T M S C A   H I G H   S C H O O L
M A T H E M A T I C S
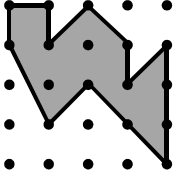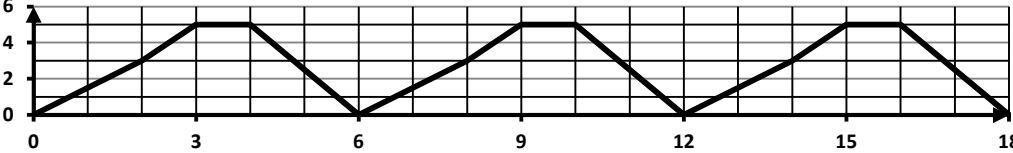# T E S T   # 8   ©
J A N U A R Y   2 5 , 2 0 1 4

GENERAL DIRECTIONS

1. About this test:
  A. You will be given 40 minutes to take this test.
  B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
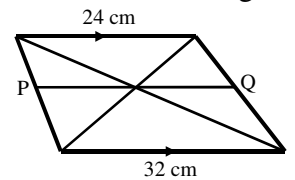9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $(0.525252...)^{-1} + (0.0171717...)^{-1} =$

A) $\dfrac{179}{330}$ B) $\dfrac{537}{1000}$ C) $\dfrac{52173}{364}$ D) $\dfrac{53163}{884}$ E) $\dfrac{13425}{221}$

2. Events A and B are independent such that $p(A)=0.32$ and $p(A\cup B)=0.66$. Calculate $p(A'\cap B)$.

A) 0.50 B) 0.68 C) 0.21 D) 0.34 E) 0.16

3. The full cost of a bunch of flowers is \$12.99. Lesley bought 5 bunches with a 60% sale discount. The sales clerk then applied a 15% student discount to Lesley's total bill. How much did Lesley pay for the flowers including sales tax of 8.25%?

A) \$22.08 B) \$33.12 C) \$17.58 D) \$23.90 E) \$35.86

4. The quality control worker at the peppermint factory selected a sample of eight bags. The bags contained 57, 64, 55, 68, 59, 57, 63, and 60 peppermints respectively. What is the sum of the mean, median and mode of this data set?

A) 174.625 B) 176.875 C) 175.375 D) 58.968 E) 177.500

5. The coordinates of points P and Q are $(10,9)$ and $(-14,3)$ respectively. Which of the following is an equation of the perpendicular bisector of $\overline{PQ}$?

A) $x-4y=-26$ B) $4x+y=-2$ C) $4x+y=49$ D) $x-4y=-22$ E) $4x+y=-53$

6. 52 kilometers per hour = _____ centimeters per second. (nearest whole number)

A) 8667 B) 867 C) 144 D) 87 E) 1444

7. $\dfrac{4x^2+12x+9}{x^2-16} \div \dfrac{6x^2+19x+15}{3x^2+17x+20} =$

A) $2x+3$ B) $\dfrac{x+4}{2x+3}$ C) $\dfrac{2x+3}{x-4}$ D) $\dfrac{x-4}{2x+3}$ E) $\dfrac{1}{x-4}$

8. Mr. Harris finished teaching his history class at 3:07 pm. What was the smaller angle formed by the hour and minute hands of his clock at that time?

A) 42° B) 55° C) 48° D) 38.5° E) 51.5°

9. $\dfrac{3x+5}{(x+3)(x-1)} = \dfrac{A}{x+3} + \dfrac{B}{x-1}$. Calculate $A+B$.

A) 8 B) 5 C) 3 D) 1 E) 2

10. Given $f(x)=3x+2$, $g(x)=x^2-1$ and $h(x)=\dfrac{x}{4}$ evaluate $h\big(g(f(3))\big)$.

A) 6.5 B) 40 C) 13 D) 30 E) 26

11. Anthony's test grades vary directly with the amount of time he studies. He studied for 5 hours for the last test and earned a 75%. How long should he plan to study if he wants to make a grade of 95% on his next test?

