, find $g(f(x))$.

A) $8x^3+10x^2+50x+125$ C) $8x^3+60x^2+150x+124$ E) $8x^3+124$

B) $8x^3+20x^2+50x+124$ D) $8x^3+125$

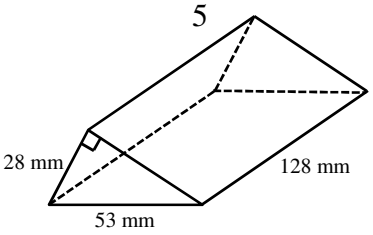
40. Determine the equation of the directrix of the parabola $x^2=-4y$.

A) $x=1$ B) $y=4$ C) $y=-1$ D) $y=1$ E) $y=-4$

41. Box A contains 5 pieces of paper numbered 1, 3, 5, 7 and 9. Box B contains 3 pieces of paper numbered 1, 4 and 9. One piece of paper is drawn at random from each box. What is the probability that the two numbers obtained will have a product divisible by 3?

A) $\frac{2}{15}$ B) $\frac{13}{15}$ C) $\frac{7}{15}$ D) $\frac{3}{5}$ E) $\frac{2}{5}$

42. The total surface area of the right triangular prism shown is _____mm^2.



A) 16870 B) 17388 C) 19096 D) 94976 E) 17612

43. Evaluate $\log_4 32+\log_9 27+\log_2 8$.

A) 7 B) $\frac{11}{2}$ C) $\frac{9}{2}$ D) 6 E) $\frac{27}{4}$

44. What is the length of the chord formed when the line $x-y=1$ intersects the circle $(x-2)^2+(y-2)^2=25$?

A) 14 B) $5\sqrt{2}$ C) 10 D) 7 E) $7\sqrt{2}$

45. If $(3-4i)-(2-5i)^2\times(7+3i)=a+bi$, then $a+b=$

A) 109 B) 304 C) 289 D) 64 E) 84

46. If $f(x)=Ax^4+Bx^2+3x-6$ and $f(4)=375$, then $f(-4)=$

A) 360 B) 342 C) 369 D) 351 E) 381

47. Find the number of positive integral divisors of 1440.

A) 28 B) 75 C) 32 D) 40 E) 36

48. Each of the twenty people in a conference room shakes hands with everyone else exactly once. How many handshakes take place?

A) 400 B) 210 C) 153 D) 190 E) 381

49. Given a sequence with Fibonacci characteristics *a*, 9, *b*, 24, find the value of $a+b$.

A) 15 B) 18 C) 14 D) 6 E) 21

50. Given that $(x-2)$ and $(x+2)$ are factors of $f(x)=x^3+px^2+qx+4$, find the value of $p+q$.

A) -5 B) 3 C) 5 D) -3 E) 0

51. $\int_{-a}^{a}\left(\frac{2x^2+5}{3}\right)dx =$

A) $\frac{4a^3+30a}{3}$ B) $\frac{4a^3+30a}{9}$ C) 0 D) $\frac{4a^3+10}{9}$ E) $\frac{4a^3+10a}{3}$

52. What is the area of the largest isosceles triangle that can be inscribed in a circle $x^2+y^2+4x+6y-3=0$?

A) $6\sqrt{3}$ B) 18 C) 9 D) $15\sqrt{3}$ E) $12\sqrt{3}$

53. Find the constant term in the expansion of $\left(2x^3-\frac{3}{x}\right)^8$.

A) 2916 B) 6561 C) 90720 D) 1296 E) 81648

54. How many distinct arrangements are there of three letters chosen from the word COMMON?

A) 48 B) 42 C) 180 D) 24 E) 56

55. If $\frac{x-7}{x+29}+\frac{x+29}{x-7}$ is equal to the mixed number $A\frac{B}{(x+29)(x-7)}$, then $B=$

A) 324 B) 484 C) 1296 D) 242 E) 406

56. A curve has equation $xy^3+2x^2y=3$. Find the slope of the tangent to this curve at the point (1, 1).

A) -5 B) -1 C) 2 D) 5 E) 1

57. The length of a rectangular picture is three times the width. The picture is surrounded by a frame which is 4 inches wide. If the perimeter of the outside of the frame is 80 inches, what is the length of the picture in inches?

A) 8 in. B) 10 in. C) 24 in. D) 18 in. E) 30 in.

58. What is the smallest angle formed by the hour and minute hand on the clock at 11:15?

A) 120° B) 112.5° C) 247.5° D) 240° E) 97.5°

59. What is the 10^{-9} digit in the sum $x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}-\frac{x^{14}}{7!}$..... when $x=9$?