A) 360 minutes B) 365 minutes C) 340 minutes D) 390 minutes E) 380 minutes

12. If $4\times(9+3)=36+12$ and $36+12=48$ then $4\times(9+3)=48$. This is an example of _____ property.

A) Transitive B) Associative C) Substitution D) Reflexive E) Symmetric

13. Solve $\sin^2 x + \sin x \cos x = 0$ for $0 < x \le \pi$.

A) $0, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ B) $\dfrac{3\pi}{4}, \pi$ C) $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$ D) $\dfrac{\pi}{4}, \pi$ E) $\dfrac{2\pi}{3}, \pi$

14. A total of $12000 is invested in two bonds that pay 10.5% and 12% simple interest. The investor wants annual interest of $1400. What is the most that can be invested in the 10.5% bond?

A) $8667.00   B) $867.00   C) $2666.66   D) $9333.33   E) $1444.00

15. $\angle A$ and $\angle B$ are complementary and $m\angle B$ is 18° less than three times $m\angle A$. Find the measure of the supplement of $\angle A$.

A) 153°   B) 27°   C) 135°   D) 117°   E) 63°

16. A belt joins two pulleys. The smaller with a radius of 24 inches is rotating at 45 rpm. The larger has a radius of 30 inches and rotates at _____rmp.

A) 36   B) 30   C) 48   D) 56   E) 42

17. A cylindrical tank with a radius of 4 ft. and depth of 8 ft. is 40% full of water. How many gallons are in the tank? (nearest gallon)

A) 1804   B) 3008   C) 2106   D) 1930   E) 1203

18. Given $y = x + 7$ and $xy = 22$, calculate $x^3 - y^3$.

A) -497   B) -805   C) -651   D) -189   E) -959

19. The dots are 5 units apart vertically and horizontally. Find the area of the shaded region.

A) 200 units^2 B) 193.5 units^2 C) 181.5 units^2 D) 190.5 units^2 E) 187.5 units^2

20. Given the Fibonacci-type sequence: -5, a, b, 7, c, d, 23, calculate a + b + c.

A) 24   B) 15   C) 22   D) 14   E) 8

21. Evaluate: $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$.

A) 1   B) -1   C) ∞ D) $-\infty$ E) does not exist

22. The function $f(x)$ shown below is periodic and has a domain $0 \le x \le 40$. How many solutions are there to the equation $f(x) = 2$?

A) 6   B) 12   C) 13   D) 14   E) 9

23. Find the value of k for which the system of equations $\begin{matrix} 2x - ky = 2 \\ 3x + (k+1) = 4 \end{matrix}$ has no solutions.

A) $\dfrac{5}{2}$ B) $-\dfrac{2}{5}$ C) $-\dfrac{5}{2}$ D) $\dfrac{2}{5}$ E) 0

24. The graph of the equation $4x^2 + 3y^2 + 8x - 24y + 51 = 0$ is a(n) _____.

A) Parabola   B) Circle   C) Rational   D) Hyperbola   E) Ellipse

25. Find the constant term in the binomial expansion of $\left(x - \dfrac{2}{x^3}\right)^{12}$.

A) -13824   B) 220   C) 4096   D) -1760   E) 59136

26. Simplify: $(\cos x + \sin x)(\cos x - \sin x)$.

A) 1   B) $\cos 2x$ C) $\sin 2x$ D) $\tan 2x$ E) 0

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27. Find PQ.

24 cm

A) $\dfrac{192}{7}$ cm   B) 28 cm   C) $\dfrac{200}{7}$ cm   D) $16\sqrt{3}$ cm   E) $\dfrac{56+16\sqrt{3}}{3}$ cm

32 cm

28. Eight circles in a rectangle are congruent. Each circle is tangent to the adjacent circles and the outer six circles are tangent to the rectangle. A dart lands somewhere inside the rectangle. What is the probability it lands in a circle?

A) 77%   B) 67%   C) 70%   D) 65%   E) 72%
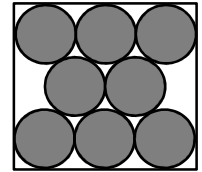
29. Determine the concavity of the graph of $f(x) = \sin x - \cos x$ at $x = \dfrac{3\pi}{4}$.

A) $-\dfrac{1}{2}$ B) $-\dfrac{\sqrt{3}+1}{2}$ C) $-\sqrt{2}$ D) $\dfrac{\sqrt{2}}{2}$ E) $\dfrac{1+\sqrt{3}}{2}$

30. P and Q are the zeros of $f(x) = 6x^2 - x - 35$. Calculate $P^4 - 4P^3Q + 6P^2Q^2 - 4PQ^3 + Q^4$.

A) $\dfrac{1}{1296}$ B) $\dfrac{707281}{1500625}$ C) $\dfrac{707281}{50625}$ D) $\dfrac{1}{50625}$ E) $\dfrac{707281}{1296}$

31. $1331_b = \underline{\hspace{1cm}}_{b+1}$, where $b > 4$.

A) 4000   B) 401   C) 1000   D) 1220   E) 1001

32. The real number solution set for $3|4 - 5x| \le 9$ is

A) $\left(-\infty, -\dfrac{1}{5}\right] \cup \left[\dfrac{7}{5}, \infty\right)$ B) $\left[-\dfrac{7}{5}, \dfrac{1}{5}\right]$ C) $\left(-\infty, -\dfrac{7}{5}\right] \cup \left[-\dfrac{1}{5}, \infty\right)$ D) $\left[\dfrac{1}{5}, \dfrac{7}{5}\right]$ E) $\left(-\infty, -\dfrac{7}{5}\right] \cup \left[-\dfrac{1}{5}, \infty\right)$

33. A 15-oz bag of ground coffee is made up of 30% decaf grounds. How many ounces of decaf grounds need to be added to create a 50% mixture?

A) 6   B) 7   C) 7.5   D) 5   E) 10.5

34. A standard 52-card deck of playing cards is thoroughly shuffled and 3 cards are dealt face up on a table. What are the odds that all three are spades?

A) $\dfrac{11}{839}$ B) $\dfrac{1}{64}$ C) $\dfrac{1}{63}$ D) $\dfrac{169}{10031}$ E) $\dfrac{11}{850}$

35. Given triangle ABC has an area of 82 cm^2, $AB = 20$ cm, $BC = 32$ cm and $m\angle CAB > 90°$, find $m\angle ABC$. (nearest degree)

A) 15°   B) 83°   C) 7°   D) 75°   E) 22°

36. $\log_a 7 = P$, $\log_a 3 = Q$ and $\log_a 2 = R$. Write $\log_a 36.75$ in terms of P, Q and R.

A) $\dfrac{2P+Q}{2R}$ B) $\dfrac{P^2+Q}{R^2}$ C) $P^2 + Q - R^2$ D) $-\dfrac{PQ}{R}$ E) $2P+Q-2R$

37. What is the angle between the vectors $\langle -8,17 \rangle$ and $\langle 11,5 \rangle$? (nearest degree)
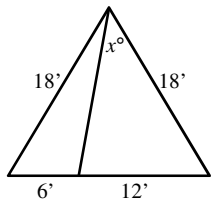
A) 84°   B) 89°   C) 91°   D) 132°   E) 96°

38. The graph of the polar equation $r = 7 + 7\sin\theta$ is a

A) Ellipse   B) Rose Curve   C) Cardioid   D) Lemniscate   E) Circle

39. What is the slope of the line normal to the graph of $2x^2 + y^2 = 34$ at $(3,4)$?

A) $-\dfrac{2}{3}$ B) $\dfrac{3}{2}$ C) $\dfrac{2}{3}$ D) $\dfrac{8}{3}$ E) $-\dfrac{3}{2}$

40. $\sin\theta<0$ and $\sec\theta<0$. The terminal ray for θ lies in _____.

A) QI   B) QII   C) QIII   D) QIV   E) indeterminate

41. A fair coin is tossed six times. What is the probability of at least four consecutive heads?

A) $\dfrac{15}{64}$ B) $\dfrac{11}{32}$ C) $\dfrac{1}{8}$ D) $\dfrac{11}{21}$ E) $\dfrac{3}{32}$

42. The coordinates of triangle PQR are $(-3,2)$, $(1,5)$ and $(1,-1)$. The coordinates of the incenter are (x,y).

Find y.

A) 2   B) 1.5   C) 1   D) 0.5   E) 0.25

43. How many solutions exist to the equation $7x+5y=116$ such that x and y are both non-negative integers?

A) 4   B) 3   C) 11   D) 5   E) 0

44. Three workers can plaster a wall in 5 hours.   How long will it take for 4 workers to plaster a wall that is the same length but twice as high?