A) 5 B) 3 C) 8 D) 4 E) 2

60. How many positive perfect cubes are factors of $(3!)(5!)(6!)$?

A) 5 B) 3 C) 6 D) 4 E) 7

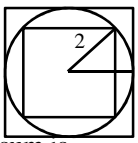
2014-2015 TMSCA Mathematics Test Eight Answers

1. B	21. C	41. D
2. C	22. A	42. B
3. E	23. C	43. A
4. A	24. A	44. E
5. B	25. E	45. C
6. A	26. C	46. D
7. E	27. D	47. E
8. D	28. B	48. D
9. E	29. A	49. E
10. C	30. D	50. A
11. D	31. A	51. B
12. B	32. D	52. E
13. A	33. C	53. E
14. D	34. A	54. B
15. C	35. B	55. C
16. E	36. C	56. B
17. C	37. E	57. D
18. E	38. C	58. B
19. D	39. C	59. D
20. C	40. D	60. C

2014-2015 TMSCA Mathematics Test Eight Select Solutions

3. $\dfrac{0.3x+0.8(2-x)}{2}=0.5$ so $x=1.2$

5. The radius of the circle (2) is also half the diagonal of the smaller square, so the area of the second square is $\frac{1}{2}d_1d_2=\frac{1}{2}(4)(4)=8$



This pattern continues as a geometric sequence, so the sum is $\dfrac{16}{1-\frac{1}{2}}=32$

10. Treat the 4 books as if they were a single object to get $6!=720$ possible arrangements of the 5 other books and the group of Dickens. Then multiply by 4! which is the number of possible arrangements of just the Dickens to get 17280.

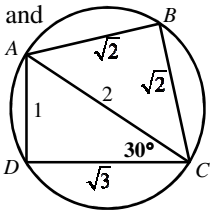
11. The length of the longest rod would be between opposite corners, which is the hypotenuse of the right triangle formed by a diagonal and side, so $3^2+\left(3\sqrt{2}\right)^2=27$, and $\sqrt{27}=3\sqrt{3}$

12. $3a^2+8a+6=2b^2+7b+2$, and $a^2+4a+6=b^2+2$. The second equation can be factored to solve for $a+2=b$. Substitute back into the first equation and $a=9$.

13. $2k+3k=\dfrac{25}{6}$ (sum of the roots), so the roots are $\frac{5}{3}$ and $\frac{5}{2}$, so $c=25$.

18. $2^{18}=262144$ because the sum of the numbers in each row of Pascal's triangle is 2^n.

25. Let the radius of the circle be 1, so the area of the circle is π. The areas of the triangles are 1 and $\frac{\sqrt{3}}{2}$, and $\dfrac{1+\frac{\sqrt{3}}{2}}{\pi}=\dfrac{2+\sqrt{3}}{\pi}$.



31. The matrix for a 90° counter-clockwise rotation is $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$, which for $\theta=90^\circ$, is $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$.

32. There will be no change in the tens place for any addition above 10!, so the ten's digit can be found with just the first four terms, 4.

36. Multiply both sides by the common denominator to obtain: $A(x+2)+B(x+5)=2x-28$, then $A+B=2$ as the coefficients of the x terms.

44. The line can be written as $x-y-1=0$, and the center of the circle is at $(2,2)$. The distance between the center and line is $\dfrac{|1(2)-1(-2)-1|}{\sqrt{1+1}}=\dfrac{1}{\sqrt{2}}$. Use this distance as one leg of a right triangle and 5 (radius of the circle) as the hypotenuse to find the other leg, $\frac{7\sqrt{2}}{2}$. The length of the chord is twice this or $7\sqrt{2}$.

46. Let $g(x)=Ax^4+Bx^2$, then $g(4)=369$, and $f(-4)=g(-4)-12-6=369-18=351$.

48. If $n=$ the number of people in the room, then the number of handshakes will be the $(n+1)^{th}$ triangular number or in this case: $\dfrac{19(20)}{2}=190$.

53. ${}_8C_2\left(2x^3\right)^2\left(\dfrac{-3}{x}\right)^6$ will be the term with the variables that divide to one, so all that will be left is $28(4)(729)=81648$.

54. The number of arrangements with O's repeating is (3 letter selections)(3 arrangements) = 9. Similarly, the number with M's repeating is 9. The number of arrangements with no letter repeating is 4•3•2, so the total is 42.

59. This is the expansion of $\sin x^2$, so $\sin 81\approx -0.6298879943$ and the 10^{-9} digit is 4.