A) 10 hours   B) 30 hours   C) 8.5 hours   D) 7.5 hours   E)   9   hours

45. Find the value of x in the triangle diagram. (nearest degree)

A) 40°   B) 39°   C) 42°   D) 43°   E) 41°

46. Determine the number of non-negative integer solutions to $p+q+r=10$.

A) 14   B) 66   C) 210   D) 231   E) 55

47. How many distinguishable arrangements can be made from the letters "PEPPERMINT"?

A) 302400   B) 907200   C) 3628800   D) 1209600   E) 10886400

48. The discriminate of $f(x)=(2x-10)(5x+3)$ is

A) 736   B) 4336   C) 1936   D) 3136   E) 2700

49. Find the sum of all two-digit numbers such that reversing the digits results in another two-digit number that

is $58\dfrac{2}{31}\%$ less than the original.

A) 204   B) 93   C) 186   D) 164   E) 154

50. $f(x)=1+x-\dfrac{x^2}{2}-\dfrac{x^3}{3!}+\dfrac{x^4}{4!}+\dfrac{x^5}{5!}-\dfrac{x^6}{6!}....$ Find the 10^{-8} place of $f(4)$.

A) 1   B) 6   C) 9   D) 5   E) 2

51. A line passes through the point $(2.5,-1.3)$ and has an x-intercept of 4. Which of the following is an equation of the line?

A) $53x+25y=100$ B) $25x-53y=-212$ C) $53x-25y=-100$ D) $25x+53y=-153$ E) $13x-15y=52$

52. Point P has polar coordinates $\left(11,\dfrac{7\pi}{6}\right)$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian plane?

A) QI   B) QII   C) QIII   D) QIV   E) x-axis

53. $f(x)=xe^{2x}$. Find $f'(x)$.

A) $e^{2x}(x+1)$ B) $2e^{2x}$ C) $e^{2x}(2x+1)$ D) $2xe^{2x-1}$ E) $2xe^{2x}$

54. Calculate the area of the region defined by $y < 2^{3x}$, $y > 0$, $x > 0$, and $x < a$.

A) $\dfrac{3(2^{3a} - 1)}{\ln 2}$ B) $\dfrac{2^{3a-1}}{3}$ C) $\dfrac{2^{3a} - 1}{3\ln 2}$ D) $\dfrac{2^{3a} - 1}{3}$ E) $\dfrac{2^{3a-1}}{3\ln 2}$

55. The table below shows the effective interest rates on an investment of $2700 for each of 5 years. What was the average annual interest rate for the 5-year period? (nearest hundredth)

Year	1	2	3	4	5
Percent	2.5%	-4%	3.7%	1.1%	-2%

A) 0.26%   B) 2.66%   C) 2.26%   D) 0.22%   E) 2.20%

56. If $\dfrac{x-7}{x+4} + \dfrac{x+4}{x-7}$ is equal to the mixed number $A + \dfrac{B}{(x+4)(x-7)}$, then $B =$

A) 64   B) 8   C) 121   D) 24   E) 9

57. The repeating decimal 0.363636… in base 7 can be written as which of the following fractions in base 7?

A) $\dfrac{6}{11_7}$ B) $\dfrac{36}{343_7}$ C) $\dfrac{12}{22_7}$ D) $\dfrac{12}{343_7}$ E) $\dfrac{6}{66_7}$

58. If $f(x) = ax^5 + bx^3 + cx + 9$ and $f(8) = 15$, find $f(-8)$.

A) -3   B) 3   C) 7   D) -7   E) 6

59. The areas of the bases of a frustum are A_1 and A_2 and the height is 15 inches. The Heronian mean of A_1 and A_2 is 311π in^2. Find the volume of the frustum.

A) 4665π in^3 B) 1555 in^3 C) $\dfrac{1555\pi}{3}$ in^3 D) 1555π in^3 E) $\dfrac{4665\pi}{2}$ in^3

60. On the circle with center F, EC = 7AE. The area of triangle ABE is 108 cm^2. Find the area of the shaded region.

A) 324 cm^2 B) 486 cm^2 C) 648 cm^2 D) 864 cm^2 E) 756 cm^2
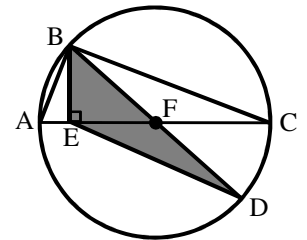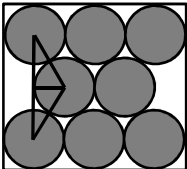
2013-2014 TMSCA Mathematics Test Eight Answers

1. D	21. A	41. C
2. D	22. C	42. A
3. D	23. B	43. B
4. B	24. E	44. D
5. B	25. D	45. E
6. E	26. B	46. B
7. C	27. A	47. A
8. E	28. A	48. D
9. C	29. C	49. C
10. D	30. B	50. A
11. E	31. C	51. E
12. A	32. D	52. C
13. B	33. A	53. C
14. C	34. A	54. C
15. A	35. A	55. D
16. A	36. E	56. C
17. E	37. C	57. C
18. B	38. C	58. B
19. A	39. C	59. A
20. B	40. C	60. C

2013-2014 TMSCA Mathematics Test Eight Select Solutions

2. $0.66 = 0.32 + p(B) - 0.32\, p(B)$, so

$p(B) = 0.5$ and $p(A') = 0.68$.

$p(A' \cap B) = 0.5(0.68) = 0.34$.

9. Multiplying both sides of the equation by the common denominator and dividing leaves the equation:

$3x + 5 = A(x-1) + B(x+3)$. Letting $x = 1$, yields $8 = 4B$, so $B = 2$. Letting $x = -3$, yields $-4 = -4A$, so $A = 1$.

$A + B = 3$.

19. $A = \dfrac{4+14}{2} - 1 = 8$ units^2 on the diagram. Since each linear unit on the diagram is 5, each square unit is scaled to 25. Area $= 8(25) = 200$ units^2.

21. $\displaystyle\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x} = 1 \bullet 1 = 1$

23. If the system has no solutions, then the determinate of the matrix

$\begin{pmatrix} 2 & -k \\ 3 & k+1 \end{pmatrix} = 0$. So, $2k + 2 + 3k = 0$.

$k = -\dfrac{2}{5}$

27. Harmonic mean of the bases.

$PQ = \dfrac{2(24)(32)}{(24+32)} = \dfrac{192}{7}$

28.

Let the radius of each circle be 1. The small triangles on the diagram are 30-60-90 with hypotenuse 2, so the long leg of each is

$\sqrt{3}$. This makes the height of the rectangle $2 + 2\sqrt{3}$, while the base is 6. So,

$p = \dfrac{8\pi}{6\left(2 + 2\sqrt{3}\right)} \approx 77\%$

30. $P^4 - 4P^3Q + 6P^2Q^2 - 4PQ^3 + Q^4$ is the binomial expansion of $(P-Q)^4$ or $(Q-P)^4$, which will be $\left(\dfrac{29}{6}\right)^4$ or $\left(-\dfrac{29}{4}\right)$ which both equal $\dfrac{707281}{1296}$.

42. The lengths of the sides are 5, 5, and 6. For the incenter,

$(x, y) = \left(\dfrac{ax_1 + bx_2 + cx_3}{perimeter}, \dfrac{ay_1 + by_2 + cy_3}{perimeter}\right)$,

where a, b and c are the lengths of the sides across from their coordinate multipliers. So

$y = \left(\dfrac{2(6) + 5(5) + (-1)(5)}{16}\right) = 2$

46. The number of non-negative solutions to the equation $p + q + r = n$ is the $(n+1)^{th}$ triangular number. So for $p + q + r = 10$, the number of solutions will be $\dfrac{11(12)}{2} = 66$.

49. $1 - \left(58\dfrac{2}{31}\%\right) = \dfrac{13}{31}$, so for a number AB, $\dfrac{13}{31}(10A + B) = 10B + A$. This simplifies to $A = 3B$. Original 2-digit numbers will be 31, 62 and 93 which have a sum of 186.

50. This is the MacClaurin series expansion of $f(x) = \sin x + \cos x$, so $f(4) \approx -1.410446116$ with a 1 in the 10^{-8} place.

54. $\displaystyle\int_0^a 2^{3x}\, dx = \left[\dfrac{2^{3x}}{3\ln 2}\right]_0^a = \dfrac{2^{3a} - 1}{3\ln 2}$

60. Triangles ABE, EBF and EFD all have the same height and EF = 3AE, so the area of the shaded region is $108(3)(2) = 648$ cm^2